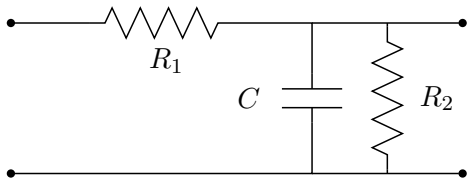
Name and ID Number: _____

Ask for guidance if you are confounded by a question.

1. (35 points) Kirchoff's Laws and the potential divider.



(a) Suppose that a constant (DC) potential $V_\circ$ is applied to the left side of this circuit. Use the concepts of conservation of energy and charge to derive the expression for the potential V_2 across R_2.

(b) Now consider the $R_2||C$ section and derive the expression for the total impedance of this section as function of frequency ω.

(c) Using this result, write the expression for the potential $V_2(\omega)$ across R_2 when $V(\omega) = V_\circ e^{i\omega t}$ is applied to the left side of the circuit.

(d) If $R_1 = 100\ \Omega$, $R_2 = 1000\ \Omega$ and $C = 1\mu$F, at what frequency will $|V_2| = V_\circ/2$?

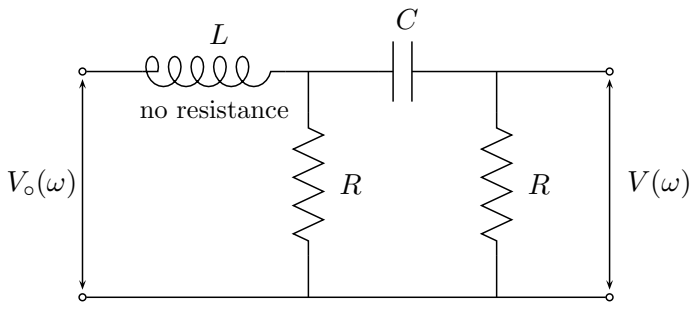
(e) At this frequency, how much power is dissipated in each element of the circuit when $V_\circ = 10$ V?

2. (20 points) Input and output impedance.

   (a) Explain how to measure the open-circuit potential and the output impedance of some signal source at a particular frequency. Draw a clear picture, and explictly write any equations you would use.

   (b) Explain how to measure the input impedance of an instrument at a particular frequency. Draw a clear picture, and explictly write any equations you would use.

3. (45 points) Analyze the following circuit in the frequency domain, assuming that the inductor is ideal.



(a) Consider the circuit to be two consecutive filters with transmission functions $A_1(\omega)$ and $A_2(\omega)$. Use physical reasoning and the concept of impedance to explain thoroughly the behavior of the each filter at low, intermediate and high frequencies. Clearly indicate where current is or is not flowing at low, intermediate and high frequencies.

(b) Derive the expressions for the transmission or response functions $A_1(\omega)$ and $A_2(\omega)$. Write the expression for $A_{total}(\omega)$ in terms of an amplitude $|A_{total}(\omega)|$ and a phase factor $e^{i\alpha}$.

(c) Sketch $|A_1(\omega)|$, $|A_2(\omega)|$ and $|A_{total}(\omega)|$in dB versus $\log(\omega)$. Sketch the phase of $A_{total}(\omega)$.

(d) Prove that the frequency at which the maximum in the transmission occurs is $\omega_\circ = 1/\sqrt{LC}$.

(e) What is the slope of $|A_{total}(\omega)|$ as $\omega \to \infty$ and as $\omega \to 0$?