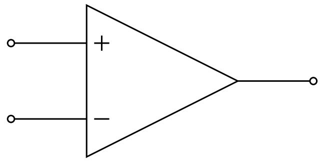
Name and ID Number: ____________________________________

Ask for guidance if you are confounded by these answers.

1. (25 points) The ideal op amp and the noninverting amplifier.

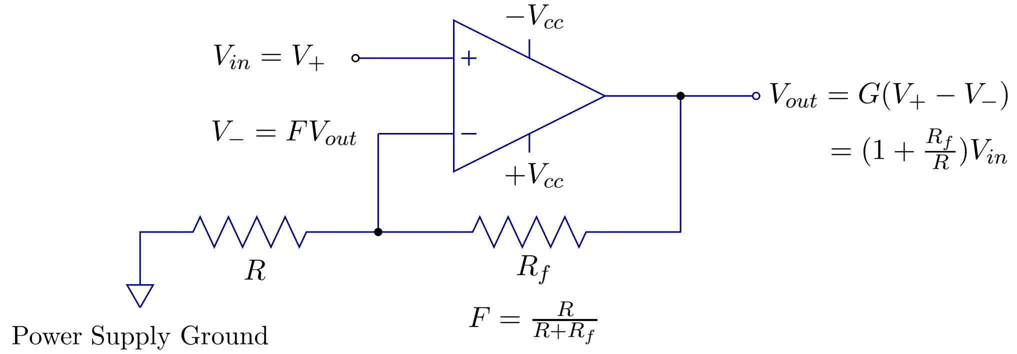


(a) What are the rules governing the behavior of an ideal op amp?
(3 points)

1. $G >>> 1$, so a finite $V_{out} = G(V_+ - V_-)$ implies $V_+ \simeq V_-$.
2. The input resistance of both inputs is infinite, or, for an even stronger statement, no current flows through either input.

(There are other minor ideal properties such as equal response at all frequencies and zero output resistance.)

(b) Draw the configuration of a noninverting op amp circuit with a feedback fraction F defined in terms of resistances.
(3 points)



(c) Using the op amp rules and conservation of energy and charge, derive the gain A of this noninverting amplifier when $GF >> 1$.
(3 points)

$$V_{out} = G(V_+ - V_-) = G(V_{in} - FV_{out}) \rightarrow V_{out}(1 + GF) = GV_{in}$$

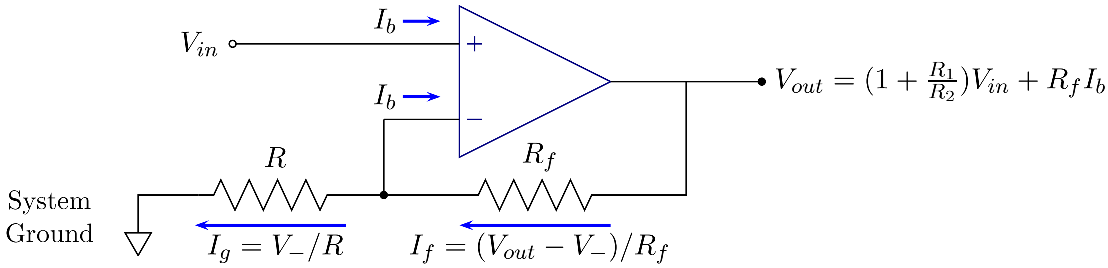
$$V_{out} = \frac{G}{1+GF}V_{in} \simeq \frac{V_{in}}{F} \text{ since } GF >> 1 \ .$$

(d) For a real op amp at a sufficiently high frequency, GF is not so large. Derive the first-order correction to the result of part (C).
(3 points)

$$\frac{G}{1+GF} = \frac{G}{GF(1+1/GF)} \simeq \frac{G}{GF}(1 - 1/GF) = \frac{1}{F} - \frac{1}{GF^2} \ .$$

(e) A real op amp also has input bias currents $I_{b+} = I_{b-} = I_b$. Assuming again that $GF >> 1$, use the op amp rules to determine how I_b appears in the output potential.

(3 points)



Begin with $V_{out} = G(V_+ - V_-) = G(V_{in} - V_-)$, so $V_{in} \simeq V_-$. Find the relation between V_- and V_{out} using $I_f = I_b + I_g$.

$$\frac{V_{out} - V_-}{R_f} = I_b + \frac{V_-}{R} \rightarrow \frac{V_{out}}{R_f} = V_- \left(\frac{1}{R_f} + \frac{1}{R}\right) + I_b$$

$$V_{out} = \left(1 + \frac{R_f}{R}\right) V_{in} + R_f I_b$$

(f) An internal input offset potential $V_o = V_+ - V_-$ is another reality. Assuming that $I_b = 0$ and $GF >> 1$, determine how V_o appears in the output potential.

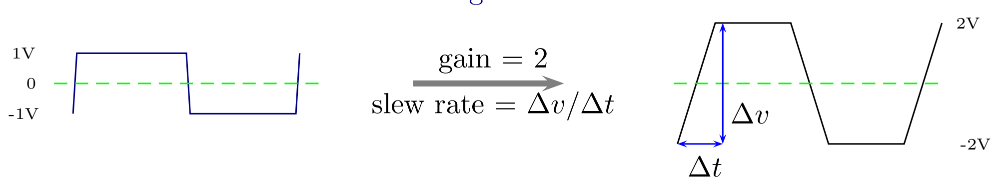
(3 points)

$$V_{out} = G(V_+ - V_-) = G(V_{in} + V_o - V_-) = G(V_{in} + V_o - FV_{out})$$

$$V_{out} = \frac{G}{1 + GF}(V_{in} + V_o) \simeq \frac{1}{F}(V_{in} + V_o) = \left(1 + \frac{R_f}{R}\right)(V_{in} + V_o)$$

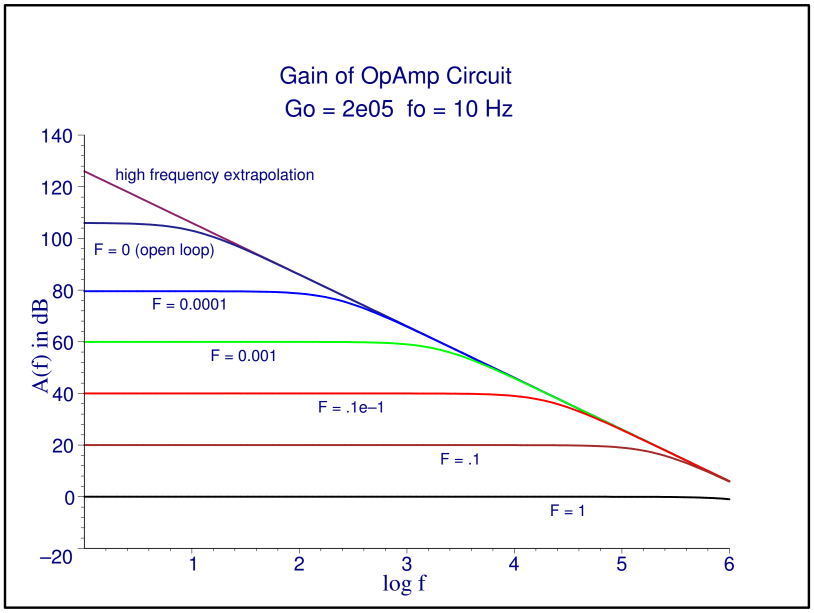
(g) Explain the slew rate of an op amp. How does it effect the output of an otherwise ideal op amp?

(3 points) Real op amps have a finite *slew rate*, that is the maximum rate at which the output potential can change. The slew rate is measured by using a very sharp square wave input and measure the rate of change of the output in units of V/μsec. A slew rate-limited amplifier can turn a sine wave into a triangle or sawtooth waveform.

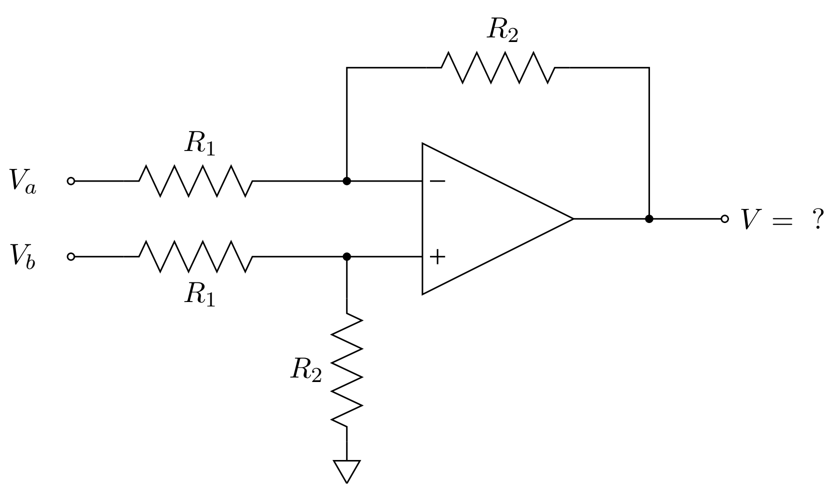


(h) Sketch the gain $|A(\nu)|$ in dB versus $\log \nu$ for a typical 741 op amp with an open-loop gain of 10^5 for $|A(0)| = 1$, 10, 10^2, 10^3 and 10^4. Label the axes properly.

(4 points) For the noninverting amplifier, $F = R/(R + R_f)$ and $A = 1/F = 1 + R_f/R$. The graph was drawn using opamp_gain.mws.



2. (20 points) An interesting amplifier.



(a) Assuming that the op amp is ideal, use the op amp rules and conservation of energy and charge to derive the output potential V in terms of V_a, V_b, R_1 and R_2.

(10 points) Assume that $I_b = 0$. Begin by realizing that there are three unknowns, V, V_- and V_+. The first op amp rule states $V_- = V_+$. We now need two equations to solve for V and V_+. First, at the noninverting side there is a simple potential divider,

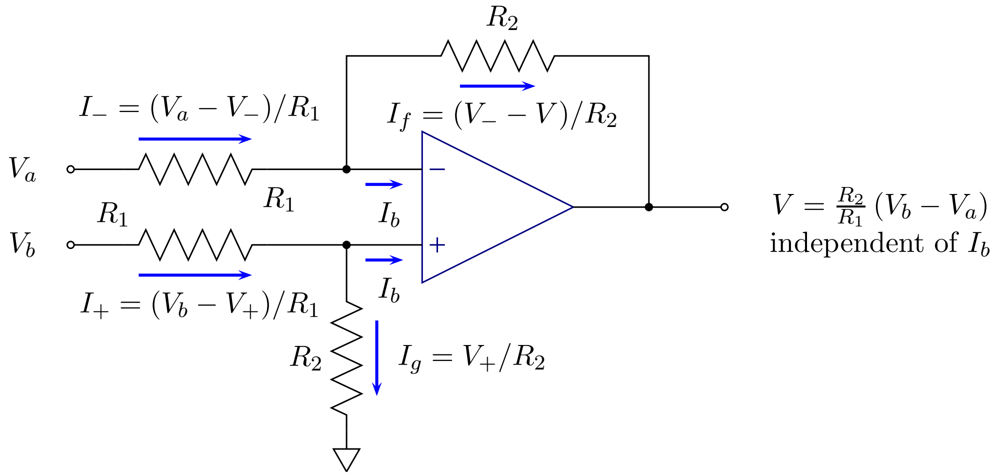
$$V_+ = \frac{R_2}{R_1 + R_2} V_b \ .$$

Second, by conservation of current at the inverting side,

$$I_- = I_f \rightarrow \frac{(V_a - V_+)}{R_1} = \frac{V_+ - V}{R_2} \rightarrow V = -\frac{R_2}{R_1}V_a + R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_+ \ .$$

Combining both equations, we arrive at the expression for a *difference amplifier*,

$$V = -\frac{R_2}{R_1}V_a + R_2\frac{R_1 + R_2}{R_1 R_2}\frac{R_2}{R_1 + R_2}V_b = \frac{R_2}{R_1}(V_b - V_a) \ .$$



(b) Now suppose that both the inverting and noninverting inputs draw a small input bias current I_b. Using the same approach, derive an expression for V, and determine whether or not I_b appears in the output.

(10 points) Applying conservation of current at the noninverting side, $I_+ = I_b + I_g$. So,

$$\frac{(V_b - V_+)}{R_1} = I_b + \frac{V_+}{R_2} \rightarrow V_+\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_b}{R_1} - I_b \rightarrow V_+ = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}\left(\frac{V_b}{R_1} - I_b\right) \ .$$

By conservation of current at the inverting side,

$$I_- = I_f + I_b \rightarrow \frac{(V_a - V_+)}{R_1} = \frac{V_+ - V}{R_2} + I_b \rightarrow V = -\frac{R_2}{R_1}V_a + R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_+ + R_2 I_b \ .$$

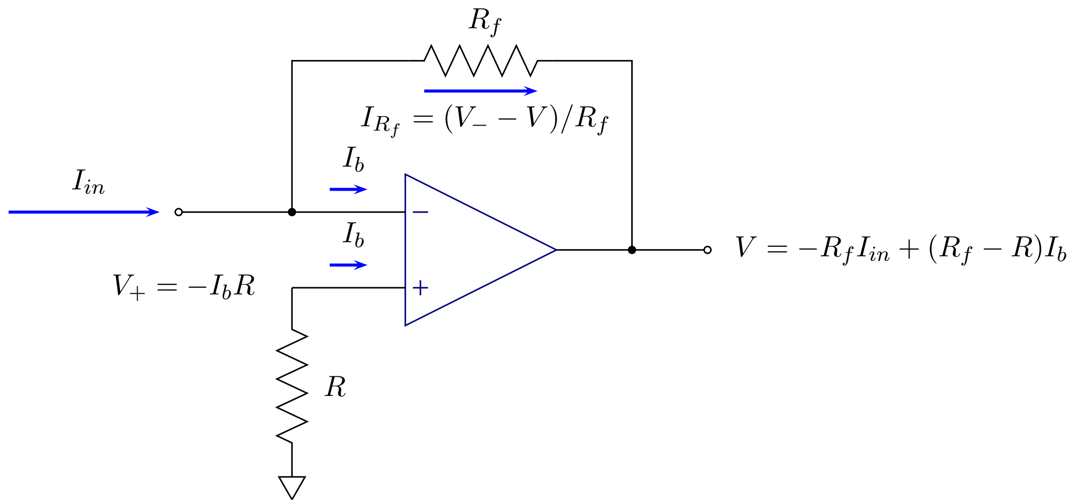
Combining both equations, we see that the output of the difference amplifier is independent of I_b if $I_{b+} = I_{b-}$,

$$V = \frac{R_2}{R_1}(V_b - V_a) + R_2 I_b - R_2 I_b = \frac{R_2}{R_1}(V_b - V_a) \ .$$

3. (15 points) Photodetectors generate a current proportional to the absorbed optical power, and hence they are ideal current sources. Consider the situation in which you would like to transform a small photocurrent I_p which is oscillating between 0 and 10 nA at 10kHz into a potential oscillating between 0 and -1 V.

(a) Design a circuit based upon an ideal op amp to accomplish this.

(5 points) To achieve the specified output, the inverting transimpedance amplifier is a good choice. For an ideal op amp, $I_b = 0$, $R = 0$ and $V_{out} = -R_f I_{in}$. So, $R_f = 1\ \text{V}/1 \times 10^{-8}\ \text{A} = 1.0 \times 10^8\ \text{V/A} = 1.0 \times 10^8\ \Omega$.



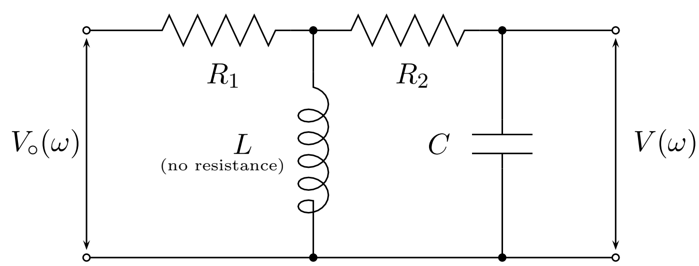
(b) If a real op amp is used, which non-ideal properties would effect your ability to successfully build such a circuit? Explain why.

(5 points) The most serious problem is the current drawn by the inputs, due to finite input resistance and bias current. If $R_{input} > 10R_f$ and $I_b = 0$, then the amplifier will work well. If $I_b \neq 0$, then the resistor to ground, $R = R_f$, must be used.

(c) In a realistic situation, there is always some bothersome background photocurrent arising from the room lights oscillating at 120 Hz. Suppose this background current oscillates between 0 and 1 μA at 120 Hz. Devise a modification to the circuit so that this background signal is eliminated from the output signal of the circuit. Be specific about the values of all components.

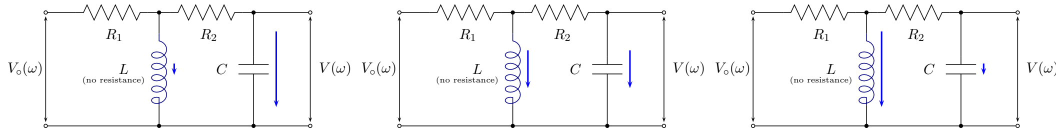
(5 points) A high pass filter (CR or RL) type will provide the necessary differential attentuation between the two frequency components. Since the background signal is 100 times greater than the signal of interest, the filter must be placed before the amplifier. To avoid attenuating the photo-signal much, the - 3 dB frequency could be as high as 5 kHz. For the high-pass CR filter, $RC = 1/(2\pi \times 5 \times 10^3 \text{ Hz }) \simeq 3.3 \times 10^{-5}$ sec .

4. (30 points) Analyze the following circuit in the frequency domain, assuming that the inductor is ideal.



(a) Consider the circuit to be two consecutive filters with transmission functions $A_1(\omega)$ and $A_2(\omega)$. Use physical reasoning and the concept of impedance to explain thoroughly the behavior of the each filter at low, intermediate and high frequencies. Clearly indicate where current is or is not flowing at low, intermediate and high frequencies. Pictures would useful.

(9 points) Since $Z_L = i\omega L$, the inductor presents a short-circuit at low frequency and an open circuit at high frequency. Since $Z_C = 1/i\omega C$, the capacitor presents a short-circuit at high frequency and an open circuit at low frequency. So, at low frequency, current flows through the inductor but not the capacitor. At high frequency, current flows through the capacitor but not the inductor. At intermediate frequency, current flows through both objects. Shown below are representations of the time-varying current at high, medium and low frequencies.



(b) Derive the expressions for the transmission or response functions $A_1(\omega)$ and $A_2(\omega)$.
(6 points)

$$A_1(\omega) = \frac{Z_L}{R_1 + Z_L} = \frac{i\omega L}{R_1 + i\omega L} = \frac{1}{1 - i\frac{R_1}{\omega L}} = \frac{1}{\sqrt{1 + \left(\frac{R_1}{\omega L}\right)^2}} e^{i\tan^{-1}\frac{R_1}{\omega L}} = |A_1(\omega)|e^{i\phi_1(\omega)}$$

$$A_2(\omega) = \frac{Z_C}{R_2 + Z_C} = \frac{1/i\omega C}{R_2 + 1/i\omega C} = \frac{1}{1 + i\omega R_2 C}$$
$$= \frac{1}{\sqrt{1 + (\omega R_2 C)^2}} e^{-i\tan^{-1}\omega R_2 C} = |A_2(\omega)|e^{i\phi_2(\omega)}$$

(c) Each function has a frequency ν_1 or ν_2 for which $20\log|A(\nu_{1 \text{ or } 2})| = -3$ dB. Determine both of these for the case $R_1 = 2\pi \times 100\ \Omega$, $L = 1.0 \times 10^{-3}$ H, $R_2 = 100/2\pi\ \Omega$, and $C = 1.0 \times 10^{-8}$ F.
(4 points)

$$-3 \text{ dB} = 20\log\frac{1}{\sqrt{2}} \rightarrow |A_1(\nu_1)| = |A_2(\nu_2)| = \frac{1}{\sqrt{2}}$$
$$\nu_1 = \frac{1}{2\pi}\frac{R_1}{L} = 10^5 \text{ Hz}$$
$$\nu_2 = \frac{1}{2\pi R_2 C} = 10^6 \text{ Hz}$$

(d) Write the symbolic (no numbers) expression for $A_{total}(\omega)$ in terms of an amplitude $|A_{total}(\omega)|$ and a phase factor $e^{i\alpha}$.
(4 points)

$$V_{out}(\omega) = A_2(\omega)\left(A_1(\omega)V_{in}\right) = A_{total}(\omega)V_{in}$$

So,

$$A_{total}(\omega) = A_1(\omega)A_2(\omega) = |A_1(\omega)||A_2(\omega)|e^{i(\phi_1(\omega)+\phi_2(\omega))}\ ,$$

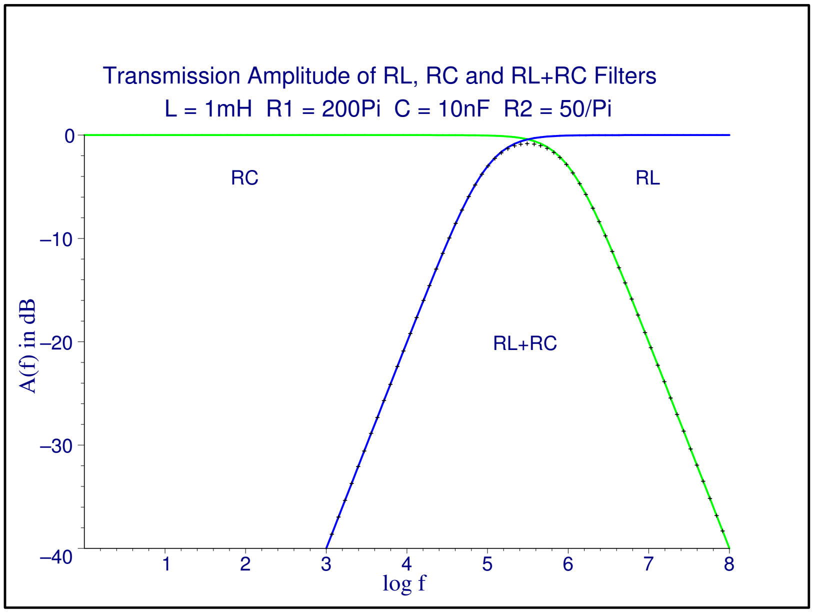
and

$$\phi_{total} = \tan^{-1}\frac{R_1}{\omega L} - \tan^{-1}\omega R_2 C\ ,$$

which varies from$\pi/2$ to $-\pi/2$ as ω varies from 0 to ∞.

(e) Accurately sketch $|A_1(\nu)|$, $|A_2(\nu)|$ and $|A_{total}(\nu)|$in dB versus $\log \nu$. Indicate the -3dB points and make the slopes of the functions reasonably accurate.

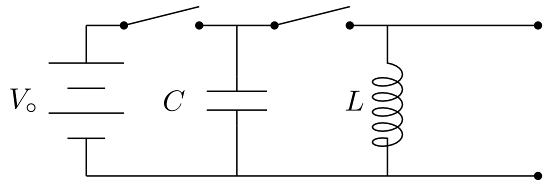
(4 points) This graph was made using rl+rc_filters.mws. Notice that at the peak, the transmission is large, experiencing a change of only -1 dB.



(f) Since this circuit has an inductor and a capacitor, one might consider the possibility of a resonance frequency. Is there a resonance frequency? Explain briefly why or why not. If this is a resonant circuit, is it a lossy oscillator or does it have a high Q?

(3 points) In this circuit, energy can flow back and forth between the capacitor and the inductor, so there is a resonance frequency. However, the presence of the resistor R_2 makes it a very lossy (low Q) oscillator with a frequency slightly shifted from $\omega_\circ = 1/\sqrt{LC}$. This low Q is manifested in the broad bandpass function of this circuit and in the lack of a peak in transmission that is greater than 0 dB.

5. (10 points) Time-domain analysis of the LC oscillator.



(a) Write the differential equation governing the behavior of this circuit when the battery have been used to establish an initial condition but is otherwise disconnected from the circuit.
(10 points)

$$V_L(t) = V_C(t) \rightarrow L\frac{dI(t)}{dt} = \frac{Q(t)}{C} \rightarrow -\frac{dQ(t)}{dt} = \frac{Q(t)}{LC} \ .$$

(b) Assuming an oscillatory solution for the charge $Q(t) = Q_\circ e^{i\omega_\circ t}$, find the expression for the natural frequency of oscillation $\omega_\circ$ by solving the differential equation when the initial condition is $Q(0) = CV_\circ$.
(10 points) Since

$$\frac{dQ(t)}{dt} = -\frac{Q(t)}{LC} \text{ and } Q(t) = Q_\circ e^{i\omega_\circ t} \rightarrow -\omega_\circ^2 Q(t) = -\frac{Q(t)}{LC} \ ,$$

then

$$\omega_\circ = \frac{1}{\sqrt{LC}} \ .$$

The value $Q_\circ = CV_\circ$ certainly satisfies the differential equation and initial condition.