# Thermal Conduction and Insulation

## Thermal Conduction

Energy flows spontaneously as heat from a region of higher temperature to a region of lower temperature, as depicted in Figure 1. Given that the cross sectional area (the area perpendicular to the direction of heat flow) is $A$ and that the length of the object is $L$, the power flowing from the high temperature side to the low temperature side is

$$P = \kappa A \frac{T_h - T_c}{L} = \kappa A \frac{\Delta T}{L}\,.$$

The quantity $(T_h - T_c)/L$ is the *temperature gradient*, and $\kappa$ is the *thermal conductivity* in W/m/K. Materials exhibit a wide variation in thermal conductivity, as shown in Table 1.

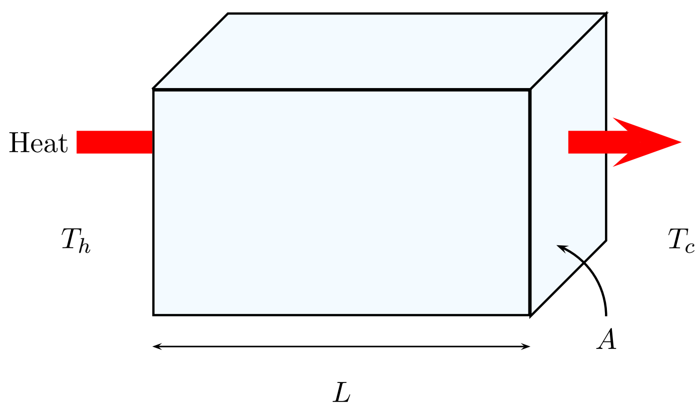


Figure 1: Heat conduction through a uniform object. The crossectional area is $A$, and, since $T_h > T_c$, the temperature gradient is $(T_h - T_c)/L$ to the left.

| material | $\kappa$ (W/mK) | $R_f$(mK/W) | $\kappa$ (btu/hr ft °F) | $R_f$(hr ft$^2$ °F/btu in) |
|---|---|---|---|---|
| copper | 400 | 0.0025 | 2780 | 0.00036 |
| steel | 46 | 0.022 | 320 | 0.003 |
| concrete | 1.7 | 0.59 | 12 | 0.083 |
| brick | 0.7 | 1.43 | 5 | .02 |
| water | 0.6 | 1.67 | 4.15 | 0.25 |
| glass | 0.59 | 1.69 | 4 | 0.25 |
| $H_2$ | 0.17 | | | |
| wood | 0.13 | 7.7 | 0.9 | 1.1 |
| sawdust | 0.059 | 16.9 | 0.4 | 2.5 |
| glasswool | 0.038 | 26.3 | 0.27 | 3.7 |
| air | 0.023 | 43.5 | 0.16 | 6 |
| argon | 0.016 | | | |
| $CO_2$ | 0.014 | | | |
| $Cl_2$ | 0.007 | | | |
| vacuum | 0 | $\infty$ | | |

In the context of energy conservation through insulation, a more common expression is

$$P = \frac{A \triangle T}{R} \,,$$

where $R = R_f L$ is the $R$ *value* and $R_f = 1/\kappa$ is the $R$ *factor*. If the object consists of layers of different materials oriented perpendicular to the direction of heat flow, the expression for the power flowing through the object can be written in the general form

$$P = A\frac{\triangle T}{R} = A\frac{\triangle T}{R_{f1}L_1 + R_{f2}L_2 + R_{f3}L_3 + \cdots} \,.$$

The $R$ value for the object is simply the sum of the products of the $R$ factors and thicknesses for the different layers. The larger the R value, the greater the insulation property of the object.

## Heat loss through a window

Heat passes by conduction through glass from a warm interior environment to a cold exterior region, resulting in the loss of energy over a period of time. Consider a double pane window shown in Figure 2 with $g = 6$mm and $s = 1$cm.

The power lost by conduction is

$$P = A\frac{\triangle T}{R_{fglass}g + R_{fair}s + R_{fglass}g} \,.$$

For this case, $R = 2 \times 1.69$ mK/W $\times .006$ m + 43.5 mK/W $\times$ .01 m = 0.455 m$^2$K/W. Assuming a temperature difference $\triangle T = 20$°C, we arrive at the result $P = 43.9$W/m$^2$ $\times A$. If the second pane of glass and the intervening air are removed, then $P = A\triangle T/gR_{fglass} = 1972$ W/m$^2$ $\times A$. This is a whopping 45 times greater power loss per square meter of window. If electricity is used to provide heating at a cost of 6 cents per kWhr, then the heat lost through the 1 m$^2$ single pane window costs 11.8 cents per hour versus 0.26 cents per hour for the double pane window. Conservation can be quite cost effective.

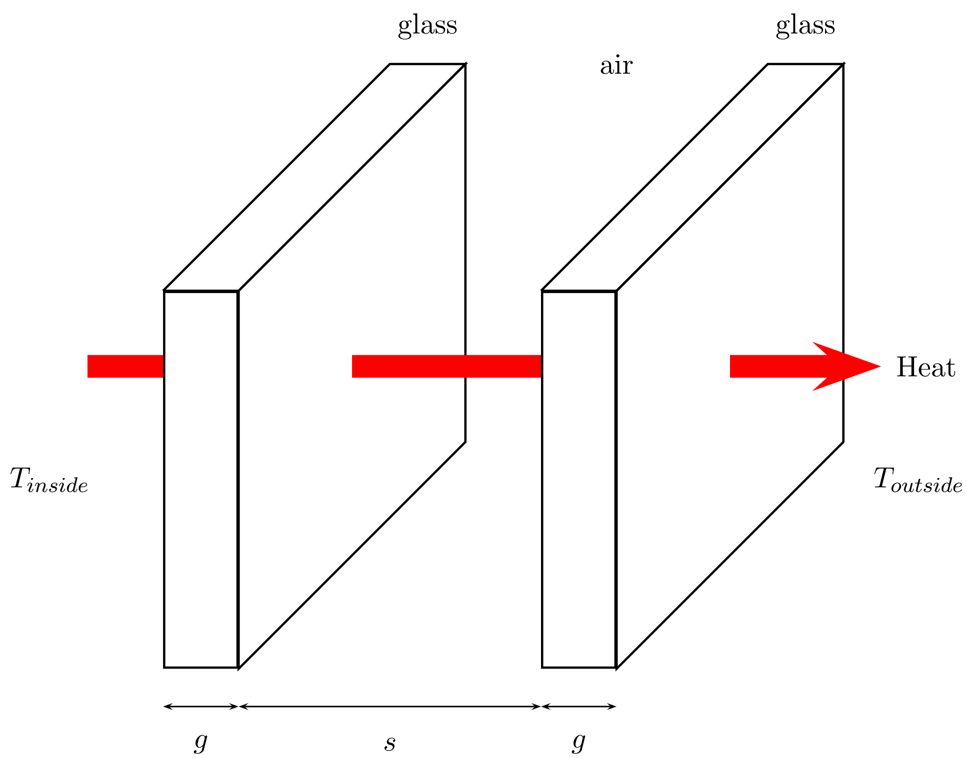


Figure 2: Heat conduction through a dual pane window.