Hydroelectric Power Generation

Basic Concepts

Heat from the absorption of solar radiation provides the energy to evaporate water and carry the vapor to great heights via convection. The heat of vaporization of water at 1 atm pressure and at a temperature of 298 K (25°C) is [1]

$$\triangle G = 2.063\text{kcal/mole} = 8.62\text{kJ/mole} = 479\text{kJ/kg}\,.$$

This energy is released as heat in clouds when the vapor condenses to form drops of water again, so none of that energy can be converted into mechanical work. However, the increase in gravitational potential energy can be tapped. If 1 kg of liquid water lies 3,000 m (9,842 ft) above the surface of the sea, then the change in gravitational potential energy is

$$\triangle PE = mgh = 1\text{kg} \times 9.8\text{m/s}^2 \times 3000\text{m} = 2.94 \times 10^4\text{J}\,.$$

When the water falls to sea level the potential energy is converted to kinetic energy,

$$\triangle PE = \triangle KE = \frac{1}{2}mv^2\,,$$

at least in principle. In reality, air resistance would limit the velocity to a smaller value than that indicated by the above equation, and energy would be lost to the air through friction. The point to be made is that when precipitation collects in a reservoir at some elevation, the gravitational potential energy will be converted to kinetic energy as the water runs downhill, and useful mechanical work can be extracted from the flowing water.

The power in flowing water, which is kinetic energy passing by per unit time, is

$$P = \frac{1}{2}\rho A v^3\,,$$

where $\rho = 1 \times 10^3\text{kg/m}^3$ is the density of water, A is the cross-sectional area in the plane perpendicular to the direction of motion and v is the speed of the water. As an example, for $v = 1\text{m/s}$ the power per unit area is 500 W/m^2. From an analysis based upon laminar flow, incompressibility of air and conservation of energy, the maximum efficiency for extraction of power from the flowing water is 0.529 for an idealized turbine. The table lists the power applied to turbines of various radii for various water speeds. Note that, since the density of water is 770 times that of air, the power per unit area of a river is 770 times that of wind flowing at the same speed.

radius (m)	area (m^2)	water speed (m/s)	Power (W)
1	3.14	1	1,570
1	3.14	2	12,560
1	3.14	5	196,250
5	78.5	1	39,250
5	78.5	2	314,000
5	78.5	5	4,906,250

A hydroelectric reservoir produces power proportional to both the *head*, or height through which the water drops, and the *flow*, or rate at which water drains from the reservoir. Figure 1 depicts

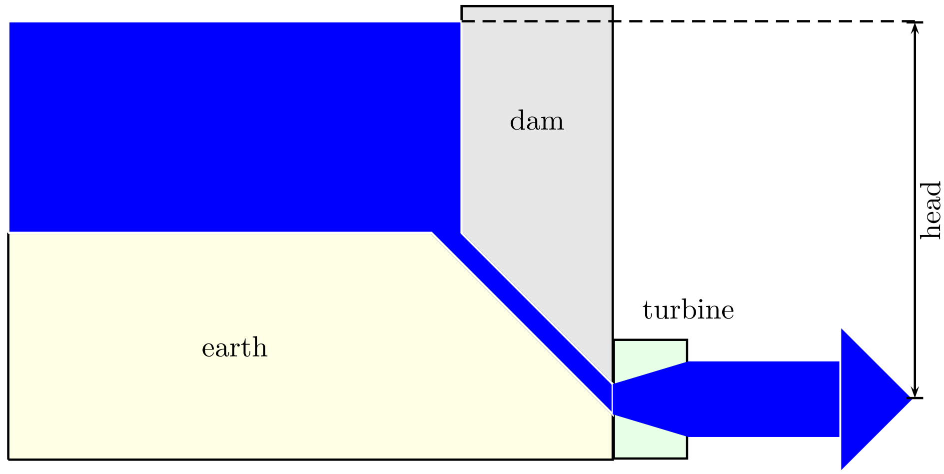

Figure 1: Hydroelectric dam and reservoir. The power is proportional to the *head* and the flow, or rate at which water drains from the reservoir.

a typical dam, reservoir and turbine hydroelectric system. Ignoring frictional losses, the electrical power produced is

$$P_e = \eta F \rho g h \,,$$

where η is the efficiency of the turbine, F is the flow, g is the acceleration due to gravity at the surface of the earth and h is the head. A real dam was found to produce 235 kW at a flow of 386 ft^3/s and a head of 13 ft. The ideal power in the water was

$$F\rho g h = 10.4\frac{\mathrm{m}^3}{\mathrm{s}} \times 10^3\frac{\mathrm{kg}}{\mathrm{m}^3} \times 9.8\frac{\mathrm{m}}{\mathrm{s}^2} \times 3.9\mathrm{m} = 400\mathrm{kW} \,.$$

So the efficiency was $\eta = 235/400 = 0.58$, suspiciously close to the ideal maximum value of 0.59.

[1] *Understanding Chemistry*, G. C. Pimentel and R. D. Spratley, Holden-Day, 1971, page 365.