Conversion of the Kinetic Energy of Wind, Rivers and Tides

The Earth as a Heat Engine

When the earth absorbs sunlight, electromagnetic energy is converted to heat. This is a conversion from an organized form of energy to a disorganized form. Through evaporation, convection and large scale weather phenomena, the earth acts as a heat engine to convert some of this thermal energy into the kinetic energy of the wind and rivers. Human conversion of this energy of motion in a particular direction into mechanical work on an electrical generator and then into electrical energy is governed by a few simple principles.

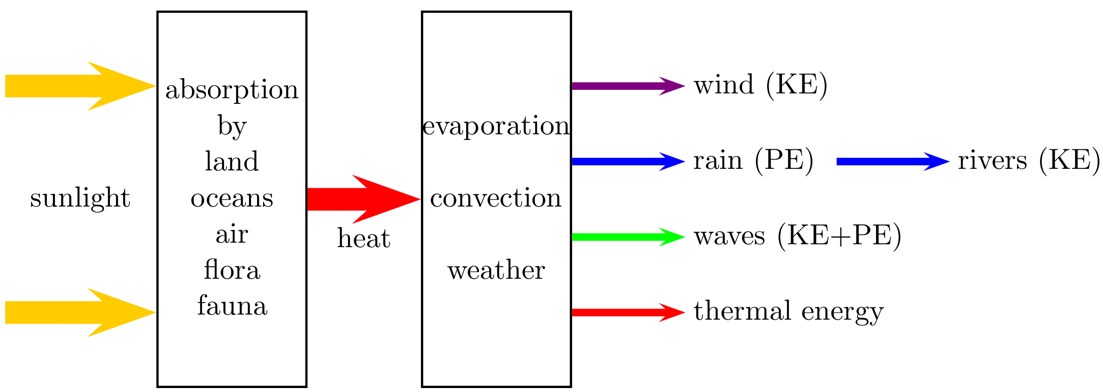

Figure 1: The great earth heat engine converts thermal energy to kinetic energy of the wind and rivers and the energy of waves.

Extraction of Mechanical Power from a Moving Fluid

To approach the topic of tapping the kinetic energy and power of a moving fluid, such as water, it is convenient to begin by considering the fluid to consist of moving blocks. A rectangular volume of water of cross-sectional area A and length L in the direction of motion has the mass

$$m = \rho V = \rho A L \,,$$

where ρ is the density and V is the volume. The kinetic energy of the object moving at speed v is

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} \rho A L v^2 \,.$$

If t is the time for the object to pass by, the power of the moving object is

$$P = \frac{KE}{t} = \frac{1}{2} \rho \frac{L}{t} A v^2 \,.$$

Since the length of the object is just the speed multiplied by the time for the object to pass by, $L = vt$, the power becomes

$$P = \frac{1}{2}\rho A v^3 \, .$$

Thus, the first important result is that the power of moving fluid is proportional to the density, the cross-sectional area and v^3.

Not all of the kinetic energy of a river or a moving steam of air can be extracted for a very simple reason. Consider water flowing in a channel with a constant cross-sectional area. If all of the kinetic energy was extracted at a particular point in the channel the water would cease to flow. Since water is not infinitely compressible, complete conversion is impossible. Suppose instead that the speed of the water, initially at v_1, is reduced not to 0 but to v_2. Assuming that the density of water is constant, the flow before the extractor, in m^3/s, must be equal to the flow after the extractor. Thus, in a given time t, the input flow V_1/t must equal the output flow V_2/t. Expressing the volumes in terms of the input and output cross-sectional areas and speeds,

$$\frac{V_1}{t} = \frac{V_2}{t} \Rightarrow \frac{A_1 v_1 t}{t} = \frac{A_2 v_2 t}{t} \Rightarrow A_1 v_1 = A_2 v_2 \, .$$

This is known as *Pascal's Law*, which states that if an incompressible fluid slows down the cross-sectional area must increase. If the output channel is twice the area of the input channel, then $v_2 = v_1/2$, and the extraction efficiency would be

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\frac{1}{2}\rho A_2 v_2^3}{\frac{1}{2}\rho A_1 v_1^3} = \frac{v_2^2}{v_1^2} = \frac{1}{4} \, .$$

The maximum efficiency of conversion is found by determining the power extracted from the fluid by an idealized rotor, dividing by the power in the incident fluid and maximizing this ratio by varying a parameter describing the rotor. Begin by defining the *axial interference factor, a,* as a number between 0 and 1. If $a = 0$ then the rotor has no effect on the wind, that is, it is not there. If $a = 1$ the rotor is actually a solid wall, and when the fluid is reflected straight back no energy is extracted. If $a = 1/2$ all the kinetic energy is extracted and the wind stops at the rotor, an unphysical situation. So, a must lie somewhere between 0 and 1/2.

A simplistic view of a rotor is show in Figure 2. The fluid is assumed to experience *laminar flow*, that is, a smooth flow without turbulence. The details of the rotor design will be ignored as only the parameter a will be considered. Notice that though the intial pressure is 1 atm and ultimately the final pressure in the wake is also 1 atm, there is pressure variation near the rotor.

The law of conservation of energy can be invoked by the application of Bernoulli's Principle,

$$P + \frac{1}{2}\rho v^2 = \textit{constant}$$

everywhere within the fluid, as long as no power is flowing in or out of the fluid. This equation simply states the fact that pressure-volume work on the fluid can be converted to kinetic energy and vice versa. Applying this equation on the left and the right sides of the rotor and then subtracting the two equations yields a relation for $\triangle P$.

$$P + \frac{1}{2}\rho v_f^2 = P_r + \frac{1}{2}\rho v_r^2$$

$$-(P + \frac{1}{2}\rho v_w^2 = P_r - \triangle P + \frac{1}{2}\rho v_r^2)$$

$$\frac{1}{2}\rho(v_f^2 - v_w^2) = \triangle P\,.$$

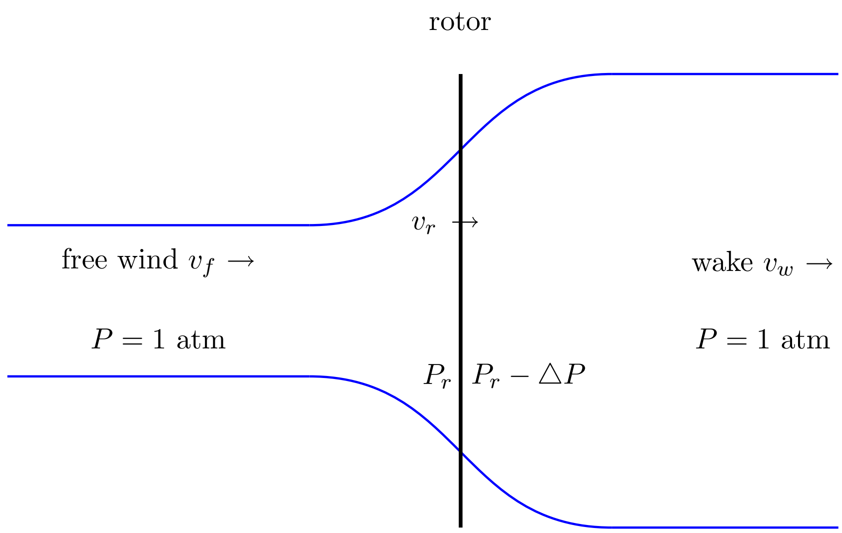

Figure 2: Extraction of kinetic energy from the wind using a rotor. The wake speed v_w is less than the free wind speed v_f, and the speed within the rotor is between the two. There is a pressure drop $\triangle P$ across the rotor.

The next step is to consider the force on the rotor from two perspectives. Since force is momentum change in the fluid per unit time,

$$F = \frac{\triangle p}{\triangle t} = \frac{\triangle p}{V} \times \frac{V}{\triangle t} \text{ or}$$

$$F = \rho(v_f - v_w) \times \pi r^2 v_r\,.$$

Here, $\triangle p$ is the change in momentum and the cross sectional area of the rotor of radius r is πr^2. This force must also be equal to the pressure drop across the rotor times the area, or

$$F = \pi r^2 \triangle P\,.$$

So,

$$\rho(v_f - v_w)\pi r^2 v_r = \pi r^2 \triangle P = \pi r^2 \frac{1}{2}\rho(v_f^2 - v_w^2)\,.$$

Thus, a simple, and somewhat obvious, expression for v_r is obtained:

$$v_r = \frac{1}{2}\frac{v_f^2 - v_w^2}{v_f - v_w} = \frac{1}{2}\frac{(v_f + v_w)(v_f - v_w)}{v_f - v_w}\,,$$

or

$$v_r = \frac{1}{2}(v_f + v_w) = \frac{1}{2}v_f\left(1 + \frac{v_w}{v_f}\right) = v_f(1 - a)\,.$$

The interference factor a has now been defined as

$$a = \frac{1}{2}\left(1 - \frac{v_w}{v_f}\right)\,.$$

Note that since

$$v_w = 2v_r - v_f = 2v_f(1-a) - v_f = v_f(1-2a)\,,$$

this definition of a agrees with the previous physical definition. If $a = 1$, $v_w = -v_f$ and the wind is reflected. If $a = 1/2$, $v_w = 0$ which is an unphysical situation.

The power applied to the rotor is

$$P = \frac{work}{time} = \frac{F\triangle x}{\triangle t} = Fv_r\,.$$

Using the earlier equation for the force on the rotor, we arrive at the important formula for the power extracted from the moving fluid by the rotor:

$$P = 2\pi r^2 \rho a(1-a)v_f^2 v_r = 2\pi r^2 \rho a(1-a)^2 v_f^3\,.$$

Thus, the power is proportional to v_f^3, the density of the fluid, the cross-sectional area of the rotor and a function of the interference parameter.

The maximum power extracted occurs for the value of a such that

$$\frac{dP}{da} = 0\,.$$

Solving this equation for a yields 1/3. Thus, the most efficient rotor has $a = 1/3$. Using this in the expression above for the power extracted yields

$$P_{max} = 2\pi r^2 \rho v_f^3 \frac{1}{3}\left(\frac{2}{3}\right)^2 = \frac{8}{27}\pi r^2 \rho v_f^3\,.$$

Finally, the maximum efficiency is

$$\eta_{max} = \frac{P_{max}}{P_{wind}} = \frac{\frac{8}{27}\pi r^2 \rho v_f^3}{\frac{1}{2}\rho \pi r^2 v_f^3} = \frac{16}{27} = 0.593\,.$$