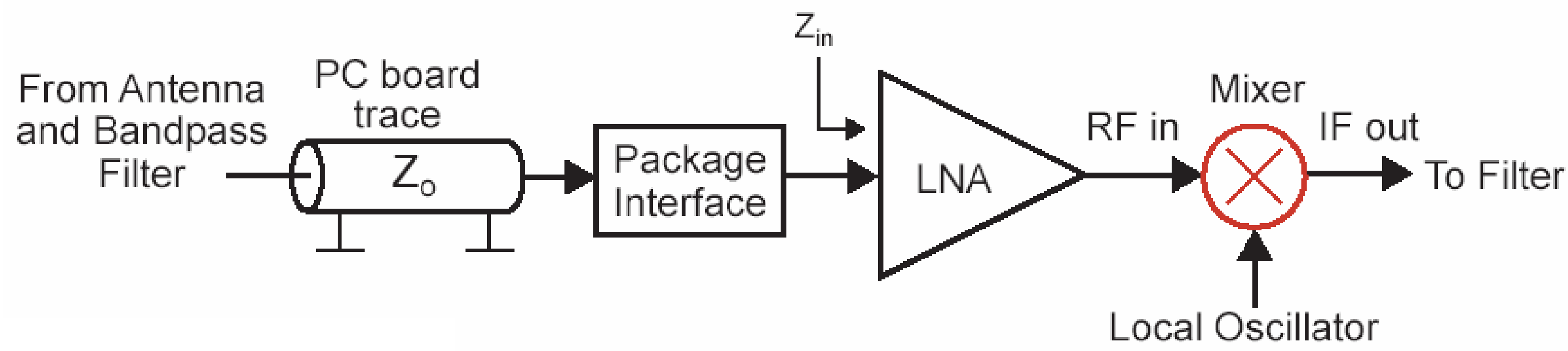
Mixer Design Overview

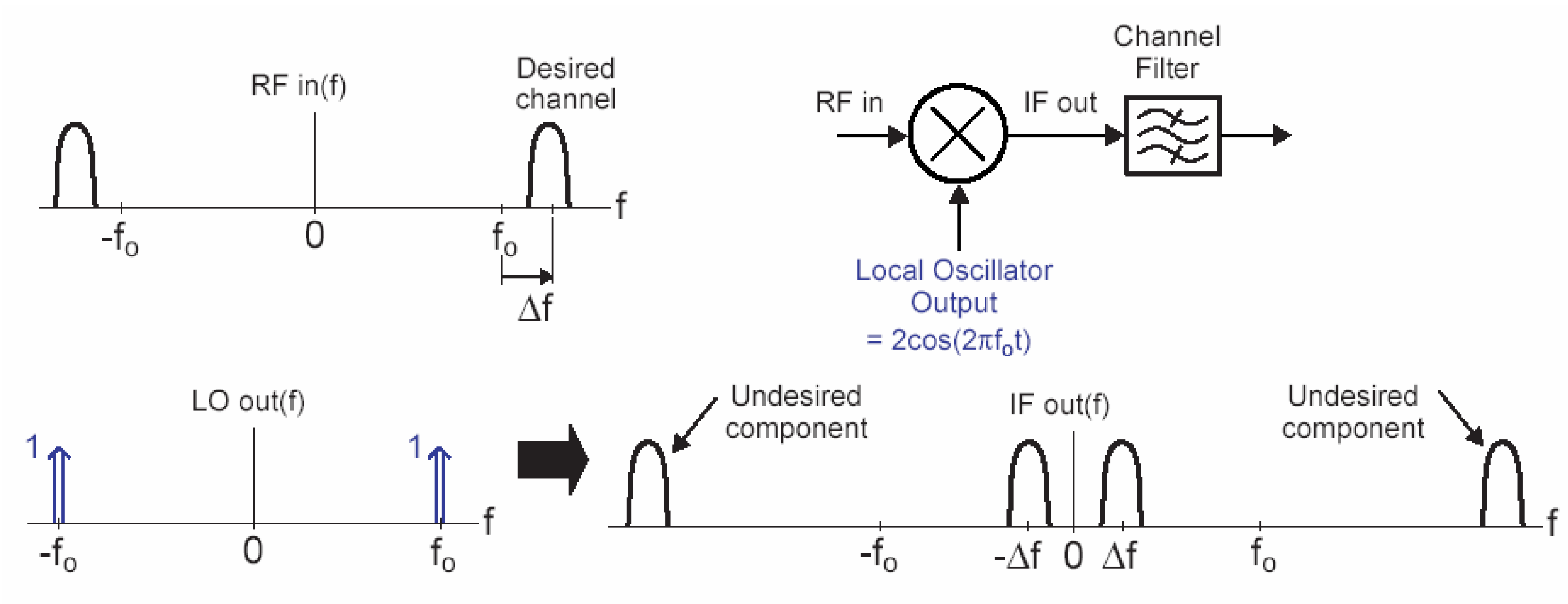


- Noise Figure – impacts receiver sensitivity
- Linearity (IIP3) – impacts receiver blocking performance
- Conversion gain – lowers noise impact of following stages
- Power match – want maximize voltage gain rather than power match for integrated designs
- Power – want low power dissipation
- Isolation – want to minimize interaction between the RF, IF, and LO ports
- Sensitivity to process/temp variations – need to make it manufacturable in high volume

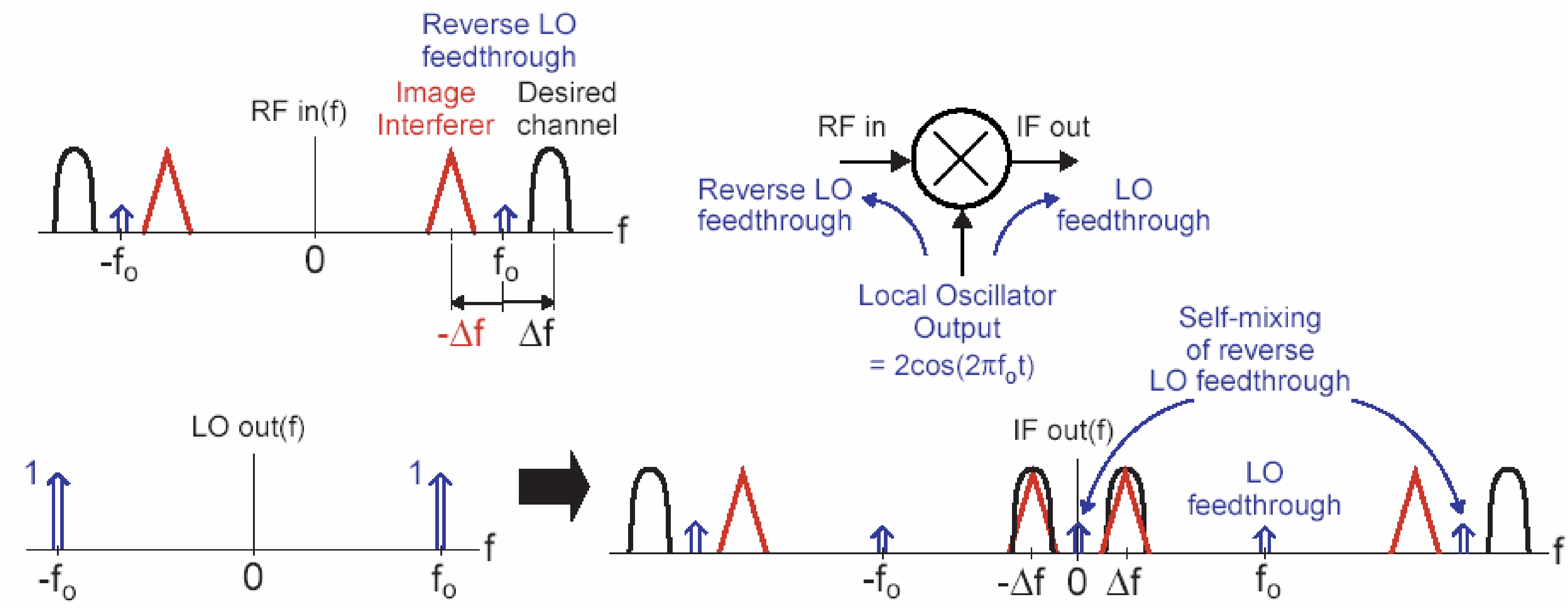
Types of Mixer

- Multiplication through device non-linearity
- Multiplication through switching
  - Active mixers
  - Passive mixers

Ideal Mixer Behavior

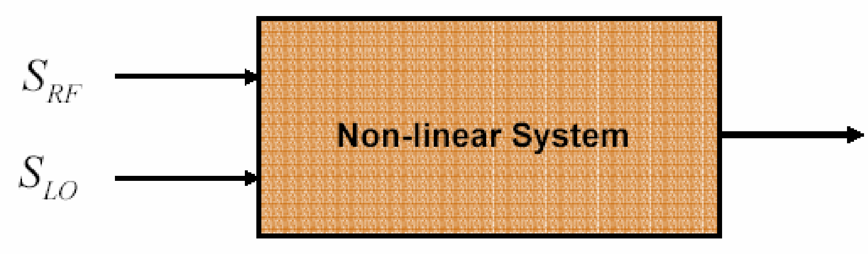


Non-Ideality in Mixers



- Image problem
- LO feedthrough
- Self mixing due to reverse LO feedthrough

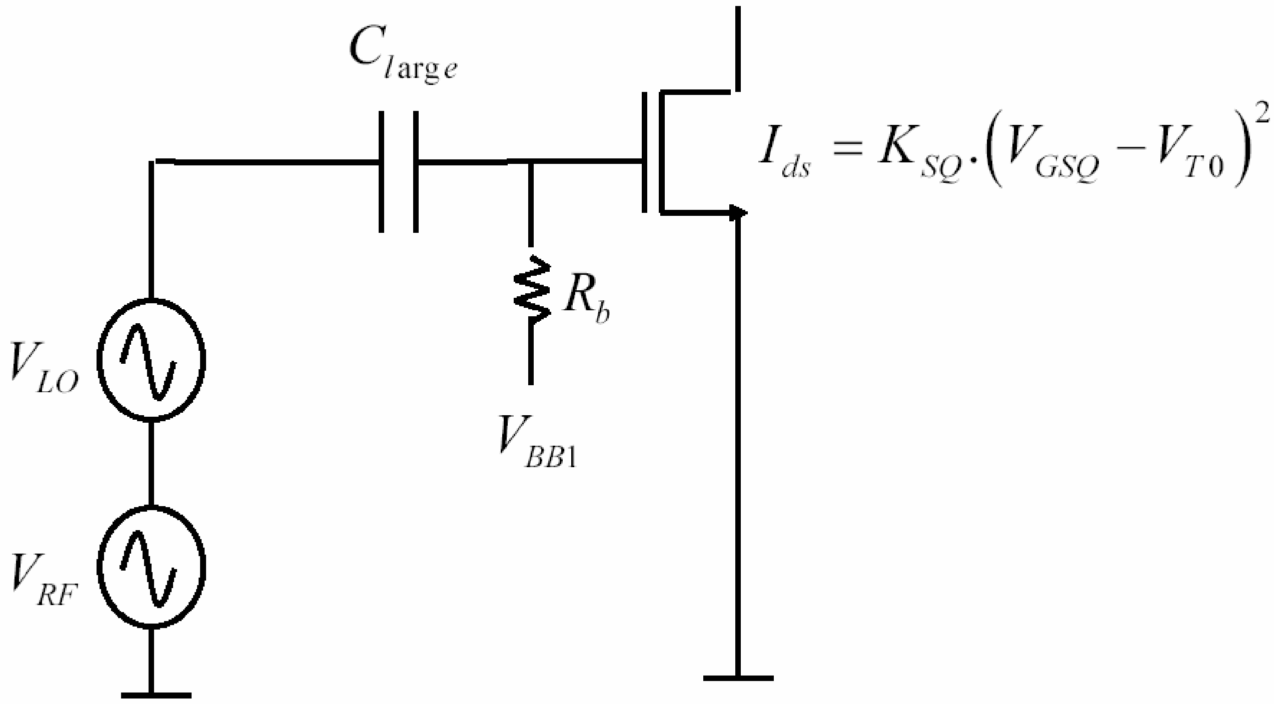
Mixer Based on Non-Linearity



$$
\begin{aligned}
S_{MIX} &= a_1 S_{RF} + a_2 S_{RF}^2 + a_3 S_{RF}^3 + \ldots \\
&+ b_1 S_{LO} + b_2 S_{LO}^2 + b_3 S_{LO}^3 + \ldots \\
&+ \underline{c_1 S_{RF} S_{LO}} + c_2 S_{RF}^2 S_{LO} + c_3 S_{RF} S_{LO}^2 + \ldots
\end{aligned}
$$

- **Drain current of an MOSFET exhibits a square dependence on gate overdrive**
- **Collector current of an BJT exhibits a exponential dependence on base-emitter voltage drive**

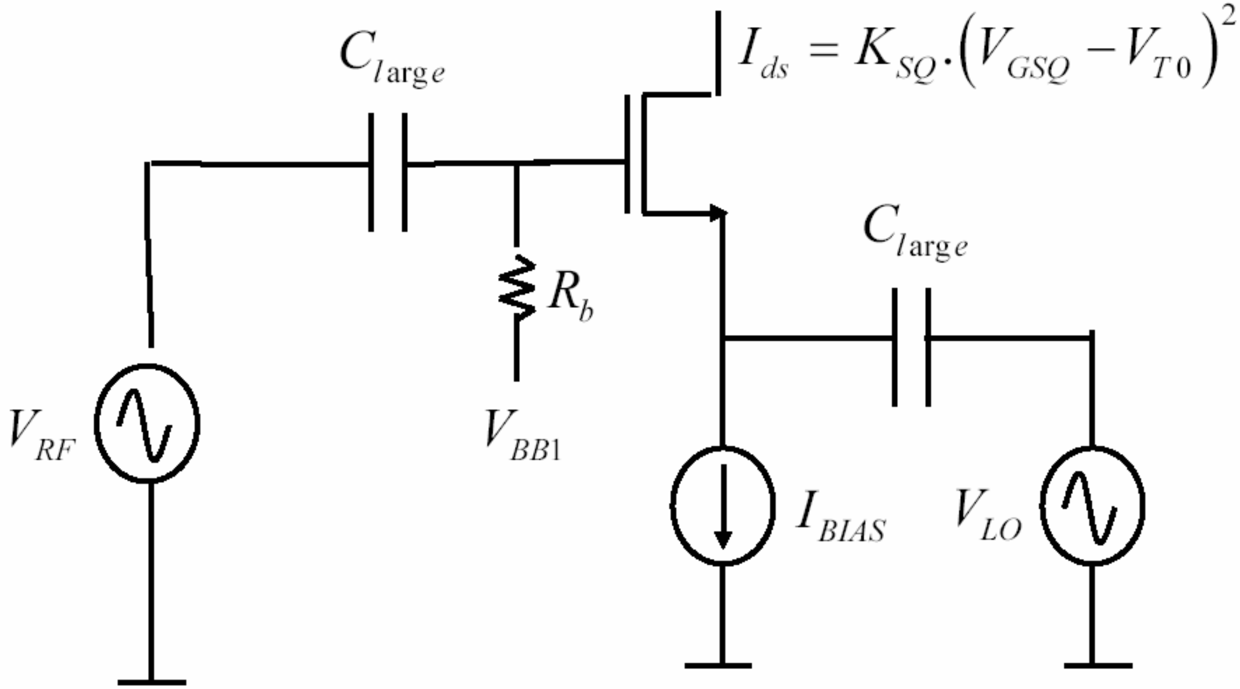
Single-Device Mixer Using MOSFET (Square-Law Mixer)



$$I_{ds} = K_{SQ}.(V_{bias} + V_{RF} + V_{LO} - V_{T0})^2$$
$$= K_{SQ}.\left\{(V_{bias} - V_{T0})^2 + (V_{RF} + V_{LO})^2 + 2(V_{bias} - V_{T0}).(V_{RF} + V_{LO})\right\}$$

$$i_{ds}(\omega_{LO} \pm \omega_{RF}) = .K_{SQ}.V_{RF}.V_{LO}\left\{\boxed{Cos(\omega_{LO} - \omega_{RF})t} + Cos(\omega_{LO} + \omega_{RF})t\right\}$$

Practical Configuration for Single-Device Mixer

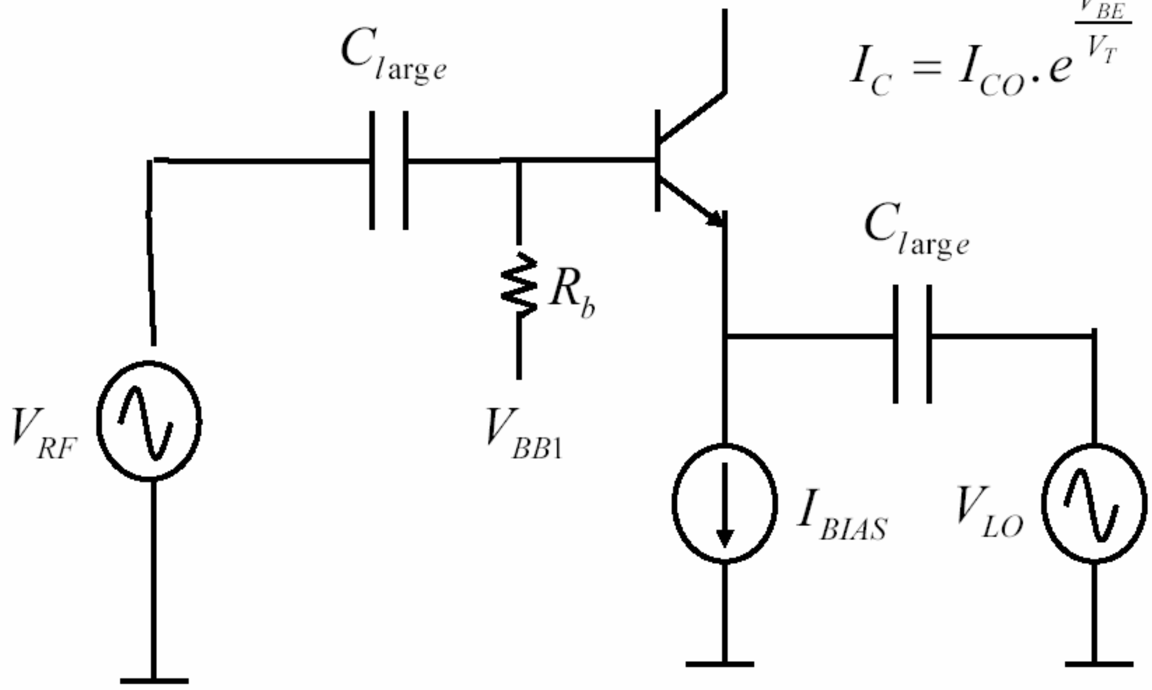


$$i_{ds}(\omega_{LO} - \omega_{RF}) = .K_{SQ}.V_{RF}.V_{LO}.Cos(\omega_{LO} - \omega_{RF})t$$

$$Transconduc\tan ce - conversion - gain = G_c = \frac{i_{ds}(\omega_{IF} = \omega_{LO} - \omega_{RF})}{V_{RF}(\omega_{RF})}$$

$$= K_{SQ}.V_{LO} = \frac{\mu C_{ox} W}{2L}.V_{LO}$$

Single-Device Mixer Using BJT



$$I_C = I_{CO}.e^{\frac{V_{BE}}{V_T}}$$

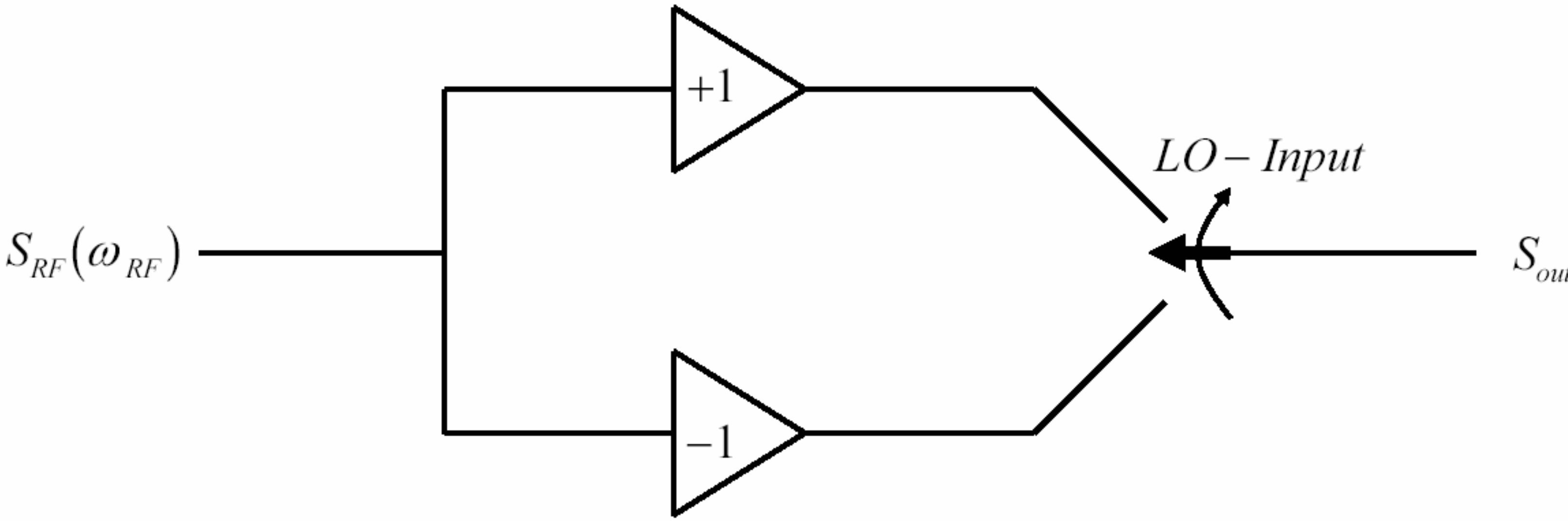
$$I_C = I_{CQ}.e^{\frac{V_{RF}-V_{LO}}{V_T}} = I_{CQ}.\left\{1+\left(\frac{V_{RF}-V_{LO}}{V_T}\right)+\left(\frac{V_{RF}-V_{LO}}{V_T}\right)^2+\ldots\right\}$$

$$Transconduc\tan ce-conversion-gain = G_c = \frac{i_c(\omega_{IF}=\omega_{LO}-\omega_{RF})}{V_{RF}(\omega_{RF})} = \frac{I_{CQ}}{V_T^2}.V_{LO}$$

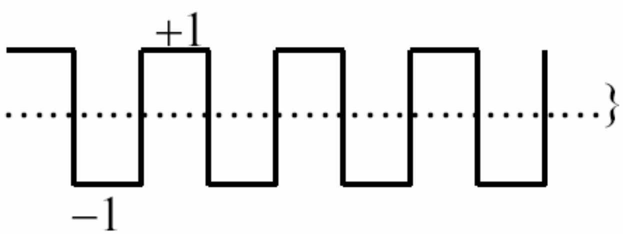
Design Considerations for Mixer Based on Device Non-Linearity

- Design simplicity
- Noise Figure
  - The square law MOSFET mixer can be designed to have very low noise figure
- Linearity
  - By operating the square law MOSFET mixer in the square law region the linearity of the mixer can be improved considerably
  - BJT mixer is less linear as it produces a host of non-linear components due to the exponential nature of the BJT mixer
- Power Dissipation
  - Very low power dissipation due to single device operation
- Power Gain
  - Reasonable power gain can be achieved
- Isolation
  - Poor isolation from LO to RF port – by far the biggest short coming

Mixing Through Switching



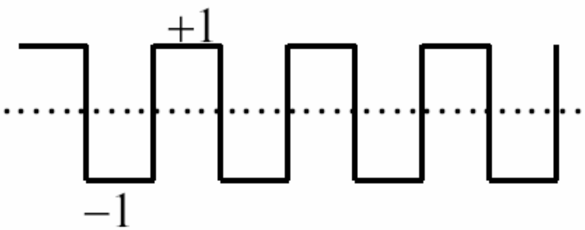
$$S_{out} = S_{RF}.Cos(\omega_{RF}t) \otimes \{\ldots\ldots\}$$



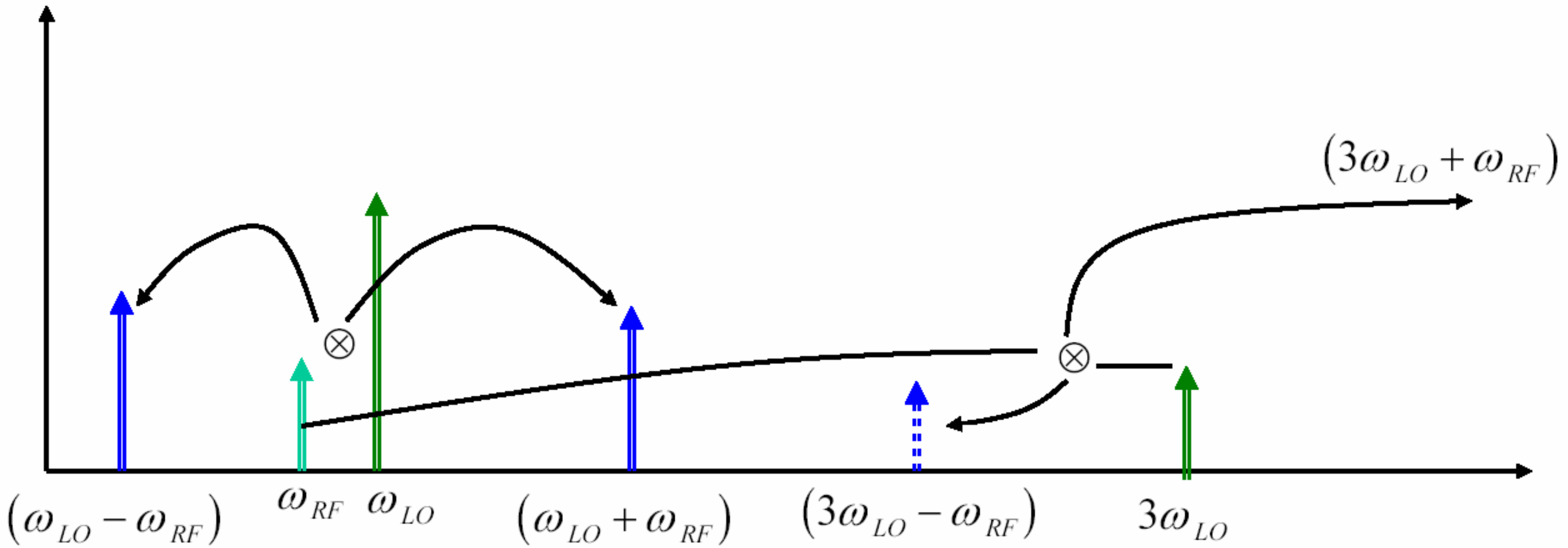
$$S_{out} = S_{RF}.Cos(\omega_{RF}t) \otimes \left\{ \frac{4}{\pi} Cos(\omega_{LO}t) - \frac{4}{3\pi} Cos(3\omega_{LO}t) + \frac{4}{5\pi} Cos(5\omega_{LO}t) - \ldots \right\}$$

Spectral Components Due to Mixing

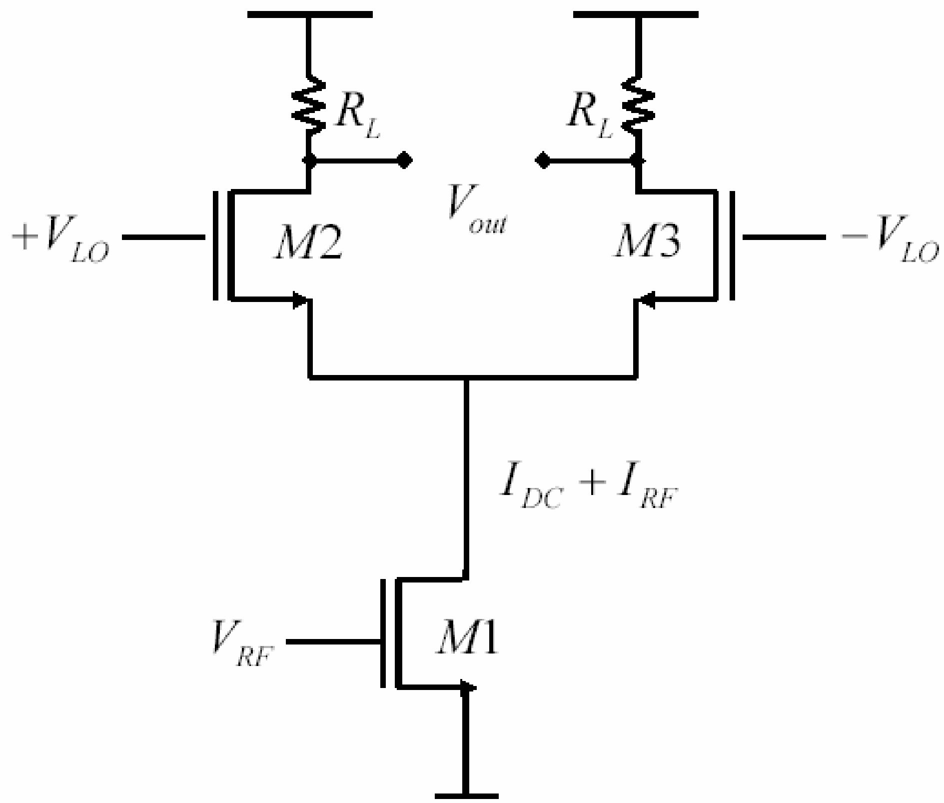
$$S_{out} = S_{RF}.Cos(\omega_{RF}t) \otimes \{\ldots\}$$



$$S_{out} = S_{RF}.Cos(\omega_{RF}t) \otimes \left\{ \frac{4}{\pi} Cos(\omega_{LO}t) - \frac{4}{3\pi} Cos(3\omega_{LO}t) + \frac{4}{5\pi} Cos(5\omega_{LO}t) - \ldots \right\}$$

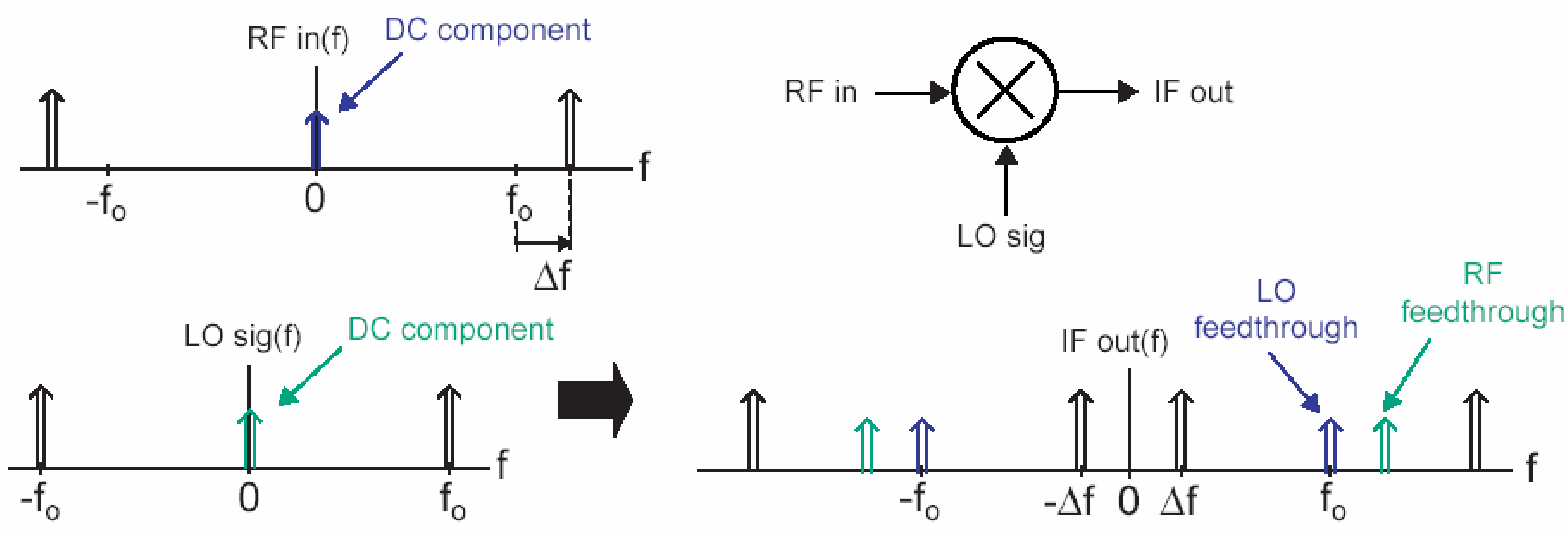


Simple Switching Mixer (Single-Balanced Mixer)



- M1 acts as a transconductance to convert the RF voltage signal to a current
- M2 and M3 commute the current between the two output branches.

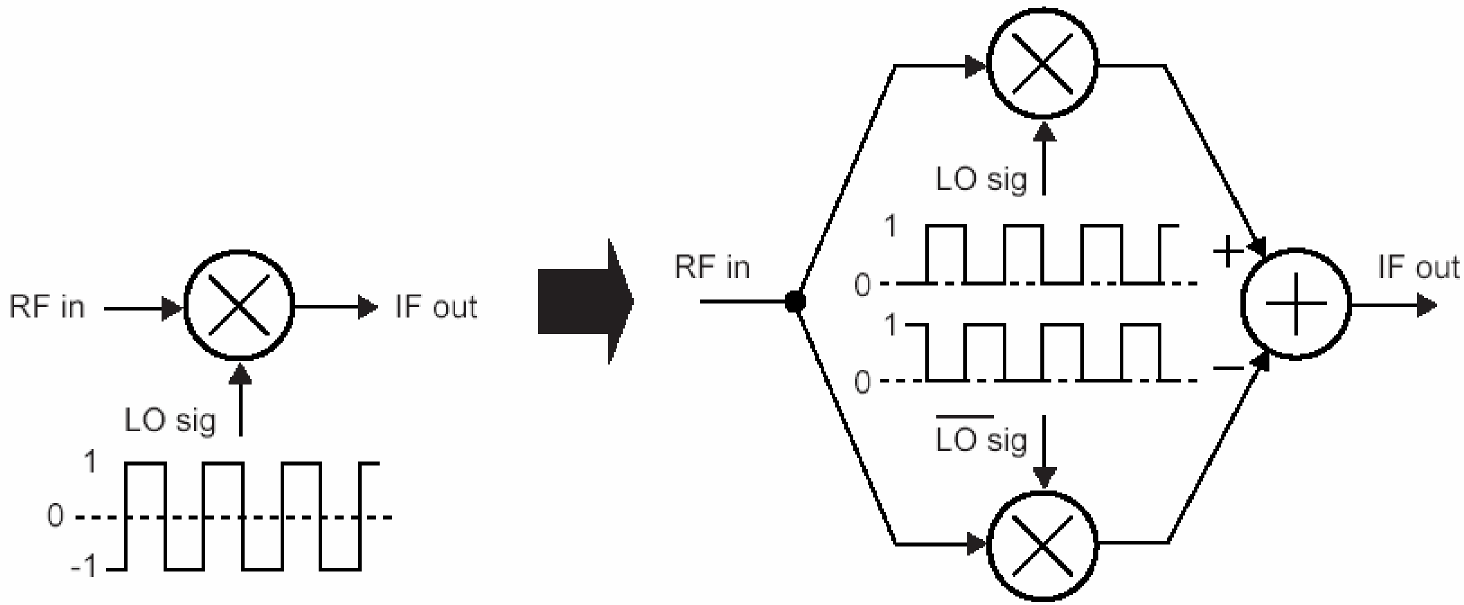
The Issue of Balance in Mixers



- A balanced signal is defined to have a zero DC component
- Mixers have two signals of concern with respect to this issue – LO and RF signals
  - Unbalanced RF input causes LO feedthrough
  - Unbalanced LO signal causes RF feedthrough
- Issue – transistors require a DC bias

Achieving Balanced LO Signal with DC Baising

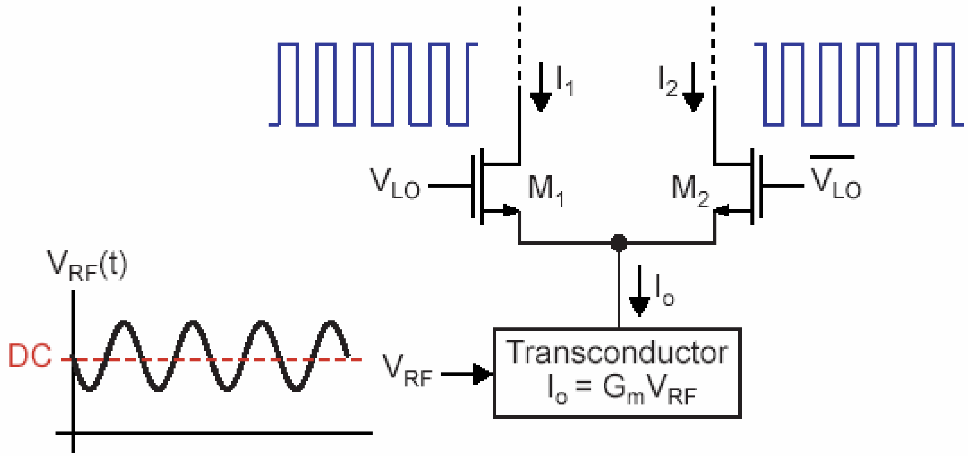
- Combine two mixer paths with LO signal 180 degrees out of phase between the paths



$$RF(t) \times [LO(t) - \overline{LO(t)}] = RF(t) \times LO(t) - RF(t) \times \overline{LO(t)}$$

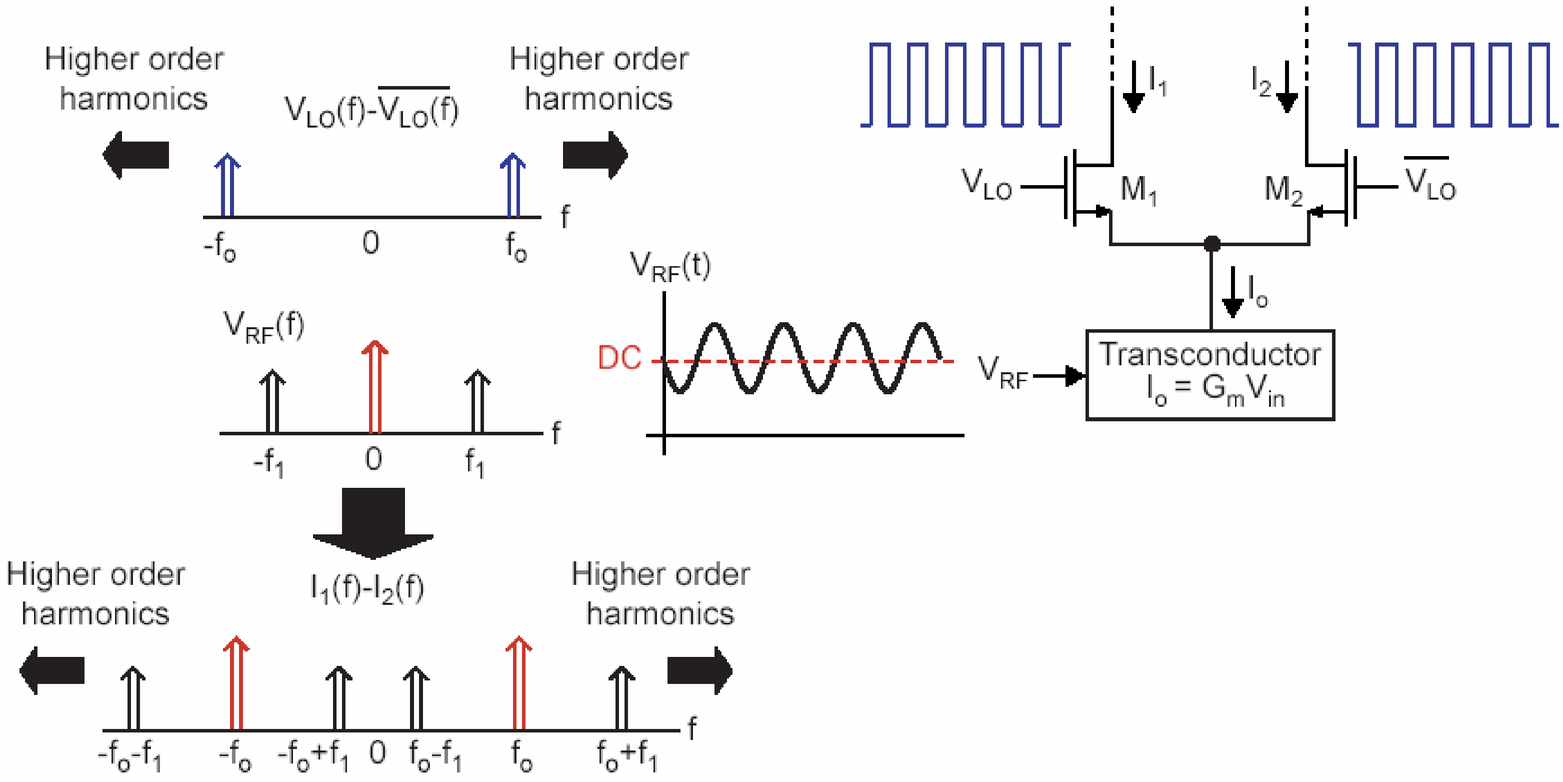
  - DC component is cancelled

Single-Balanced Mixer



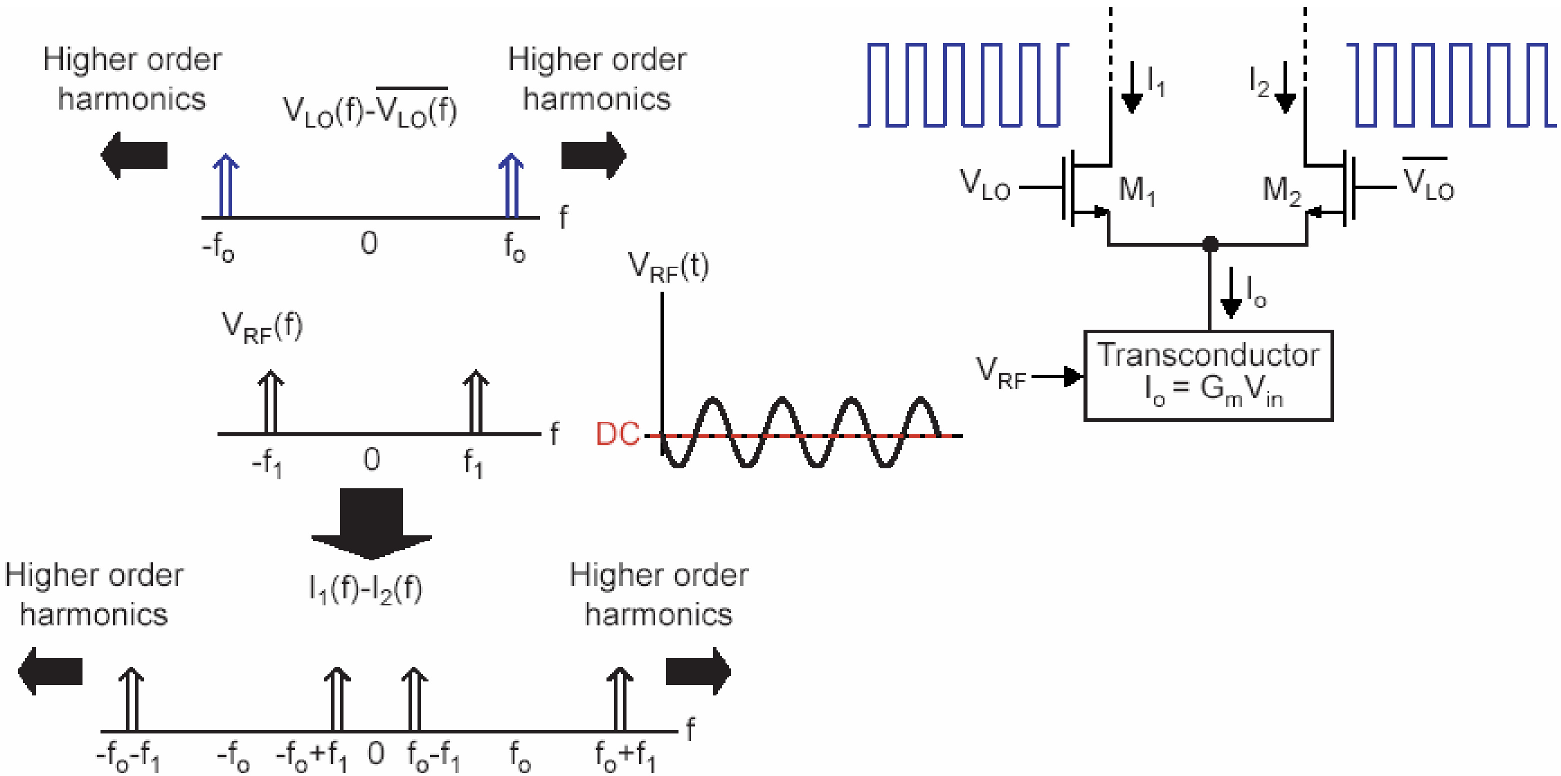
- Works by converting RF input voltage to a current, then switching current between each side of differential pair
- Achieves LO balance using technique on previous slide
  - Subtraction between paths is inherent to differential output
- LO swing should be no larger than needed to fully turn on and off differential pair
  - Square wave is best to minimize noise from M_1 and M_2
- Transconductor designed for high linearity

LO Feedthrough in Single-Balanced Mixers



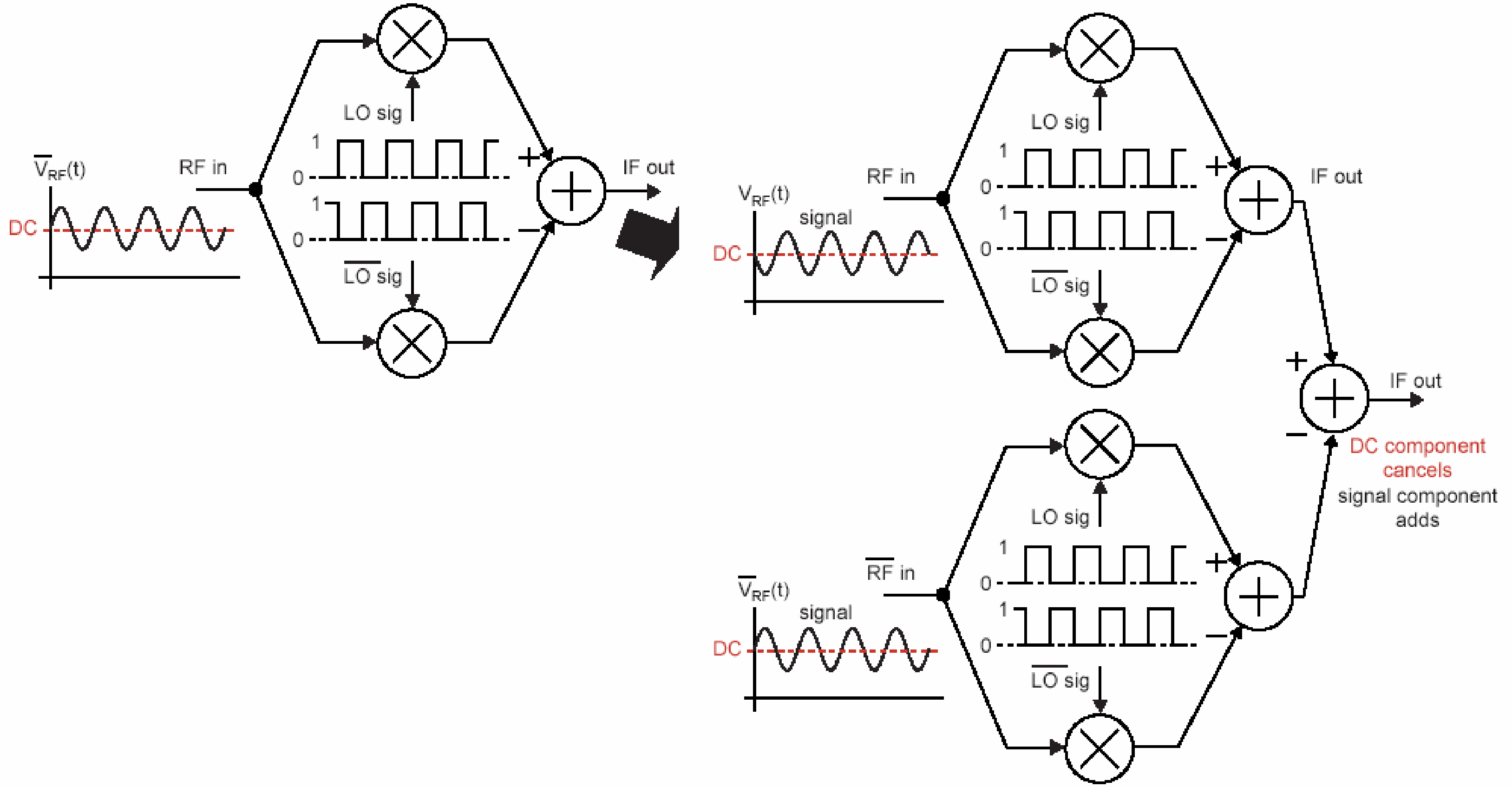
- DC component of RF input causes very large LO feedthrough
  - Can be removed by filtering, but can also be removed by achieving a zero DC value for RF input

Ideal Double-Balanced Mixer



- DC values of LO and RF signals are zero (balanced)
- LO feedthrough dramatically reduced!
- But, practical transconductor needs bias current

Achieving Balanced RF Signal with Biasing

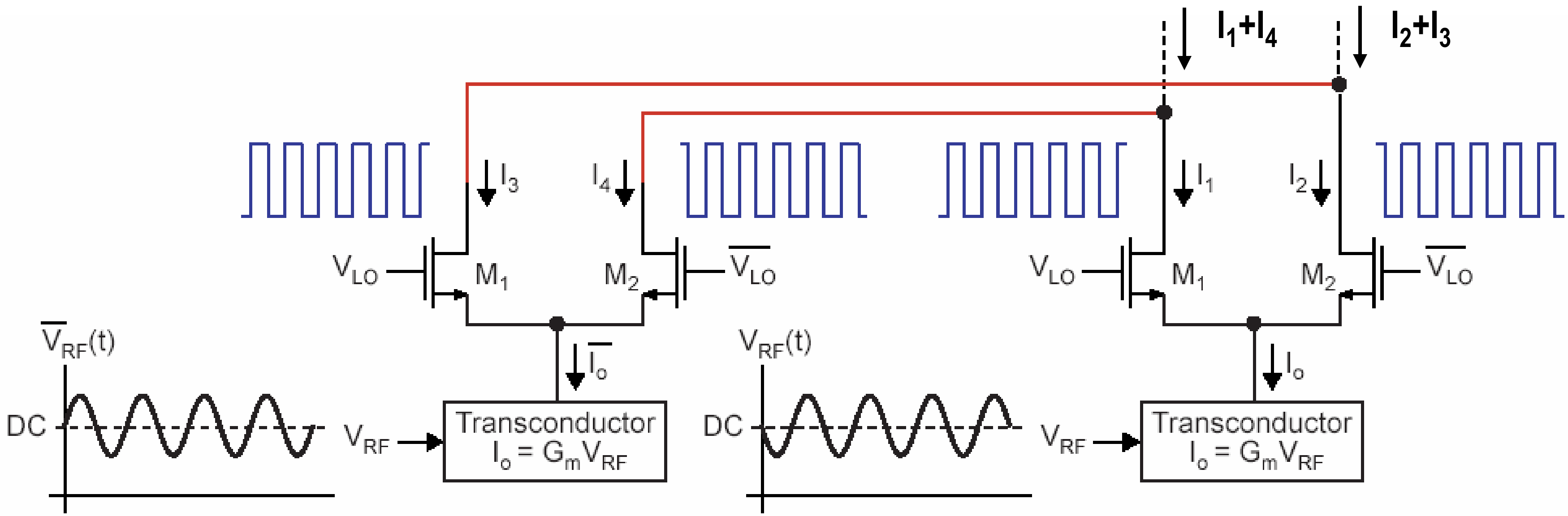


$$[RF(t) - \overline{RF(t)}] \times [LO(t) - \overline{LO(t)}]$$

$$\Rightarrow \quad RF(t) \times [LO(t) - \overline{LO(t)}] = RF(t) \times LO(t) - RF(t) \times \overline{LO(t)}$$

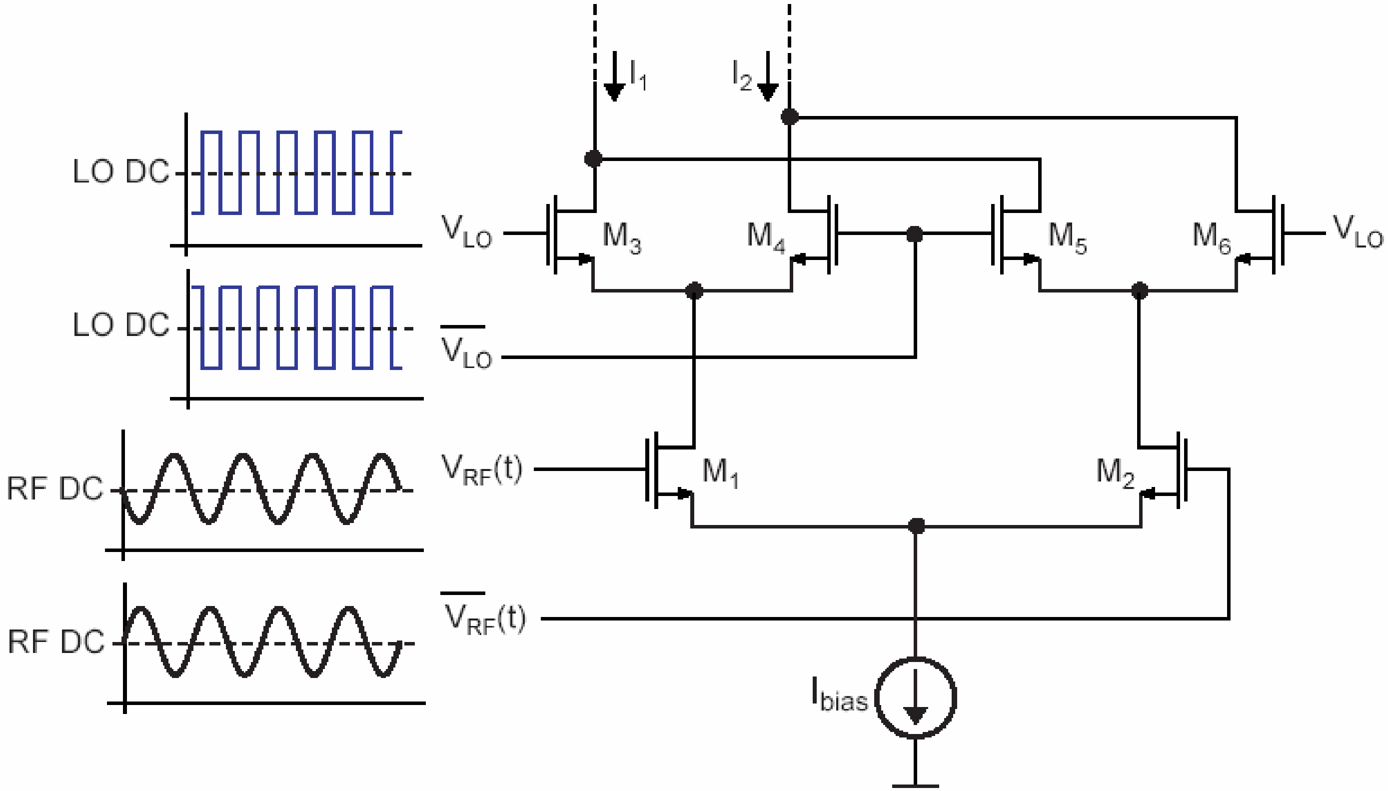
$$\Rightarrow \quad \overline{RF(t)} \times [LO(t) - \overline{LO(t)}] = \overline{RF(t)} \times LO(t) - \overline{RF(t)} \times \overline{LO(t)}$$

Double-Balanced Mixer Implementation



- Applies technique from previous slide
  - Subtraction at the output achieved by cross-coupling the output current of each stage

Gilbert Cell (Four Quadrant) Mixer

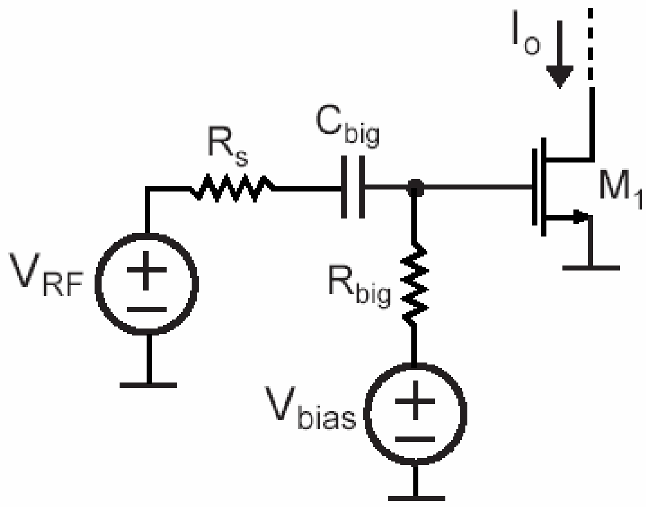


- Use a differential pair to achieve the transconductor implementation
- This is the preferred mixer implementation for most radio systems!

Mixer Voltage Conversion Gain

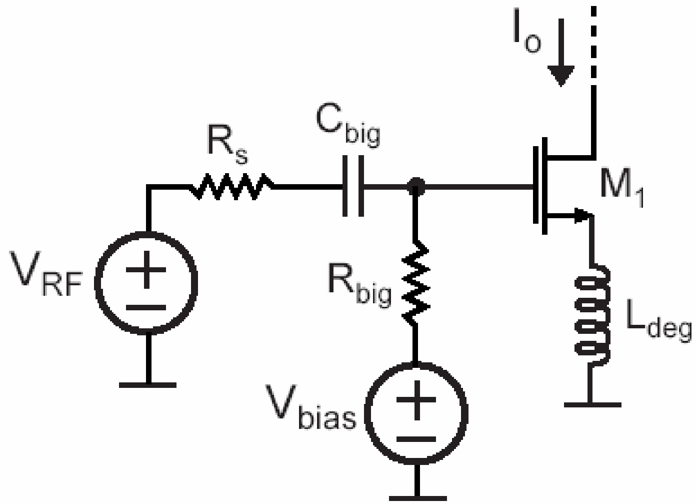
- Voltage conversion gain of a mixer depends on several factors
  - Input transconductance
  - Multiplication factor
  - Load resistance

Common-Source Transconductance Stage in Mixer



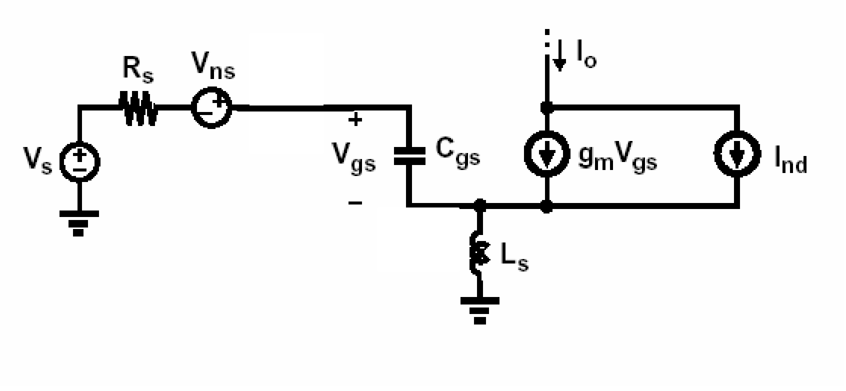
- Apply RF signal to input of common source amp
  - Transistor assumed to be in saturation
  - Transconductance value is the same as that of the transistor
- High V_{bias} places device in velocity saturation
  - Allows high linearity to be achieved

CS Transconductance Stage with Degeneration



- **Add degeneration to common source amplifier**
  - **Inductor better than resistor**
    - No DC voltage drop
    - Increased impedance at high frequencies helps filter out undesired high frequency components
  - **Don't generally resonate inductor with C_{gs}**
    - Power match usually not required for IC implementation due to proximity of LNA and mixer

Transconductor Stage in Mixer



$$g_{m\_Effective} = \frac{1/j?_0 C_{gs}}{Z_{in}\big|_{?_0}} g_m = G_m$$

$$= Q_{in} g_m$$

$$= \frac{g_m}{?_0 C_{gs}\left(R_s + \frac{g_m}{C_{gs}} L_s\right)}$$

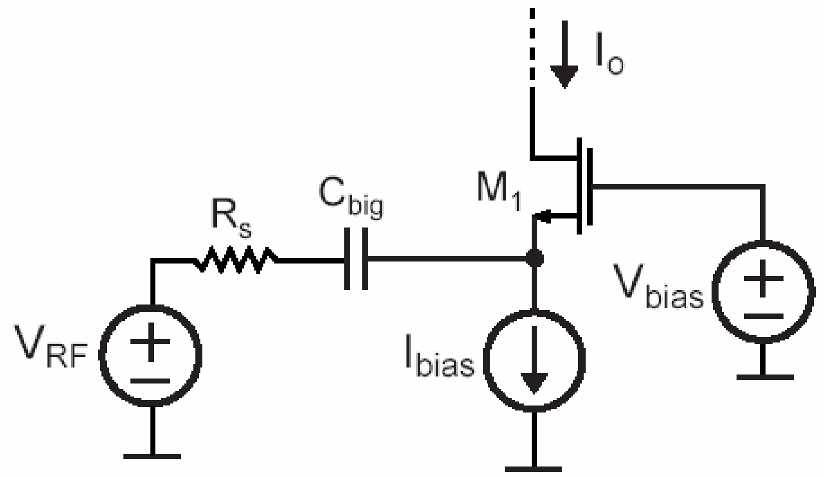
Note that : $Q_{in} = \frac{1}{?_0 C_{gs}\left(R_s + \frac{g_m}{C_{gs}} L_s\right)} = \frac{?_0 L_s}{R_s + \frac{g_m}{C_{gs}} L_s}$

$$= \frac{g_m}{?_0 C_{gs}(R_s + ?_T L_s)}$$

$$» \frac{1}{?_0 L_s}$$

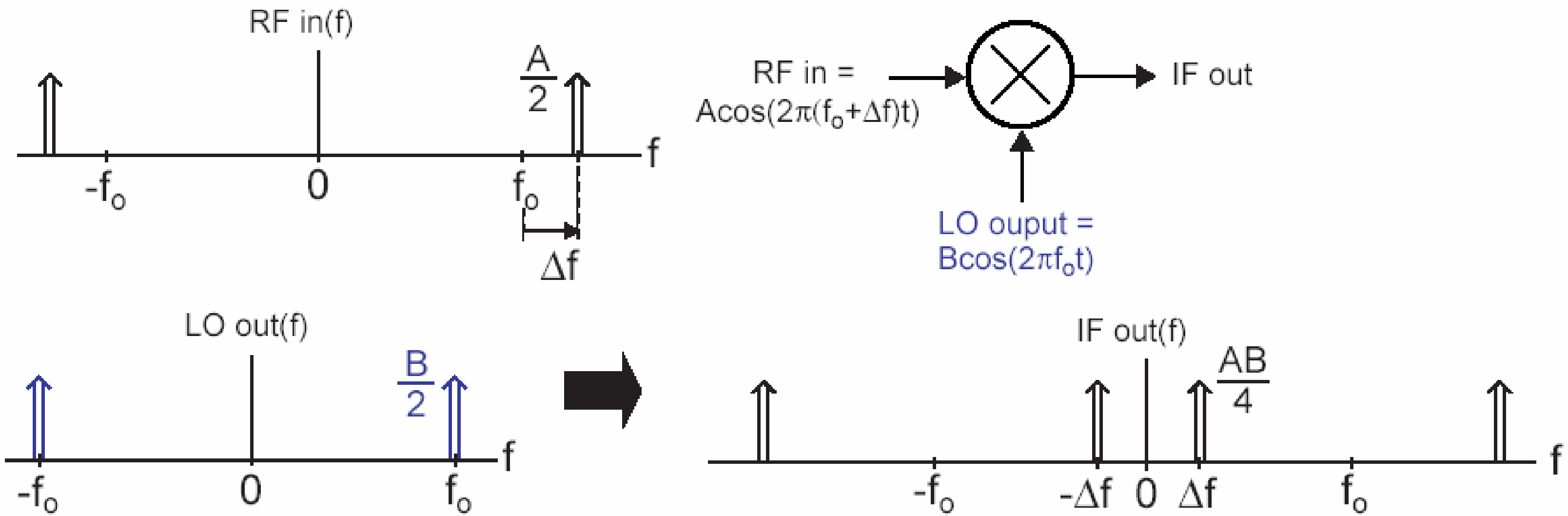
For $?_T L_s >> R_s$, *highly linearly transconductance, only depends* L_s

Common-Gate Transconductance Stage in Mixer



- Apply RF signal to a common gate amplifier
- Transconductance value set by inverse of series combination of R_s and $1/g_m$ of transistor
  - Amplifier is effectively degenerated to achieve higher linearity
- I_{bias} can be set for large current density through device to achieve higher linearity (velocity saturation)

Mixer Multiplication Factor



- **Defined as voltage ratio of desired IF value to RF input**
- **Example: for an ideal mixer with RF input = Asin(2π(f$_o$ + Δ f)t) and sine wave LO signal = Bcos(2πf$_o$t)**

$$IF\ out(t) = \frac{AB}{2}\left(\cos(2\pi(\Delta f)t) + \cos(2\pi(2f_o + \Delta f)t)\right)$$

$$\Rightarrow \text{Voltage Conversion Gain} = \frac{AB/2}{A} = \frac{B}{2}$$

Mixer Voltage Conversion Gain

- If the *sinusoidal* LO swing is sufficiently large to completely switch the current, we can approximate the LO by a square wave
- Consider only the fundamental term in LO

$$i_{out} = i_{RF}\cos(\omega_{RF} t) \times \frac{4}{\pi}\cos(\omega_{LO} t)$$

$$= \frac{1}{2}\{\frac{4}{\pi} i_{RF} \cos[(\omega_{RF} - \omega_{LO})t] + \frac{4}{\pi} i_{RF} \cos[(\omega_{RF} + \omega_{LO})t]\}$$

- After the low-pass filter,

$$i_{out} = \frac{2}{\pi} i_{RF} \cos[(\omega_{RF} - \omega_{LO})t]$$

$$= \frac{2}{\pi} g_{m\_eff} v_{RF} \cos[(\omega_{RF} - \omega_{LO})t]$$

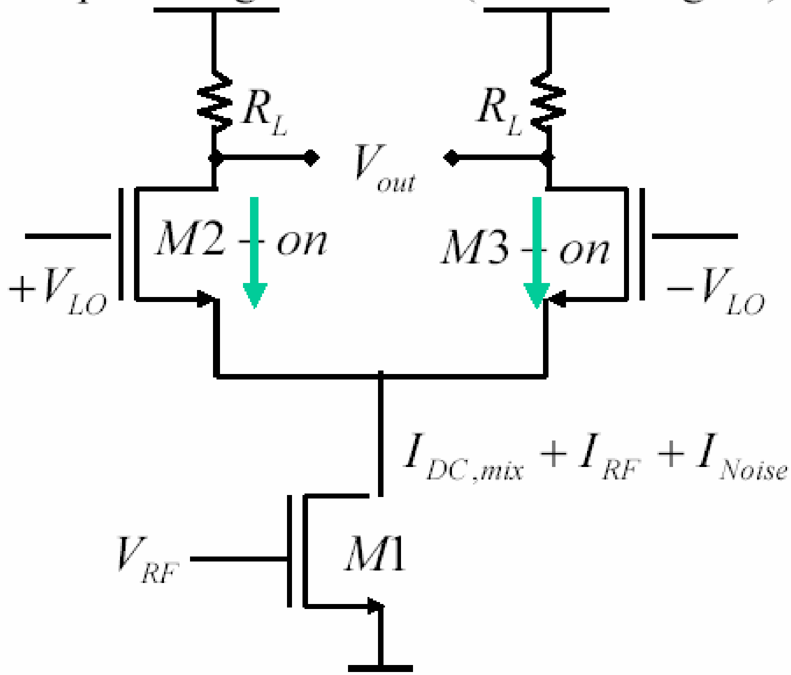
$$\Rightarrow Gain = i_{out} R_{out} = \frac{2}{\pi} g_{m\_eff} R_{out}$$

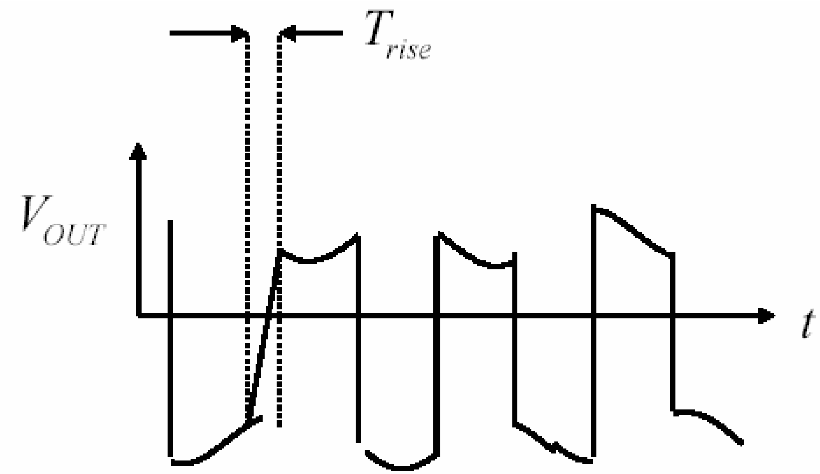
Mixer Noise Analysis

- Three contributors to mixer noise
  - Transconductance stage
  - Switching pairs
  - Load resistance

Design Consideration for Minimizing Mixer NF

- Optimizing the mixer (for noise figure):



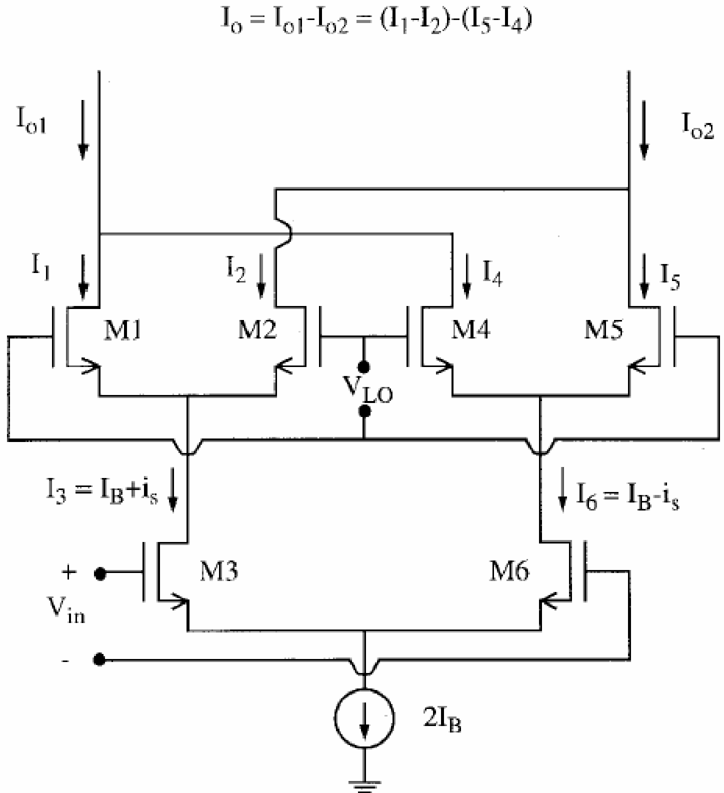


$$g_m \propto \sqrt{W} \ldots fixed - I_{DC}$$

$$\omega_T \propto \frac{1}{\sqrt{W}} \ldots fixed - I_{DC}$$

- ❑ **Design the transducer for minimum noise figure**
- ❑ **Noise from M2 and M3 can be minimized through fast switching of M2 & M3 by**
  - **making LO amplitude large to ensure complete (> 90%) current commuting**
  - **making M2 and M3 as small as possible (i.e. increasing fTof M2 and M3)**

NF Expression for Double-Balanced Mixer [1]



$$(\mathrm{NF})_{\mathrm{SSB}} = \frac{\alpha}{c^2} + \frac{2(\gamma_3 + r_{g3} g_{m3}) g_{m3}\alpha + 4\gamma_1 \overline{G} + 4 r_{g1}\overline{G^2} + \frac{1}{R_L}}{c^2 g_{m3}^2 R_s} \quad (44)$$

where

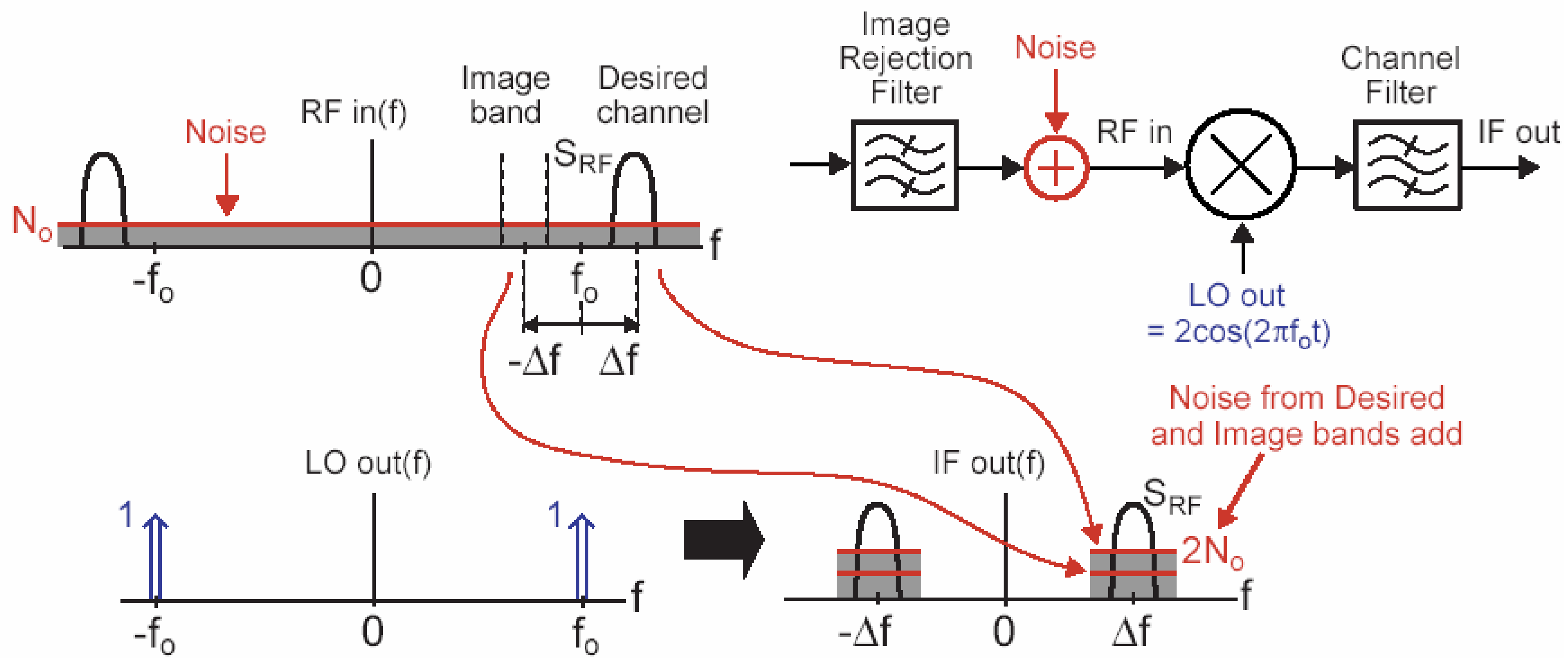
$$c \cong \frac{2}{\pi}\left(\frac{\sin(\pi\Delta f_{\mathrm{LO}})}{\pi\Delta f_{\mathrm{LO}}}\right) \quad (18)$$

$$\alpha \cong 1 - \tfrac{4}{3}(\Delta f_{\mathrm{LO}}) \quad (25)$$

$$\overline{G} = \frac{1}{\pi V_o}\int_{-V_x}^{V_x}\left(\frac{dI_{o1}}{dV_{\mathrm{LO}}}\right) dV_{\mathrm{LO}} = \frac{2I_B}{\pi V_o}. \quad (33)$$

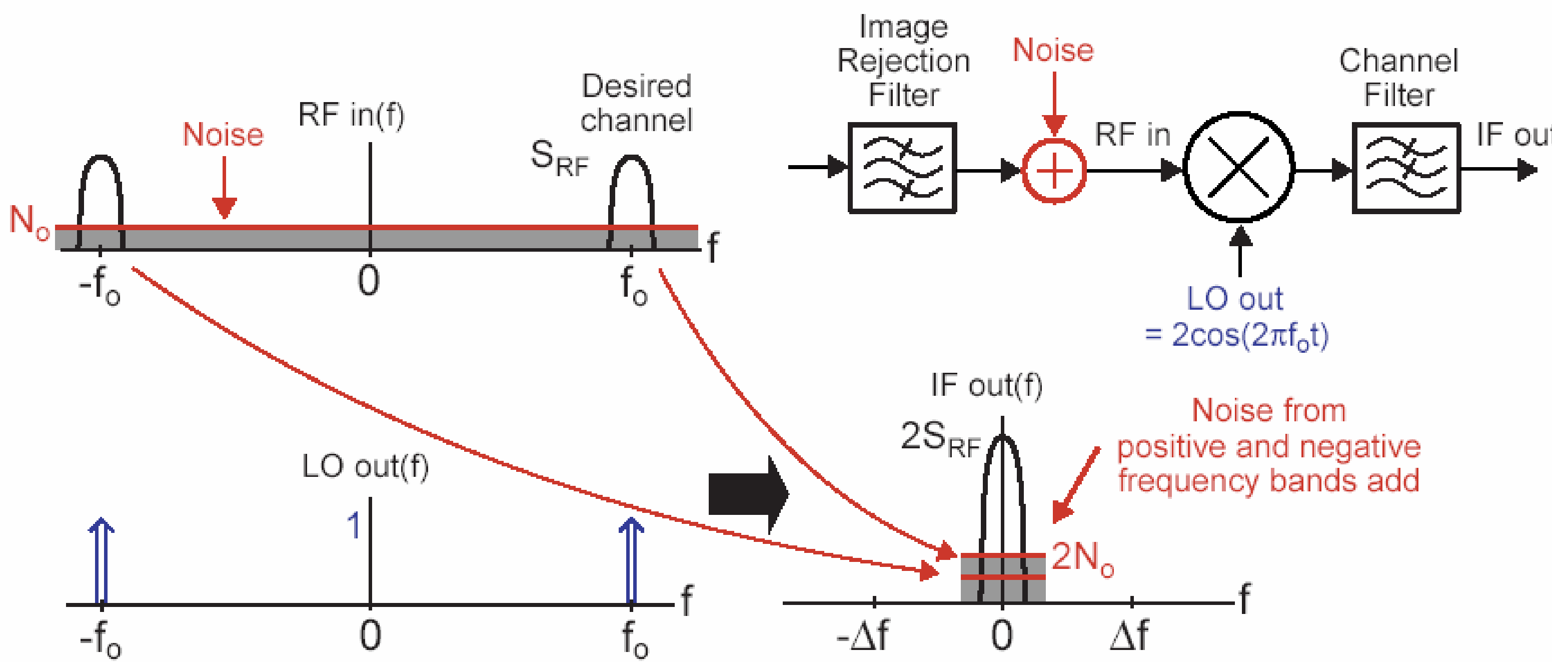
$$\overline{G^2} \approx 16\left(\frac{\ln(\sqrt{2}+1)}{\sqrt{2}} - \frac{1}{3}\right)\cdot\frac{K_1^{1/2} I_B^{3/2}}{\lambda T_{\mathrm{LO}}} = 4.64\cdot\frac{K_1^{1/2} I_B^{3/2}}{\lambda T_{\mathrm{LO}}} \quad (40)$$

Mixer NF for Single-Sideband Systems



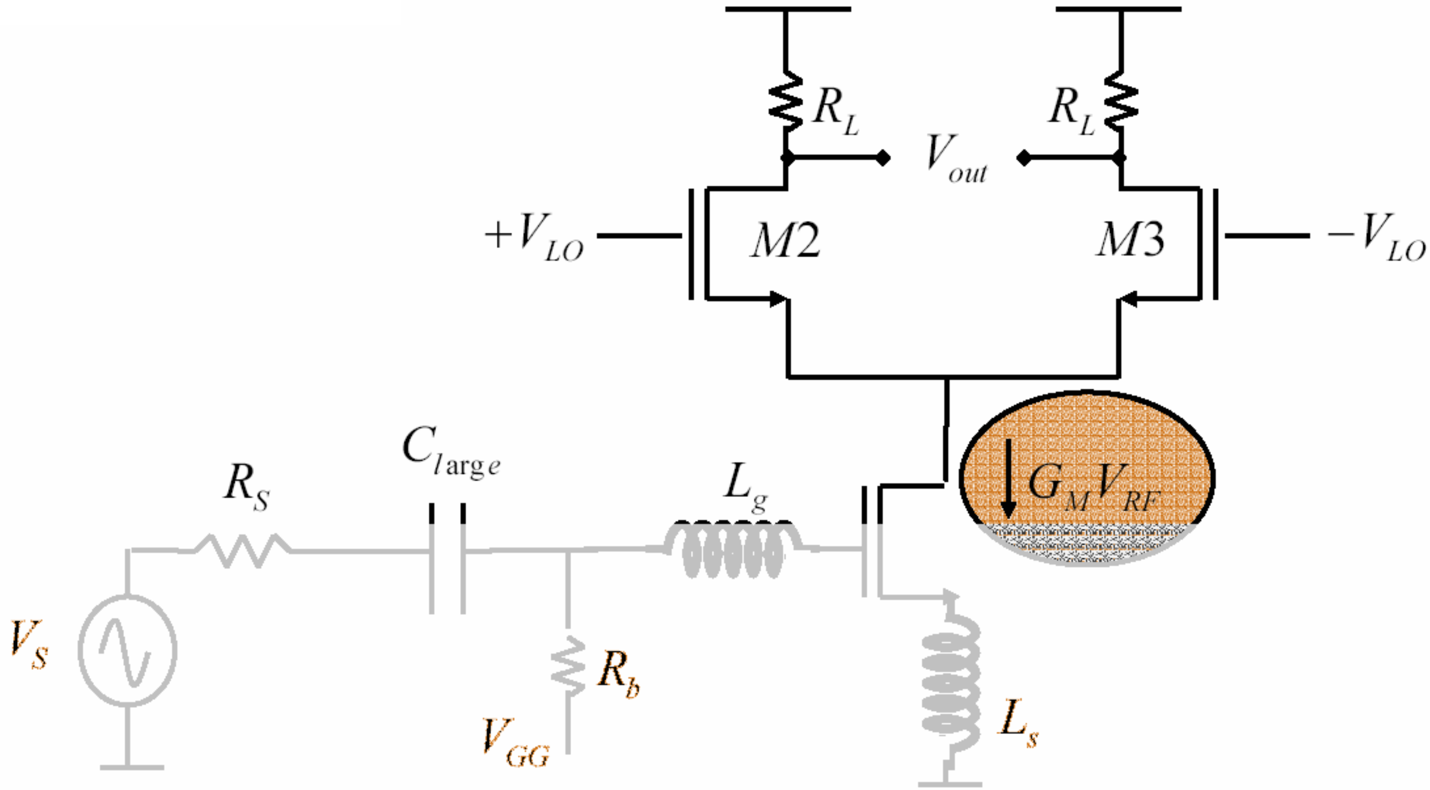
- **Issue – broadband noise from mixer or front end filter will be located in both image and desired bands**
  - **Noise from both image and desired bands will combine in desired channel at IF output**
    - Channel filter cannot remove this
  - **Mixers are inherently noisy!**

Mixer NF Double Sideband Systems



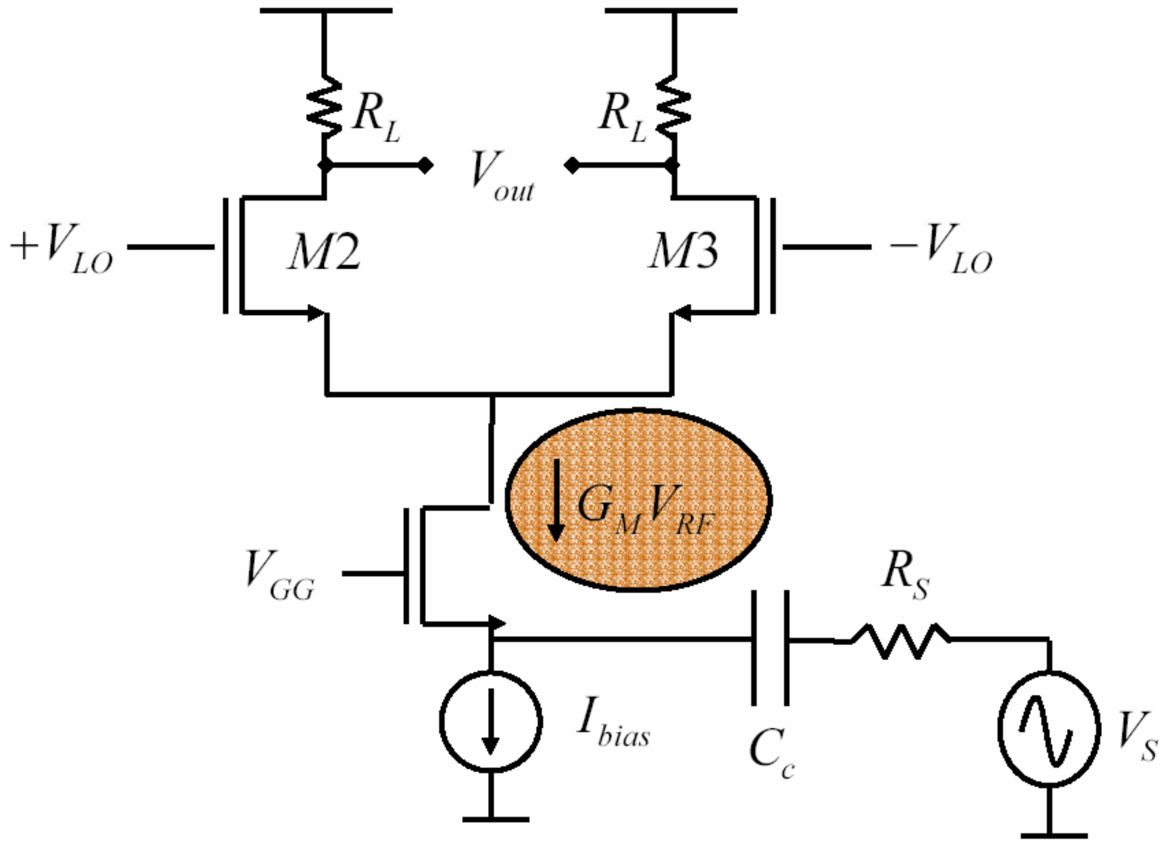
- For zero IF, there is no image band
  - Noise from positive and negative frequencies combine, but the signals do as well
- DSB noise figure is 3 dB lower than SSB noise figure
  - DSB noise figure often quoted since it sounds better
- For either case, Noise Figure computed through simulation

Design Consideration for Mixer Linearity



- Linearity of the Mixer primarily depends on the linearity of the transducer (I_tail=Gm*V_rf). Inductor Ls helps improve linearity of the transducer.
- The transducer transistor M1 can be biased in the linear law region to improve the linearity of the Mixer. Unfortunately this results in increasing the noise figure of the mixer (as discussed in LNA design).

Design Consideration for Mixer Linearity



- Using the common gate or common base stage as the transducer improves the linearity of the mixer. Unfortunately the approach reduces the gain and increases the noise figure of the mixer.

Measured IIP3 for a 0.8-mm SB Mixer [2]

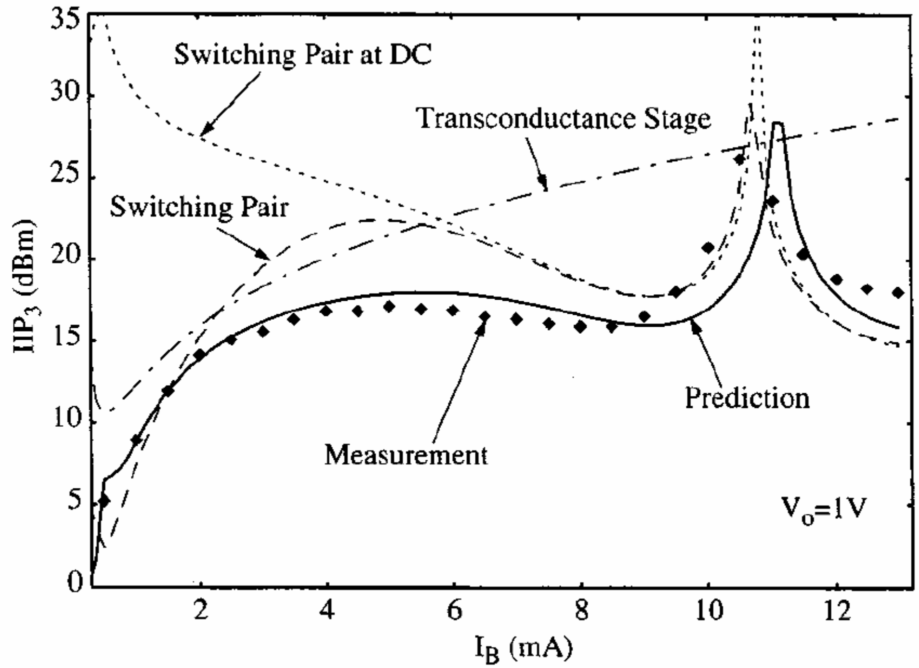
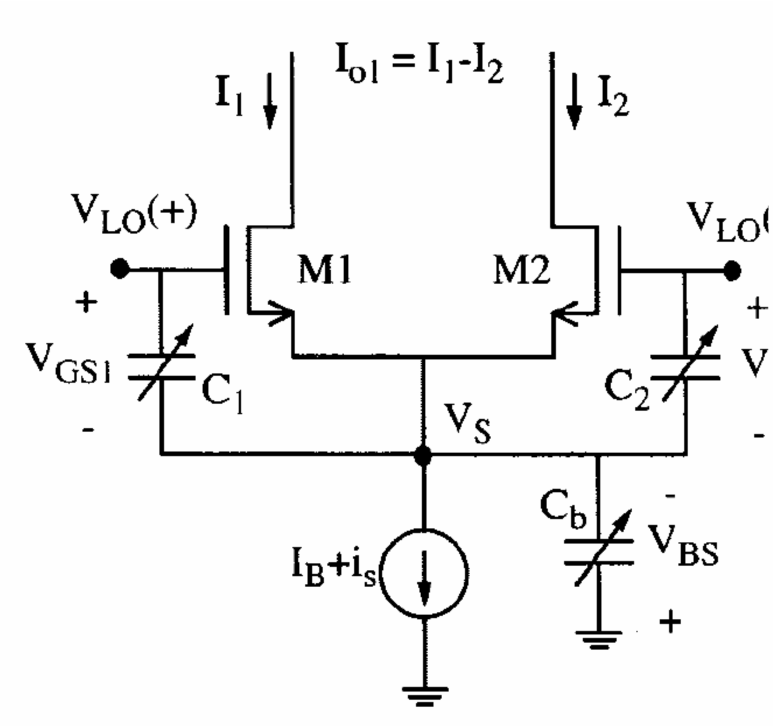


Fig. 20. Intermodulation measurements versus bias current for a fixed LO amplitude $V_o = 1$ V.

- At high bias current, the switching pair nonlinearity dominates
- At low bias current, the transconductance stage nonlinearity dominates
  - For short channel devices, the transconductance stage nonlinearity dominates
  - IIP3 is proportional to $(V_{RF\_DC} - V_{th})$

Measured IIP3 for a 0.8-mm SB Mixer [2]



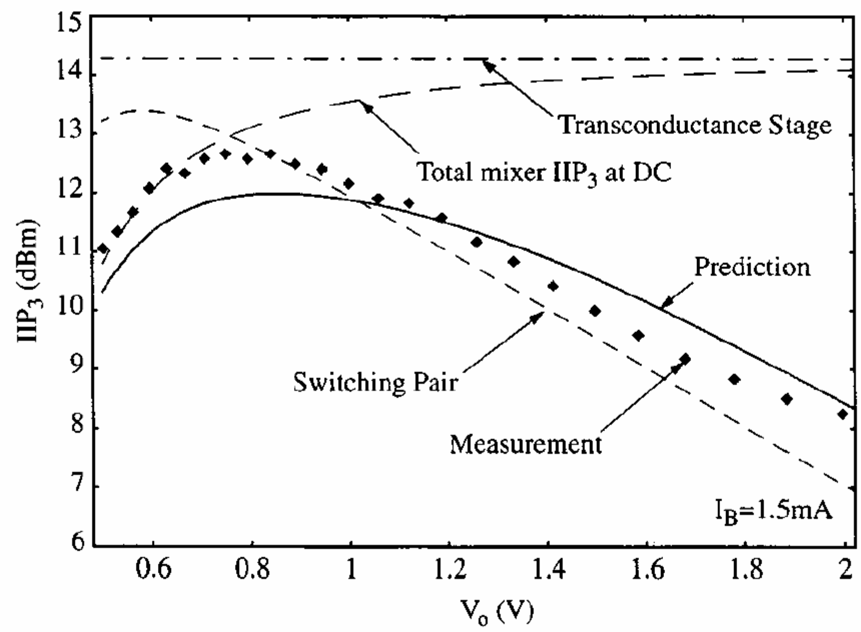
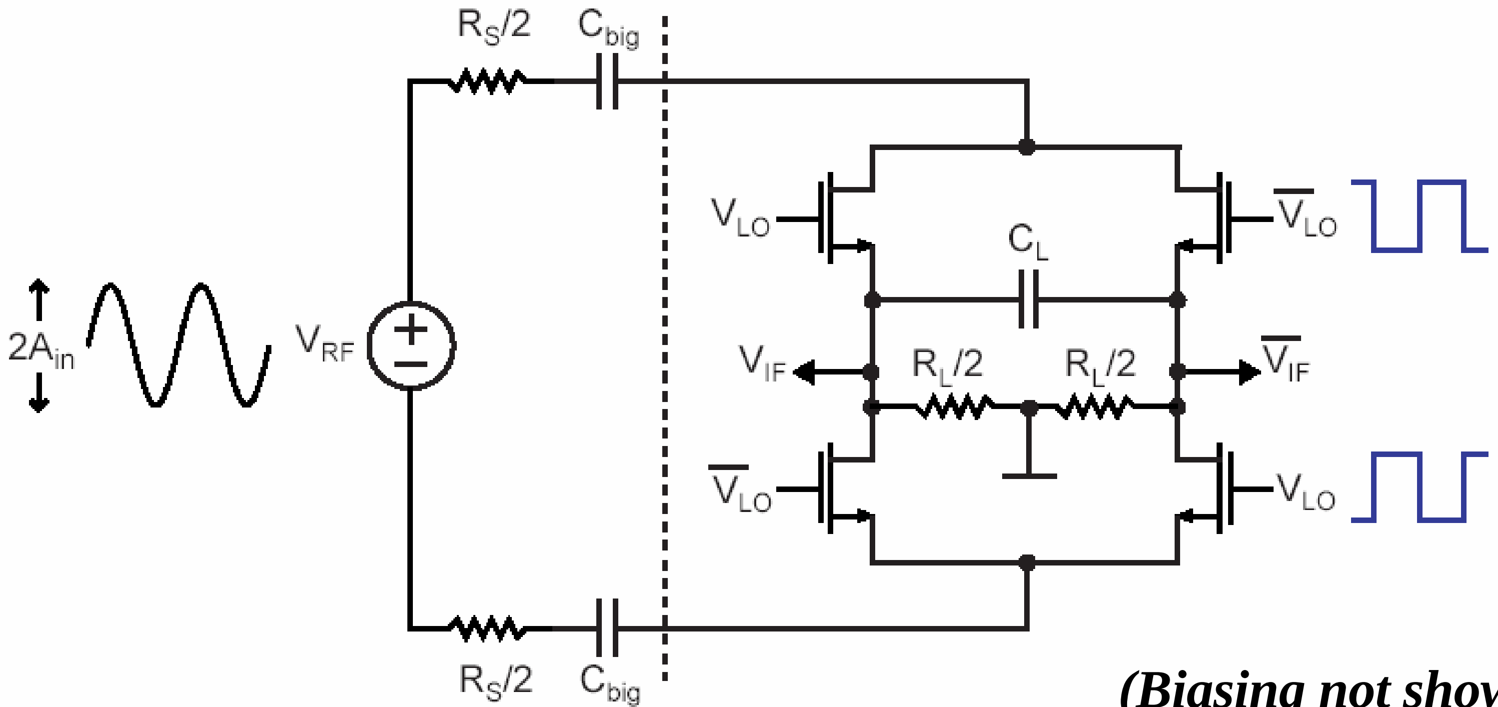


Fig. 21. Intermodulation measurements versus LO amplitude for a fixed bias current $I_B = 1.5$ mA.

- At high frequencies, excessively large LO amplitude degrades IIP3 due to parasitic capacitive coupling which is nonlinear
- For low-voltage design (< 2V), this is usually not a big concern

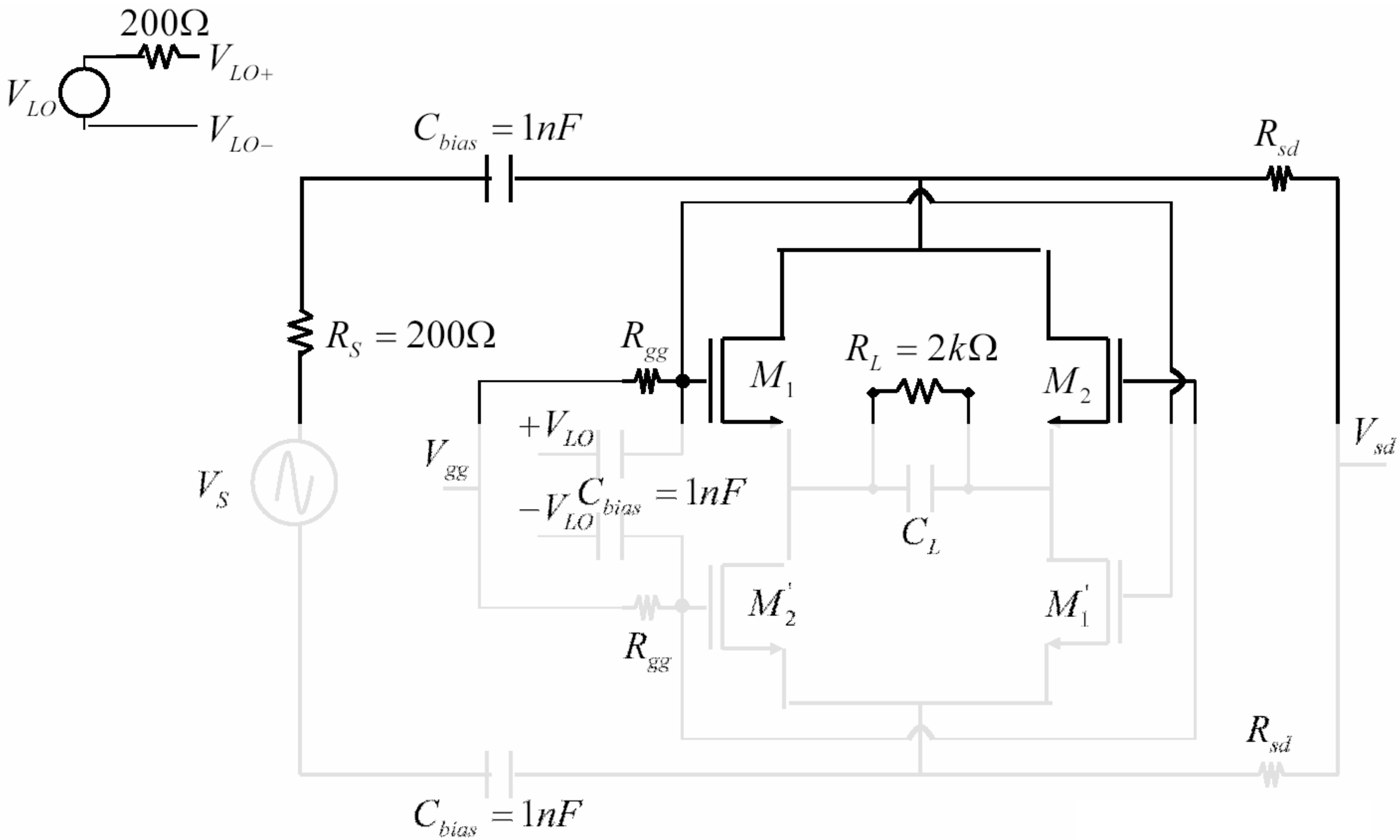
Passive Mixers



(Biasing not shown)

- Very high linearity (assuming the current are completely commuted)
  - 20–30 dBm of IIP3 achievable
- High noise figure (noise due to the the switching devices)
  - 20–30 dB of NF
- Voltage conversion loss

Passive Mixers with Biasing Shown



References

1. M. T. Terrovitis and R. G. Meyer, “Noise in Current-Commuting CMOS Mixers” *IEEE Journal of Solid-State Circuits*, Vol. 34, No. 6, June 1999.
2. M. T. Terrovitis and R. G. Meyer, “Intermodulation Distortion in Current-Commutating CMOS Mixers,” *IEEE Journal of Solid-State Circuits*, Vol. 35, No. 10, October 2000.
3. Prof. M. Perrott, MIT http://ocw.mit.edu/OcwWeb/Electrical-Engineering-and-Computer-Science/6-776Spring-2005/CourseHome/index.htm
4. Prof. L. Larson, UC San Diego ECE 265A and 265B lecture notes