IS there an easier way??

Henceforth $K \rightarrow \alpha$ (redefine spring constant)

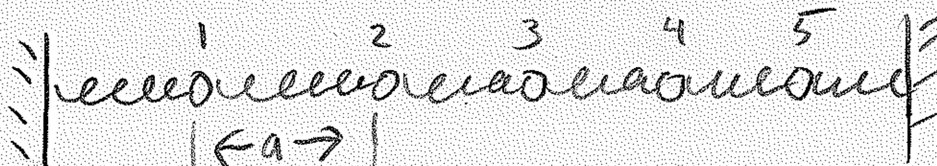
$k \equiv 2\pi/\lambda$, where λ is the wavelength of the desired normal mode.

Ansatz #2: for any periodic system we claim that a given mass n will oscillate as:

$$\boxed{x_n(t) = A \sin(\omega_k t) \sin(kna)}$$

$\hookrightarrow$ independent of n !

Example: find the natural frequencies of 5 coupled masses.



What is the longest wavelength possible?

$$\lambda = 12a \implies k = 2\pi/12a = \pi/6a$$

Step ① coupled ODEs

$$m\ddot{x}_1 = -\alpha x_1 - \alpha(x_1 - x_2)$$
$$m\ddot{x}_2 = -\alpha(x_2 - x_1) - \alpha(x_2 - x_3)$$
$$m\ddot{x}_3 = -\alpha(x_3 - x_2) - \alpha(x_3 - x_4)$$
$$m\ddot{x}_4 = \ldots$$
$$m\ddot{x}_5 = \ldots$$

step ② $K = \pi/6a$, $x_n(t) = A\sin(Kna)\sin(\omega_K t)$

$K_1 = \pi/6a$, $K_2 = \pi/3a$, $K_3 = \pi/2a$, $K_4 = \frac{2\pi}{3a}$, $K_5 = \frac{5\pi}{6a}$

$n=1$ mass $\Rightarrow x_1(t) = \frac{A}{2}\sin\omega_K t$

$n=2$ $x_2(t) = \frac{\sqrt{3}}{2}A\sin\omega_K t$

$n=3$ $x_3(t) = A\sin\omega_K t$

$n=4$ $x_4(t) = \frac{\sqrt{3}}{2}A\sin\omega_K t$

$n=5$ $x_5(t) = \frac{A}{2}\sin\omega_K t$

find ω_K by subbing $x_1(t)$ into 1st ODE:

i.e. $-m\omega_K^2 \frac{A}{2} = -\frac{A}{2}\alpha + A\left(\frac{\sqrt{3}}{2}\right)\alpha - \frac{A}{2}\alpha$

$-m\omega_K^2 = \alpha(\sqrt{3}-2)$

or $\boxed{\omega_K = \sqrt{\alpha/m}\,\sqrt{2-\sqrt{3}}}$ natural frequency of the 1st normal mode

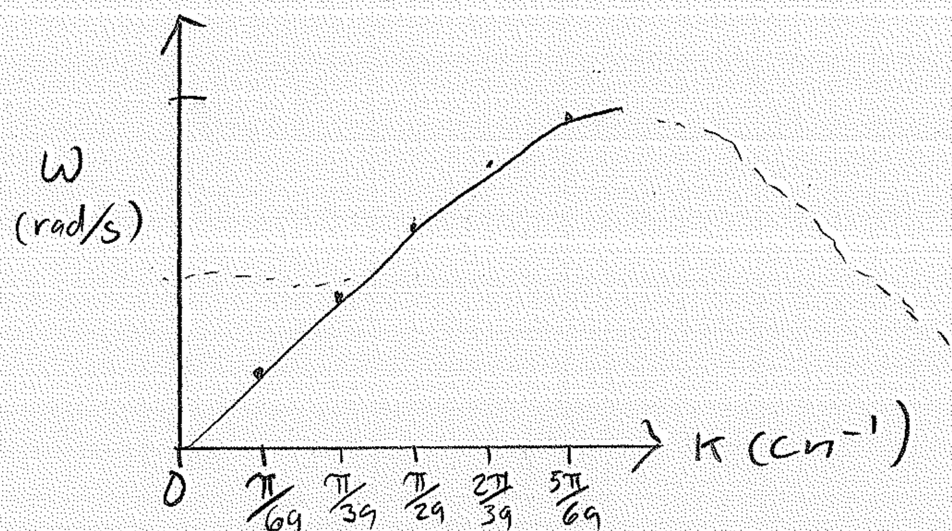
What is the frequency of the second normal mode??

First question: what is λ of the 2nd normal mode?

$\lambda = 6a \Rightarrow K = \frac{2\pi}{6a} = \pi/3a$

PhET Lab:

Part 3: goal graph ω (rad/s) vs. k (cm^{-1})



$\omega \to 2\sqrt{\alpha/m}$ as $k \to \pi/a$

Now try 10 masses on your own.

Why do we converge to $2\sqrt{\alpha/m}$?

- the masses ultimately "see" just two time constants that each contribute a $\sqrt{\alpha/m}$ natural frequency component

i.e. if the spring-masses had a frequency faster than $2\sqrt{\alpha/m}$, the springs would have to break