Spin 1/2 Reference Sheet

$S_z|+\rangle = +\frac{\hbar}{2}|+\rangle$

$S_z|-\rangle = -\frac{\hbar}{2}|-\rangle$

$S_x|+\rangle_x = +\frac{\hbar}{2}|+\rangle_x$

$S_x|-\rangle_x = -\frac{\hbar}{2}|-\rangle_x$

$S_y|+\rangle_y = +\frac{\hbar}{2}|+\rangle_y$

$S_y|-\rangle_y = -\frac{\hbar}{2}|-\rangle_y$

$\langle+|+\rangle = \langle-|-\rangle = 1$

$\langle+|-\rangle = \langle-|+\rangle^* = 0$

$|+\rangle_x = \frac{1}{\sqrt{2}}\left[|+\rangle + |-\rangle\right]$

$|-\rangle_x = \frac{1}{\sqrt{2}}\left[|+\rangle - |-\rangle\right]$

$|+\rangle_y = \frac{1}{\sqrt{2}}\left[|+\rangle + i|-\rangle\right]$

$|-\rangle_y = \frac{1}{\sqrt{2}}\left[|+\rangle - i|-\rangle\right]$

$\hat{\mathbf{n}} = \hat{\mathbf{i}}\sin\theta\cos\phi + \hat{\mathbf{j}}\sin\theta\sin\phi + \hat{\mathbf{k}}\cos\theta$

$S_n = S_x\sin\theta\cos\phi + S_y\sin\theta\sin\phi + S_z\cos\theta$

$|+\rangle_n = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}e^{i\phi}|-\rangle$

$|-\rangle_n = \sin\frac{\theta}{2}|+\rangle - \cos\frac{\theta}{2}e^{i\phi}|-\rangle$

Using the S_z basis: ($\doteq$ means "is represented by")

$S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $S_y \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$S_n \doteq \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}$

$|+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Pauli Matrices:

$$\vec{\mathbf{S}} \doteq \frac{\hbar}{2}\vec{\sigma}$$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\sigma_x|\pm\rangle_x = \pm|\pm\rangle_x$ $\sigma_y|\pm\rangle_y = \pm|\pm\rangle_y$ $\sigma_z|\pm\rangle = \pm|\pm\rangle$

 updated 12/26/18

Spin 1 Reference Sheet

$S_z|1\rangle = \hbar|1\rangle \qquad S_z|0\rangle = 0\hbar \qquad S_z|-1\rangle = -\hbar|-1\rangle$

$$|1\rangle_x = \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle \qquad |1\rangle_y = \frac{1}{2}|1\rangle + \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$$

$$|0\rangle_x = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|-1\rangle \qquad |0\rangle_y = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|-1\rangle$$

$$|-1\rangle_x = \frac{1}{2}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle \qquad |-1\rangle_y = \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$$

$$|1\rangle_n = \frac{1+\cos\theta}{2}e^{-i\phi}|1\rangle + \frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1-\cos\theta}{2}e^{i\phi}|-1\rangle$$

$$|0\rangle_n = -\frac{\sin\theta}{\sqrt{2}}e^{-i\phi}|1\rangle + \cos\theta|0\rangle + \frac{\sin\theta}{\sqrt{2}}e^{i\phi}|-1\rangle$$

$$|-1\rangle_n = \frac{1-\cos\theta}{2}e^{-i\phi}|1\rangle - \frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1+\cos\theta}{2}e^{i\phi}|-1\rangle$$

Using the S_z basis: ($\doteq$ means "is represented by")

$$S_z \doteq \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad S_x \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$S_n \doteq \hbar\begin{pmatrix} \cos\theta & \frac{\sin\theta}{\sqrt{2}}e^{-i\phi} & 0 \\ \frac{\sin\theta}{\sqrt{2}}e^{i\phi} & 0 & \frac{\sin\theta}{\sqrt{2}}e^{-i\phi} \\ 0 & \frac{\sin\theta}{\sqrt{2}}e^{i\phi} & -\cos\theta \end{pmatrix}$$

$$|1\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad |0\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad |-1\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Infinite Square Well Reference Sheet

For an infinite square well from 0 to L.

$$\hat{H}\varphi_n(x) = \frac{n^2\pi^2\hbar^2}{2mL^2}\varphi_n(x)$$

$$\varphi_n(x) = \sqrt{\tfrac{2}{L}}\sin\left(\tfrac{n\pi x}{L}\right)$$

Using the position representation:

$$\hat{x} \doteq x$$

$$\hat{p}_x \doteq -i\hbar\frac{\partial}{\partial x}$$