Electrostatic Potential for 2 Discrete Charges

Instructor Guide

Keywords: Upper-division, E and M, Electrostatic Potential, Symmetry, Discrete Charges

Brief overview of the activity

Students work in small groups to create power series expansions for the electrostatic potential due to two electric charges separated by a distance D.

This activity brings together student understanding of:

1. Electrostatic potential
2. The physical and geometric meaning of $\frac{1}{\vec{r}-\vec{r}'}$
3. Superposition
4. Power series expansion

Student prerequisite skills

Before starting this activity, through traditional lecture or the optional activities linked below, students need to acquire understandings of:

1. Electrostatic potential, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r}-\vec{r}'|}$. `Link to electrostatic potential activity`.
2. The physical and geometric meaning of $\frac{1}{\vec{r}-\vec{r}'}$. `Link to position vector activity`.
3. Superposition, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r}-\vec{r}_i|}$. `Link to superposition activity`.
4. Conceptual understanding of power series expansion and knowledge of the 4-10 most common power series formulas (or students should know where to find them in a reference book) including $(1+z)^p = 1 + pz + \frac{p(p-1)}{2!}z^2 + ...$ `Link to power series activities`.

Props

- Balls to represent point charges
- Voltmeter Coordinate system (e.g. with straws or Tinkertoys)
- Poster-sized whiteboards
- Markers
- Whiteboards around room. `Link to room set-up`.

The activity - Allow 50 minutes

Overview

Students should be given or have been reminded of the formula $V(\vec{\boldsymbol{r}}) = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}$ and be assigned to work in groups of three on the Electrostatic Potential - Discrete Charges Worksheet. This activity is designed for eight groups, but can be used with as few as two groups. If working with only two groups, have each group do two of the first four problems on the worksheet. If there are more groups do more examples or have each group just do one problem. Students do their work collectively with markers on a poster-sized sheet of whiteboard at their tables. `Link to worked solutions for power series expansions`.

What the students will be challenged by and how to facilitate their learning

1. Students are unlikely to start with the general case and work toward the specific as in Eq.2 and Eq. 3 in the solutions. Instead they are likely to treat this as a two-dimensional case from the start and ignore the z axis entirely and start with something like

$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} \tag{1}$$

   Although this is not problematic for obtaining a solution to this problem, it is problematic for visualizing a 3-dimensional field intersecting with a particular plane or axis. Frequently students are initially trying to find a formula they can plug things into to get an answer, or at least are trying to only see what is needed to obtain the required solution. By the end of the wrap-up and final whole class discussions, students should at least have considered the 3-dimensional case and be seeing their case as an example of a larger picture.

2. As an intermediate step, students will create an expression such as $V(x,y,z) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{D-x}+\frac{1}{D+x}\right)$. Each situation has a slightly different formula. The students will definitely spend several minutes thinking about this and working through it. However, because the coordinate system is set up for them, most students are successful with this part fairly quickly. Some may have trouble turning $|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|$ into rectangular coordinates or have problems with correct signs when applying the superposition principle. If students get stuck here, help should be given fairly quickly.

3. Students will take the equation from part 1 and develop a 4th order power series expansion. About 20 minutes will be needed for this portion of the activity. Almost all students will struggle with creating the power series (actually a Laurent series in some cases, but students do not need to be familiar with this concept before the beginning of the activity. The difference between power series and Laurent series emerges naturally in the wrap-up). Although our students have some experience with power series from mathematics courses, they have never before had the chance of employing the common physics strategy substituting into known series by rewriting an expression in terms of dimensionless parameters. Depending on the exact nature of prior instruction, students may encounter different challenges.

- Note: two of the eight cases on the worksheet are trivial (the potential on the $y$axis is zero for the $+Q$ and $-Q$ situation). Once these groups have established the correct answer and can justify it, they should be directed to work on one of the other six questions.

- If students have been exposed to Taylor's theorem $f(z) = f(a) + f'(a)(z-a) + f"(a)\frac{(z-a)^2}{2!} +$ and have not been told to use a known power series expansion, they will probably first attempt to apply this basic formula to this situation. This will rapidly lead to an algebraic mess. In general, we let students "get stuck" at this stage for about five minutes before suggesting that they try a known power series expansion. We don't tell them which one, but they rapidly rule out formulas for trigonometric functions and other functions that clearly don't apply.

- Once students are aware that $(1+z)^p = 1 + pz + \frac{p(p-1)}{2!}z^2 + ...$ is the expansion they need to be using, they still face a substantial challenge. It is not immediately obvious to them how an expression such as $\frac{1}{|x-D|}$ can be transformed to the form $(1+z)^p$. Simply giving students the answer at this point will defeat much of the learning possibilities of this activity. Students will need some time just to recognize that $p = -1$, but they will need much more time to determine if x or D is the smaller amount and recognize that by factoring out D they can have an expression that starts looking like $(1+z)^p$, with $z = \frac{x}{D}$ (or $\frac{D}{x}$ or) and $p = -1$. `Link to student language and conceptual problems regarding "factoring out" terms`.

  Students should not be allowed to stay entirely stuck for too long, but they must be given time to struggle with the problem in order for the learning to "stick" and be useful in future problems. Students should be given substantial time (about 20 minutes) to grapple with this portion of the problem. Some students may make algebraic errors such as they fincorrectly factoring out D or x. These should be brought to their attention quickly. Students may also have trouble dealing with the absolute value sign. Questioning strategies should be used to ensure understanding if this is the case.

- Once the correct power series expansion for each term is established, students will then need to work through the algebra to add the two power series together. Frequently students will make sign errors. Most student mistakes on this portion are careless algebra errors and help can be given liberally as needed.

Debriefing, Whole-Class Discussion, Wrap-up and Follow-up

Group sharing

Each of the eight groups should have an opportunity to present their results to the class such that everyone can see their work. If facilities permit, this is ideally done on large whiteboards around the room.

Compare and contrast

The instructor should encourage students to compare and contrast the results for the eight situations. This should include careful attention to: 1) whether the power series is odd or even and how this relates to whether the situation is symmetric or anti-symmetric 2) whether the answers "make sense" given the physical situation and what they tell you about moving in the + or - direction on the given axis

Consideration of the 3-dimensional case

Most students will have thought about this problem entirely within two dimensions. They should be asked to consider points with a non-zero z component. Envisioning the three-dimensional potential field will help students towards the types of thinking they will need to apply to future problems.

Laurent Series

Assuming that students have not yet been exposed to Laurent series, it should be brought to their attention that a "power series" with $\frac{1}{x}$ factors is called a Laurent series. We have found that by introducing Laurent series this way, students see it as no big deal and have sufficient understanding, but if introduced before this activity they are often intimidated and confused.

Suggested homework

Determine the general case for $V(\vec{\boldsymbol{r}}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}$in rectangular coordinates - Answer - $V(x,y,z) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}$

`Link to equipotential surfaces activity.`

`Link to "Visualizing voltage" Maple worksheet.`

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