Recorder: _____

Task Master: _____ **Cynic:** _____ **Other:** _____

FINDING $d\vec{r}$

Working in small groups (3 or 4 people), solve as many of the problems below as possible. Try to resolve questions within the group before asking for help. The Recorder is responsible for writing up the group's results and turning it in. Show your work! Full credit will only be given if your answer is supported by calculations and/or explanations as appropriate.

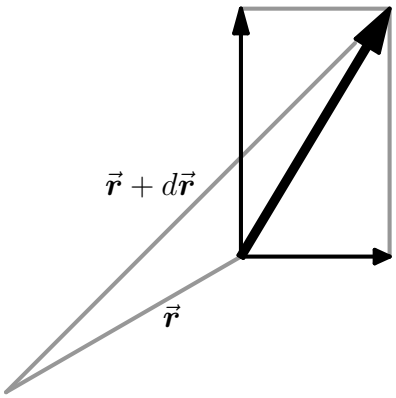
1. The diagonal of the rectangle on the left below shows (a blown-up picture of) an infinitesimal displacement from the point $(x,\, y)$ to the nearby point $(x + dx,\, y + dy)$.

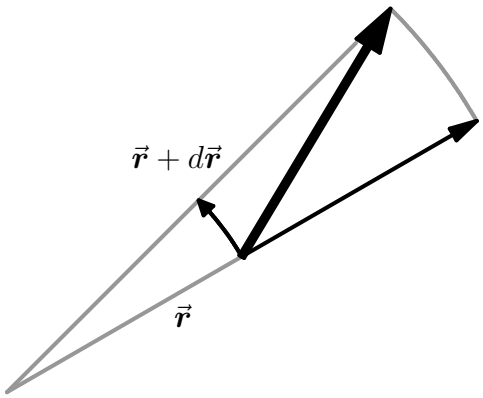
(a) Label the rectangle with the lengths of the sides.

(b) Express the sides of the rectangle indicated by arrows as vectors.
Use the unit vectors $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$.

(c) The diagonal of this rectangle is the vector differential $d\vec{\boldsymbol{r}}$. Express $d\vec{\boldsymbol{r}}$ in terms of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$.

(d) Find the length $ds = |d\vec{\boldsymbol{r}}|$ of the diagonal.





2. The diagonal of the "rectangle" on the right above shows (a blown-up picture of) the ***same*** infinitesimal displacement, now expressed in polar coordinates, from the point $(r,\, \phi)$ to the nearby point $(r + dr,\, \phi + d\phi)$.

(a) Label the rectangle with the lengths of the sides. *Careful!*

(b) Express the sides of the rectangle indicated by arrows as vectors.
Use the natural orthonormal basis defined by the picture, that is, let $\hat{\boldsymbol{r}}$ be the unit vector which points in the direction of increasing r, and let $\hat{\boldsymbol{\phi}}$ be the unit vector which points in the direction of increasing ϕ. Do not attempt to express these vectors in terms of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$! You do not need to worry about the fact that some sides of the rectangle aren't straight; the rectangle is so small that this error is negligible.

(c) The diagonal of this rectangle is again the vector differential $d\vec{\boldsymbol{r}}$. Express $d\vec{\boldsymbol{r}}$ in terms of $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\phi}}$

(d) Find the length $ds = |d\vec{\boldsymbol{r}}|$ of the diagonal.