# Group Activity 3: Finding $d\vec{\boldsymbol{r}}$

## I Essentials

### (a) Main ideas

- Introduces $d\vec{\boldsymbol{r}}$, the key to vector calculus, as a geometric object.

**Don't skip this activity if you use nonrectangular basis vectors!** [1]

### (b) Prerequisites

- Familiarity with $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\phi}}$.

### (c) Warmup

Draw a picture on the board showing $d\vec{\boldsymbol{r}}$ as the infinitesimal change in the position vector $\vec{\boldsymbol{r}}$ between two infinitesimally close points.

### (d) Props

- whiteboards and pens
- Big arrows, perhaps made of straws, which can represent an orthonormal basis, and which can be moved around a curve on the board.

### (e) Wrapup

- Emphasize that $d\vec{\boldsymbol{r}}$ is the same geometric object regardless of how it is expressed.
- Discuss the geometry of $ds$ as the magnitude of $d\vec{\boldsymbol{r}}$, that is, $ds = |d\vec{\boldsymbol{r}}|$.
- This is a good place to introduce the idea of "what sort of a beast is it"; see Chapter 8. The vector differential $d\vec{\boldsymbol{r}}$ is an infinitesimal differential having both direction and (infinitesimal) length. When writing an expression for $d\vec{\boldsymbol{r}}$, students should make sure that each term has these same properties.

[1] An alternative is to present this material in lecture, rather than as a group activity. In this case, we strongly recommend assigning the generalizations to cylindrical and spherical coordinates as homework; see Section 3.4.

## II Details

### (a) In the Classroom

Most groups will miss the factor of $r$ in the $\hat{\boldsymbol{\phi}}$ component of $d\vec{\boldsymbol{r}}$. Watch for this as you walk around the classroom. A good thing to point out is that $d\phi$ is not a length.

Some groups will then remember the formula for arclength and be able to figure out the rest on their own. Other groups will need to be reminded about the relationship between arclength and radius on a circle. A good way to do this is to ask them for the formula for the circumference of a circle, then half a circle, a quarter, etc. Make sure to give the angles in radians! Eventually, they get the point.

Some students may wonder whether the top of the (Cartesian) rectangle is $\pm dx\,\hat{\boldsymbol{\imath}}$. This question is ill-posed, since the sign of $dx$ itself depends on which way you're going; you can't change your mind in the middle of a problem. The safest way to resolve such problems is to anchor all vectors to the same point, as shown in the figures.

For the polar rectangle, many students will realize that that there are second-order differences between the two arcs, but few will realize that there are also second-order differences in the radial sides, due to changes in $\hat{\boldsymbol{r}}$.

### (b) Subsidiary ideas

- This is a good place to emphasize the relationship between the dot product and the Pythagorean Theorem.

### (c) Homework

- Have students determine $d\vec{\boldsymbol{r}}$ in 3 dimensions in rectangular, cylindrical and spherical coordinates; see Section 3.4. (Spherical coordinates are tricky; most students miss the factor of $\sin\theta$ in the $\hat{\boldsymbol{\phi}}$ component.)

- Find $d\vec{\boldsymbol{r}}$ along the diagonal of a square.

### (d) Essay questions (none yet)

## (e) Enrichment

- Emphasize that $d\vec{\mathbf{r}}$ is the concept which unifies most of vector calculus.
- It may be helpful to some students to be asked to orient the arrows (see Props) themselves at various points in the plane.