Electric Potentials for Two Charges

Two charges +Q and +Q are placed on a line at $x = -D$ and $x = +D$ respectively. What is a fourth order approximation to the electric potential $V(x,y)$ for $x = 0$ and the magnitude of y much less that the distance D? For what values of y does your series converge? For what values of y is your approximation a good one? Which direction would a test charge move under the influence of this electric potential?

The general steps in solving this problem are as follows

1. Draw a diagram containing the information given in the question
2. Recognize the problem type and choose equations accordingly
3. Set up equations for the problem including the superposition principle to reduce from two equations into a single equation
4. Convert the equations from the problem into equations that mirror common power series equations
5. Convert the equation for electrostatic potential from the problem into a power series expansion
6. Analyze the power series for ranges of convergence and divergence

In this problem we're dealing with electrostatic potential. It should be brought to attention before going into the problem that electrostatic potential is a scalar quantity and not a vector quantity. We will be using the formula for electrostatic potential (V) and the superposition principle to solve the first few steps of this problem.

Super position principle as it applies to electrostatic potential states that the total potential (V_T) is equal to the sum of the potentials involved ($V_1 + V_2 + \ldots$).

The formula for electrostatic potential is simply $V=kQ/r$ where k is a constant with a value of $(1/4\pi\epsilon_o)$, Q is in this case a point charge, and r is the distance between a test charge and the point charge Q. The total distance in this problem $r = (D^2+y^2)^{1/2}$ where D is a fixed value and is the length along the x axis the charge is from the origin. (See Figure 1 for a schematic view of the problem). The length r is found by a simple application of the Pythagorean theorem which states that for any right triangle $a^2 + b^2 = c^2$ where a and b are the opposite and adjacent sides of the triangle respectively and c is the hypotenuse.

A point charge located along the y axis will have two separate potentials acting upon it, however, due to the super position principle we can sum these potentials into a single equation for potentials in the problem.

$V(Q_1) = kQ/((-D)^2 + y^2)^{0.5})$ and $V(Q_2) = kQ/((D^2 + y^2)^{0.5})$

Notice that the two scalar quantities $V(Q_1)$ and $V(Q_2)$ are equal since $(-D)^2 = D^2$ so using the superposition principle we can now obtain an equation for V_T

$$V_T = 2kQ/((|D^2 + y^2|)^{0.5})$$

Now that we have an equation for our electric potential in the problem we need to find a way to relate it to a power series. One way would be to simply create a power series for the form that V_T is in above. However, in this case it is easier to just manipulate the equation for V_T into the form of a familiar power series. In this case it is the power series for anything of the form $(1 + Z)^p$ which can be found on the Common Power Series handout.

This power series converges for all $|Z| < 1$ which comes in to play when we decide which variable to factor out of the denominator.

$V_T = 2kQ/((|D^2 + y^2|)^{0.5})$ Simply factor a D^2 out of the denominator to obtain

$V_T = 2kQ/((D(|1+(y^2/D^2)|)^{0.5})$ Next, separate the constants from the variables

In addition to separation of constants, take the denominator and change it to the numerator by making the power negative

$$V_T = (2kQ/D)((|1+(y^2/D^2)^{-0.5})$$

So now we clearly have a problem of the form $C(1 + Z)^p$, where $C = 2kQ/D$
By observation our $p = -0.5$ and $Z = (y^2/D^2)$

Now plug these values into the power series for $(1 + Z)^p$

$(1 + Z)^p =$

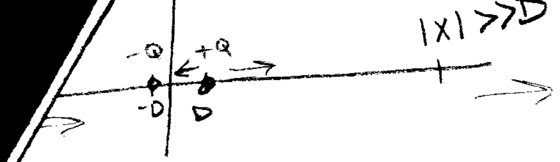
Now plugging in our values we obtain the following solution

Notice that our series converges for all $|y^2/D^2| < 1$ and in our problem $y \ll D$ therefore our series is completely convergent over the domain of our problem.
Additional Problems in Electric Potentials for Two Charges (From Electric Potentials from two charges worksheet)

I.) Problem one.

Follow the same steps as in the first example. In this problem however, notice that the charges are only located along the x-axis and the potentials $V(Q_1)$ and $VQ(_2)$ vary with respect to x only. When you're setting up your equations you get expressions for distance along the x-axis in terms of x, and D. Simply apply superposition just as in the other problems and then convert to the same power series as before, ie $(1 + Z)^p$. You end up with the first equation on the PH320: Day 5 Handout.

Note: Effects of charges on the power series expansion.

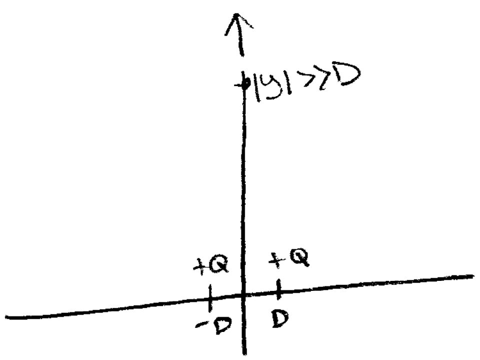


$$\frac{Q}{2\pi\varepsilon_0 |x|}\left(1+\frac{D^2}{x^2}+\frac{D^4}{x^4}+\dots\right) \quad \text{for } |x|\gg D,\ y=0$$

Converges for all $|x|\gg D$ since x is in the numerator

5.) (MAIN PROBLEM - SEE TYPED)

7.)



Series

$$\frac{Q}{2\pi\varepsilon_0 |y|}\left(1-\frac{1}{2}\frac{D^2}{y^2}+\frac{3}{8}\frac{D^4}{y^4}+\dots\right)$$

Note: Mirrors Problem 5

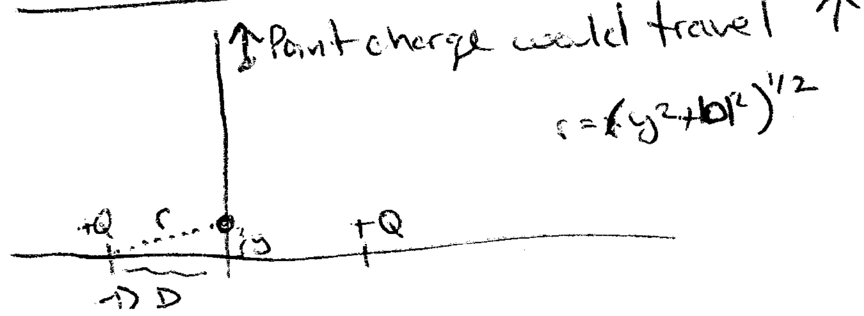
Converges for all $|y|\gg D$ since y is in the denominator.

8.) Not Sure if #8 was Included

PROBLEM 5 Solution (See PrintOut For Details)

$$V_{(x,y,z)}=\frac{Q}{2\pi\varepsilon_0 D}\left(1-\frac{1}{2}\frac{y^2}{D^2}+\frac{3}{8}\frac{y^2}{D^2}+\dots\right) \quad \text{for } |y|<D \text{ and } x=0$$

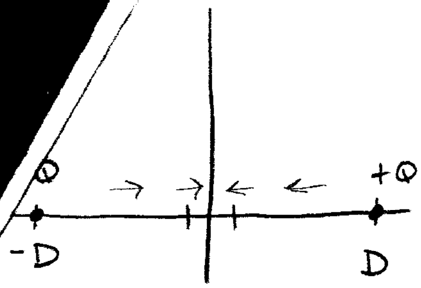
Schematic Diagram Figure 1.



$$\boxed{r=(y^2+|D|^2)^{1/2}}$$

Put Here

Find V_T 4th order approximation for $|X| << D$



$$V_T = V_1 + V_2 = \frac{KQ}{(D-|X|)} + \frac{KQ}{(D-X)}$$

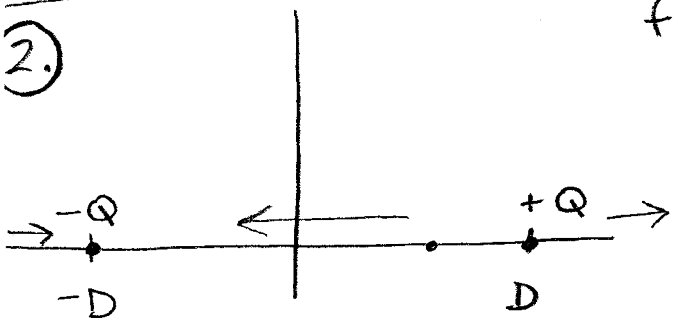
$\Rightarrow$ Summation Similar To $(1+z)^p$

After some algebraic manipulation

$$V_{(x,y,z)} = \frac{Q}{2\pi\varepsilon_0 D}\left(1 + \frac{x^2}{D^2} + \frac{x^4}{D^4} + \ldots\right)$$ for $|X| << D$ and $y=0$

Series converges for all X since $|X| << D$

2.) for $|x| << D$

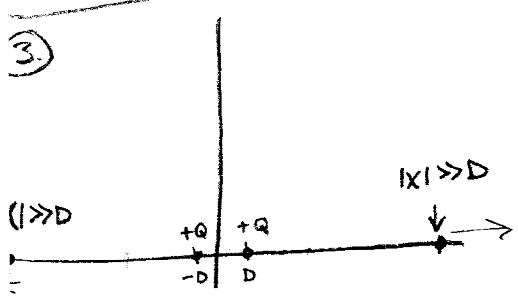


Similar To Previous Example So I'm going To Skip Setting Up V_T function. Note, the difference in charge (+/-) and observe the differences between Solution 1 and solution 2.

$$V_{(x,y,z)} = \frac{Q}{2\pi\varepsilon_0 D}\left(\frac{x}{D} + \frac{x^3}{D^3} + \frac{x^5}{D^5} + \ldots\right)$$ for $|X| << D$ and $y=0$

series converges for all X since $|x| << D$ is numerator

3.)



$$V_T = \frac{KQ}{(x-D)} + \frac{KQ}{(-x-D)}$$

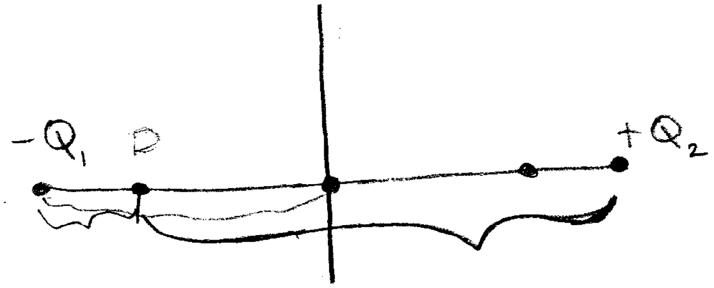
$\Rightarrow$ Series expansion

$$V_{(x,y,z)} = \frac{Q}{2\pi\varepsilon_0 |X|}\left(\frac{D}{x} + \frac{D^3}{x^3} + \frac{D^5}{x^5} + \ldots\right)$$ $x >> D$, $y=0$

series converges for all $x >> D$ since x is in the denominator

$*$ Arrows Denote Direction A $\oplus$ Test Charge Would Travel $*$ at a given location me Power Series Expansion As Above

Come Up with Equations for both



Finding Equations For Potentials is the Most Difficult Part.

$$V(Q_1) = \frac{-Q \cdot K}{|(-x-D)|} \qquad V(Q_2) = \frac{QK}{|(x-D)|}$$

$\therefore$ Sum The Potentials $V(Q_1)$ and $V(Q_2)$

$$V_T = QK\left(\frac{-1}{|(-x-D)|} + \frac{1}{(|x-D|)}\right)$$

Now Sum $\underline{V(Q_1)}$ and $\underline{V(Q_2)}$ expansions **separately**

To obtain $$\frac{Q}{4\pi\varepsilon_0} \cdot \frac{2}{|x|}\left(1 + \frac{D^2}{x^2} + \frac{D^4}{x^4} + \dots\right)$$ for $|x| > D$ and $y = 0$

3.) Exactly The Same as Problem Two with a Different Charge ($Q+$, and $Q+$) which Changes the Series as follows (Work it out on your own using the previous guidelines. for $|x| >> D$ which lies to the left and right of your points respectively.

$$V(x,y,z) = \frac{Q}{4\pi\varepsilon_0} \frac{2}{|x|}\left(\frac{D}{x} + \frac{D^3}{x^3} + \frac{D^5}{x^5} + \dots\right)$$ for $|x| > D$ and $y = 0$