Experts' understanding of partial derivatives using the Partial Derivative Machine


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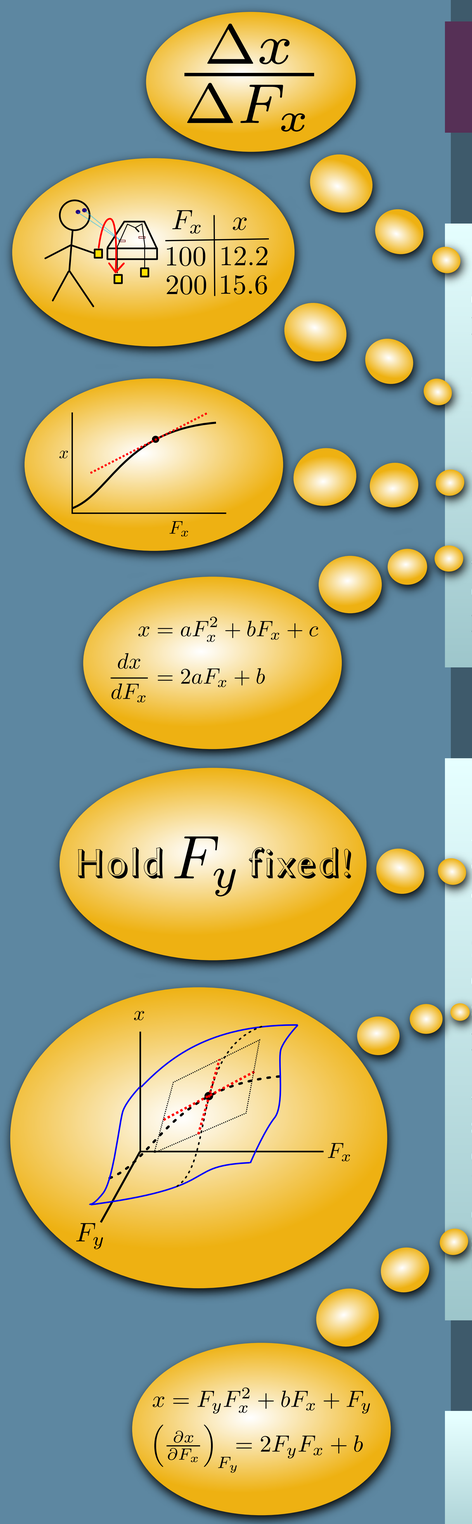
National Science Foundation
DUE-0837829
DUE-1141330



Concepts of derivatives

What is a derivative?

1. A ratio of small changes. This is the limit definition of a derivative, and seems to be the first learned by students, and the first forgotten.

2. A measurement to determine its value. This involves measuring small changes and taking a ratio.

3. The slope of the tangent to a curve

4. The result of manipulation of a symbolic expression. For many students, this is their primary understanding of "finding a derivative."

What is a *partial* derivative?

1,2. When considering a ratio of small changes or a measurement, a *partial* derivative requires that we ask not only which quantities are changing, but also which quantities to hold fixed.

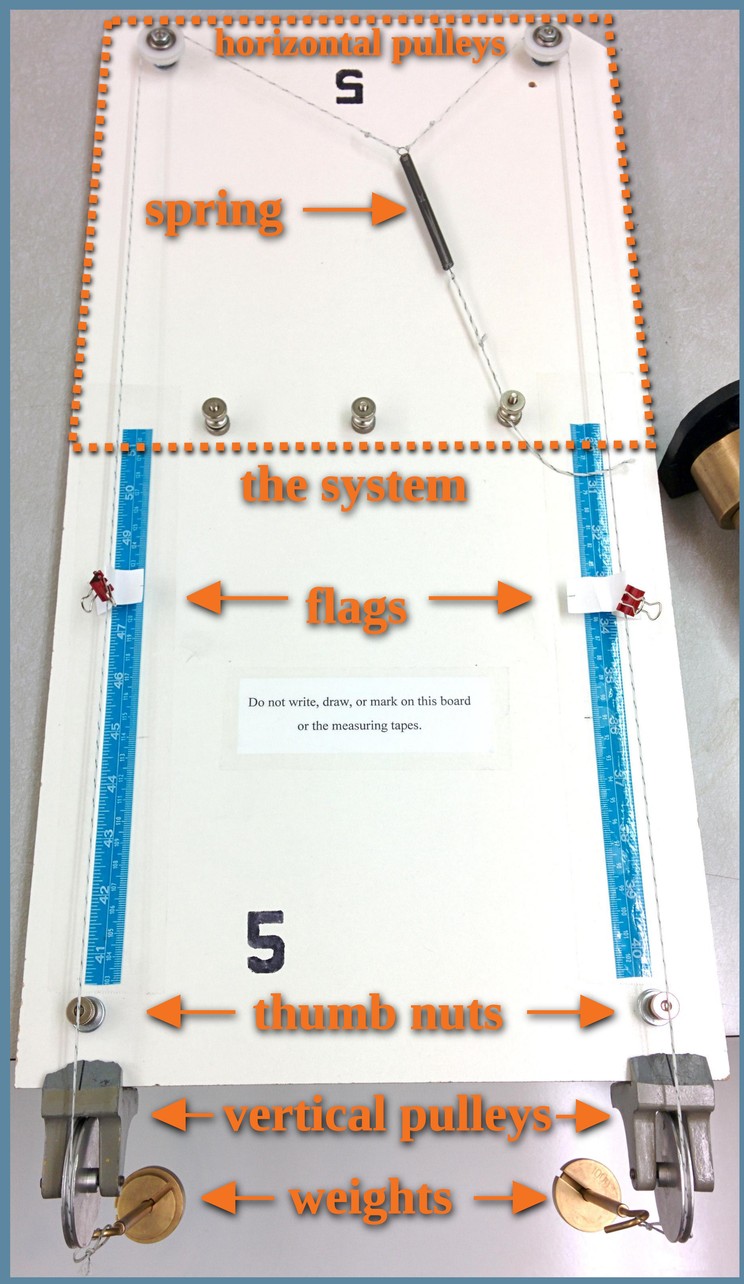
3. A tangent line turns to a tangent plane in two dimensions, and a partial derivative becomes the slope of the plane in a given direction at a given point.

4. The procedure to find a partial derivative of a symbolic expression is identical to that for an ordinary derivative, provided there are not interdependencies among the variables in the expression. It is unsurprising that many of our students believe that a partial derivative means "everything else is held fixed."

Instructions to build a PDM:

http://www.physics.oregonstate.edu/portfolioswiki/whitepapers:pdm:start

The Machine



Expert interviews

The interviewees

Physicists One associate professor and one full professor. Both use computational methods in their research in astrophysics and optics.

Engineers One chemical engineer who is a full professor with considerable research and teaching expertise in thermodynamics, and two engineers who study student thinking and epistemology in engineering.

Mathematicians Two assistant professors whose research is in mathematics education at the collegiate level.

The prompt

Find $\frac{\partial x}{\partial F_x}$

Measuring the derivative

Physicists and engineers Immediately (within two minutes of taking data) computed a "ratio of small changes" value for the derivative. Soon verified that the changes were small enough.

Mathematicians Required 30 minutes and four prompts from the interviewer before finally recognizing that they could develop an experimental measurement for the derivative. Repeatedly returned to speculation regarding the functional form of x, even after being prompted to find a numeric value.

What about y and F_y?

Physicists Assumed that F_y must be held fixed, rather than y when taking the derivative. Recognized that only two variables were independent and that one of these could be y.

Engineers Similar to physicists, in assuming that Fy should be held fixed. Two engineers had confusion in the number of independent variables.

Mathematicians Based on physical gestures, recognized that the derivative would differ if y were held fixed. Did not discuss independent variables.