FROM INDUCTION TO QUANTUM SPIN: TRAINING STUDENTS TO BECOME PHYSICISTS

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OSUPER

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- www.physics.oregonstate.edu/portfolioswiki
- www.islephysics.net

The physics major at OSU

- 1st and 2nd year: intro physics and modern physics
- 3rd year: paradigms
  - 3 week intensive short-courses grouped by concept/math
- 4th year: capstones and independent research

Paradigms in Physics <P|P>

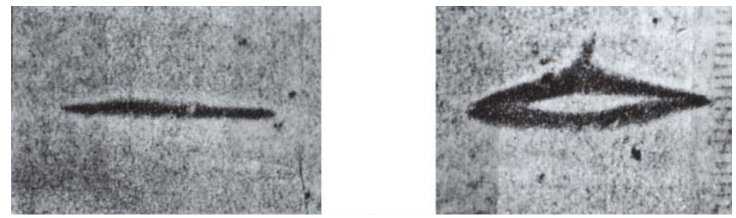
- Fast-paced, intensive structure:
  - Students meet daily: 1 hour MWF, 2 hours T and R
  - Homework/labs due once or twice per week
- Pedagogical tools used:
  - Compare and contrast activities, small white board questions, kinesthetic activities, computer simulations, small group activities, integrated labs, homework, and lecture

Paradigms in Physics <P|P>

- Goals based reform:
  - Teach physics as physicists think about it: concepts that broadly underlie multiple subfields
  - Encourage mastery of concepts
- Student quote: "The paradigms curriculum constructed my knowledge from a fundamental level, with each paradigm building on the last. The emphasis on big-picture learning and universal problem-solving carries over in my graduate studies today and the pace and difficulty prepared me for graduate school" (currently in physics grad studies at the U of Illinois)

Spins Course: Quantum measurement

- Understand and analyze sequential Stern-Gerlach (SG) measurements on spin systems, and generalize to generic systems



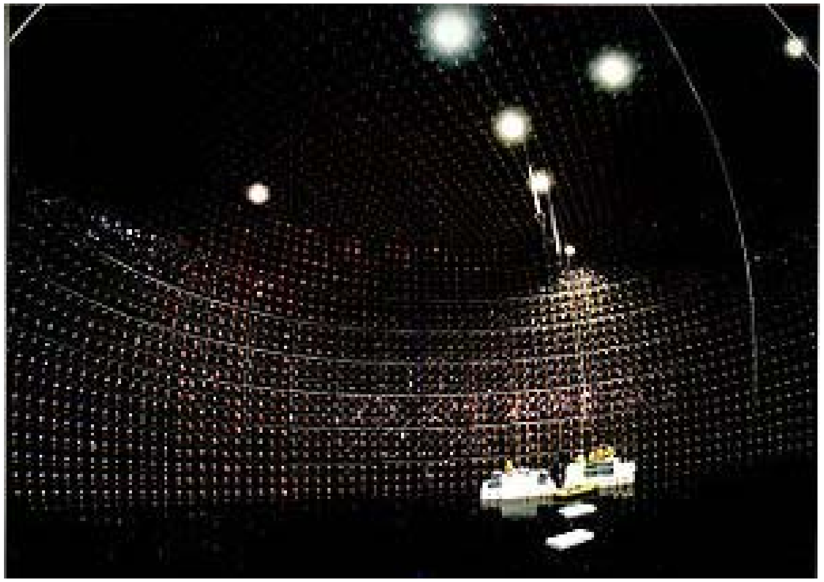
- Use time evolution to understand spin precession, and generalize to generic systems
- Plus: finding eigenvectors/values, commutation and uncertainty, probabilities, expectation value and standard deviation

Building a solid understanding of main QM postulates

1. States are defined by kets that contain all we can know
2. Physical observables are described mathematically by operators
3. The only possible result of a measurement is an eigenvalue of the corresponding Hermitian operator
4. How to calculate the probability of obtaining a specific eigenvalue
5. The system is in a new state that is the normalized projection of the original system ket onto the ket (or kets) corresponding to the result of the measurement
6. Time evolution is determined by the Schrodinger equation

Other content (2^{nd} half of the 3 weeks):

- Time evolution:
  - Solving Schrodinger equation for time-independent Hamiltonian Operator (matrix form)
  - Spin precession
  - Rabi Oscillations
  - Neutrino Oscillations
  - NMR
  - Atomic Clocks



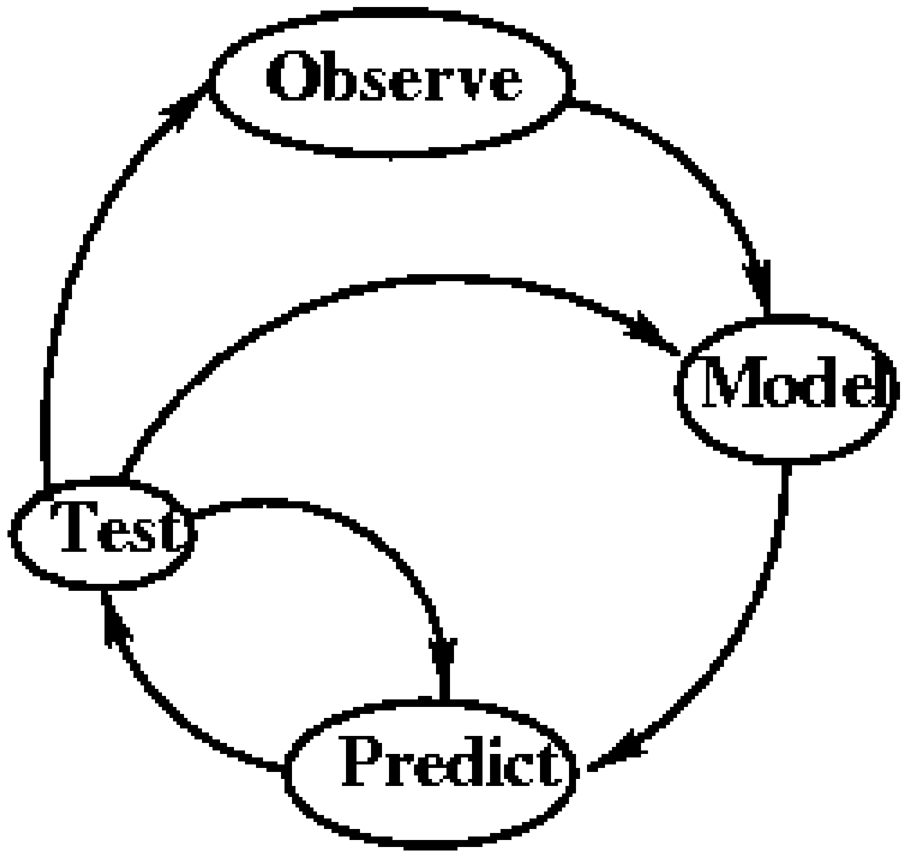
- Extend calculations to spin 1 systems

Challenges:

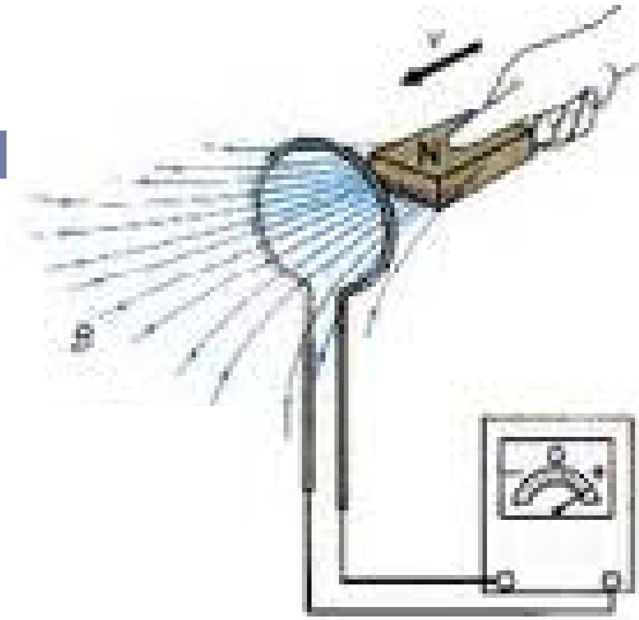
- Math
  - Many students have their first introduction to linear algebra in 1 week just before this course
- Language/Representations
  - Many new terms, and it is difficult to *speak* ‘quantum mechanics’ accurately
  - Representations for QM don’t always map to ‘real’ space
- Concepts
  - State collapse, probabilistic nature, quantization…

Course reform: Investigative Science Learning Environment (ISLE):

- Goal-based reform: Representing information, conducting experiments, thinking divergently, collecting and analyzing data, constructing, modifying and applying relationships and explanations, being able to coordinate these abilities



Example ISLE cycle in Intro physics: Induction



- Observe:
  - Watch a magnet move within a coil
  - ASK students what else they'd like to see
- Model:
  - Students brainstorm a quasi-mathematical statement for induced current BASED ON THE OBSERVATIONS
- Test:
  - Try with different coils, alignments…
- Refine model then apply: problem solving, generators…

Overlapping Goals: Paradigms and ISLE

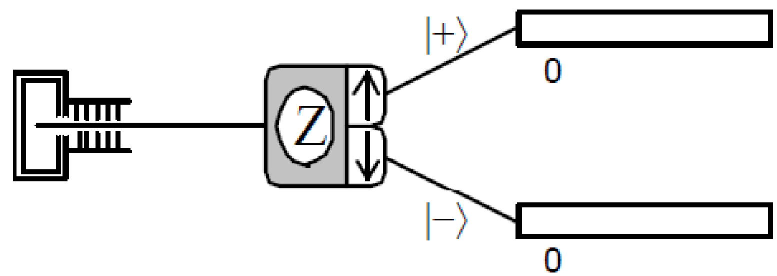
- engaging in authentic practices of physics
- mastery of key concepts
- teach physics as physicists think about it: help students see connections
- sequence activities to develop needed skills

Modifying Spins using ISLE

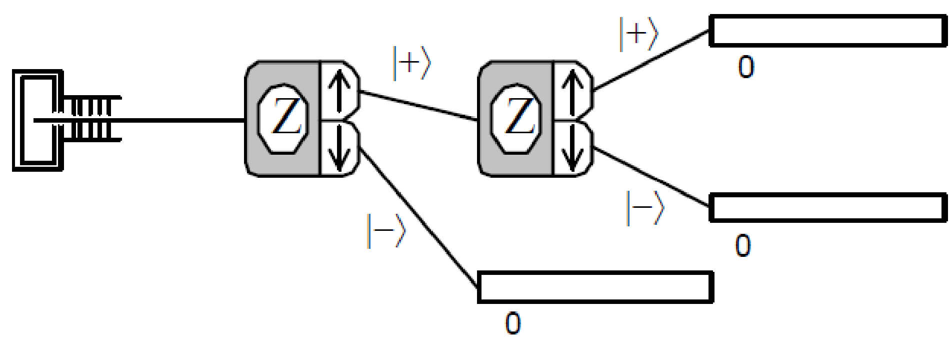
- Take existing text, labs, and homework but link it together differently
  - Make it explicit when we are doing observations and looking for patterns to devise explanations
  - Make it explicit when we are testing explanations to generate rules
  - Make it explicit when we are applying rules
- Add questions and new discussions: How do we know? How do we construct the knowledge for ourselves? How do we apply it?

Example: modifying existing lab 1 to be an observation experiment

- Lab 1 (day 2 of course)
  - Task: calculate all combinations of 2 SG devices, look for patterns



- Observe: Outcome of simple SG measurements
- Model: Using matrix math, how to calculate probabilities and 'collapsed' states

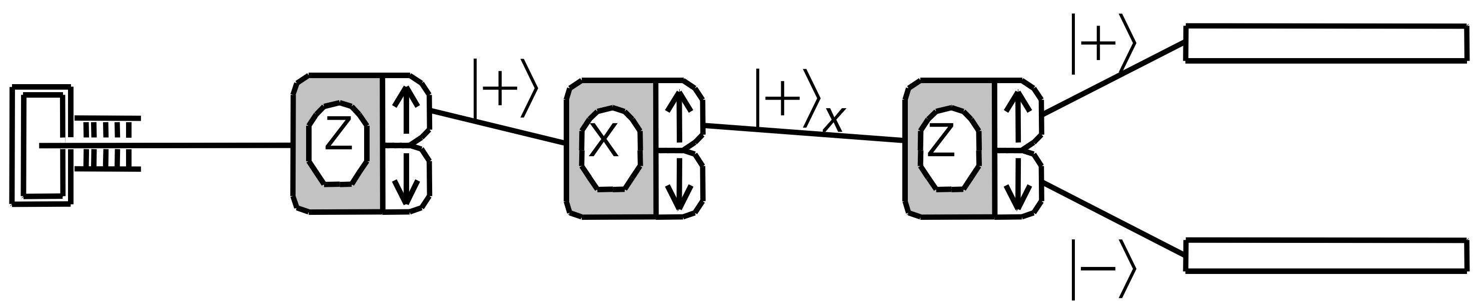


http://physics.oregonstate.edu/~mcintyre/ph425/spins/index_SPINS_OSP.html

Test findings by checking consistency with 'counter-intuitive' example

- Make sense of observations using the math they developed in lab!
- Probability calculations are easier than sense-making
- Conceptual understanding can come from crossed polarizers as an analogy: grounded physical observation!

(a)



Apply understanding to work backwards: lab 2

- Take relevant data to use probabilities to find what the initial state is
- Tests math understanding – complex manipulations, Euler's formula, ability to realize two angles can be possible, the need to take more data…

Unknown $|\psi_1\rangle$

Probabilities	Axis		
Result	x	y	z
spin up $\uparrow$			
spin down $\downarrow$			

Lab calculations are NOT easy!! (below: the easiest spin-1 unknown)

$$|\psi_1\rangle = a|1\rangle + b|0\rangle + c|-1\rangle$$

$$\mathcal{P}(1) = |\langle 1|\psi_1\rangle|^2 = |\langle 1|\{a|1\rangle + b|0\rangle + c|-1\rangle\}|^2 = |a|^2 = \tfrac{1}{4}$$

$$\mathcal{P}(0) = |\langle 0|\psi_1\rangle|^2 = |\langle 0|\{a|1\rangle + b|0\rangle + c|-1\rangle\}|^2 = |b|^2 = \tfrac{1}{2}$$

$$\mathcal{P}(-1) = |\langle -1|\psi_1\rangle|^2 = |\langle -1|\{a|1\rangle + b|0\rangle + c|-1\rangle\}|^2 = |c|^2 = \tfrac{1}{4}$$

$$|\psi_1\rangle = \tfrac{1}{2}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\alpha}|0\rangle + \tfrac{1}{2}e^{i\beta}|-1\rangle$$

$$\mathcal{P}(0_x) = |{}_x\langle 0|\psi_1\rangle|^2 = \left|\tfrac{1}{\sqrt{2}}\{\langle 1| - \langle -1|\}\left\{\tfrac{1}{2}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\alpha}|0\rangle + \tfrac{1}{2}e^{i\beta}|-1\rangle\right\}\right|^2 = \left|\tfrac{1}{2\sqrt{2}}\left\{1 - e^{i\beta}\right\}\right|^2$$

$$\mathcal{P}(0_x) = \tfrac{1}{8}\left\{1 - e^{i\beta}\right\}\left\{1 - e^{-i\beta}\right\} = \tfrac{1}{8}\left\{1 + 1 - e^{i\beta} - e^{-i\beta}\right\} = \tfrac{1}{4}\{1 - \cos\beta\} = \tfrac{1}{2}$$

$$\cos\beta = -1 \Rightarrow \beta = \pi$$

$$|\psi_1\rangle = \tfrac{1}{2}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\alpha}|0\rangle - \tfrac{1}{2}|-1\rangle$$

$$\mathcal{P}(1_y) = |{}_y\langle 1|\psi_1\rangle|^2 = \left|\left\{\tfrac{1}{2}\langle 1| - \tfrac{i}{\sqrt{2}}\langle 0| - \tfrac{1}{2}\langle -1|\right\}\left\{\tfrac{1}{2}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\alpha}|0\rangle - \tfrac{1}{2}|-1\rangle\right\}\right|^2 = \left|\tfrac{1}{4} - \tfrac{i}{2}e^{i\alpha} + \tfrac{1}{4}\right|^2$$

$$\mathcal{P}(1_y) = \tfrac{1}{4}\left\{1 - ie^{i\alpha}\right\}\left\{1 + ie^{-i\alpha}\right\} = \tfrac{1}{4}\left\{1 + 1 - ie^{i\alpha} + ie^{-i\alpha}\right\} = \tfrac{1}{2}\{1 + \sin\alpha\} = 1$$

$$\sin\alpha = 1 \Rightarrow \alpha = \tfrac{\pi}{2}$$

$$|\psi_1\rangle = \tfrac{1}{2}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\frac{\pi}{2}}|0\rangle - \tfrac{1}{2}|-1\rangle = \tfrac{1}{2}|1\rangle + \tfrac{i}{\sqrt{2}}|0\rangle - \tfrac{1}{2}|-1\rangle = |1\rangle_y$$

Example two: Observe and look for patterns to build understanding:

- As a class calculate all combinations of Sx, Sy, and Sz operators acting on all the eigenstates: $|+\rangle$, $|-\rangle$, $|+\rangle_x$, $|-\rangle_x$, $|+\rangle_y$, $|-\rangle_y$, , look for patters, and devise an explanation for what this operation "does"

- Observe: What S_i operators do to a state $|j\rangle$
- Model: What do operators do?
- Test: With more sophisticated calculations
- Apply: to understanding the Hamiltonian

Activity equations:

$$S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left[|+\rangle + |-\rangle\right] \qquad |+\rangle_y = \frac{1}{\sqrt{2}}\left[|+\rangle + i|-\rangle\right]$$

$$|-\rangle_x = \frac{1}{\sqrt{2}}\left[|+\rangle - |-\rangle\right] \qquad |-\rangle_y = \frac{1}{\sqrt{2}}\left[|+\rangle - i|-\rangle\right]$$

The equations are the spin operators for x, y and z written in matrix notation, and the eigenstates written in Dirac notation, all given in the z-basis: |+>, |->

What students DO when given this:

- Discuss notation – how do they multiple a matrix with something in Dirac notation?
- Then "plug-and-chug":
- $S_z|+> = (h_{bar}/2) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} (1/\sqrt{2}) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$= (h_{bar}/2\sqrt{2}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (h_{bar}/2)|+>$$

- (yes, they've done this one before, no, they don't usually recognize it!!!)
- This doubles as math and representation practice!!!

What students generate… (h is h_{bar}!)

	$\vert+>$	$\vert->$	$\vert+>_x$	$\vert->_x$	$\vert+>_y$	$\vert->_y$
S_z	$(h/2)\vert+>$	$-(h/2)\vert->$	$(h/2)\vert->_x$	$(h/2)\vert+>_x$	$(h/2)\vert->_y$	$(h/2)\vert+>_y$
S_y	$(h/2\sqrt{2})(0, i)$	$(h/2\sqrt{2})(-i, 0)$	$-i(h/2)\vert->_x$	$i(h/2)\vert+>_x$	$(h/2)\vert+>_y$	$-(h/2)\vert->_y$
S_x	$-(h/2)\vert->$	$(h/2)\vert+>$	$(h/2)\vert+>_x$	$-(h/2)\vert+>_x$	$i(h/2)\vert->_y$	$-i(h/2)\vert+>_y$

Typical results and student remarks:

Green: I wonder if we can simplify these, too?

Red: wait – I think there might be a mistake here (how do they see that?)

‘Fixed’ table: Look for patterns:

	$\vert +\rangle$	$\vert -\rangle$	$\vert +\rangle_x$	$\vert -\rangle_x$	$\vert +\rangle_y$	$\vert -\rangle_y$
S_z	$(h/2)\vert +\rangle$	$-(h/2)\vert -\rangle$	$(h/2)\vert -\rangle_x$	$(h/2)\vert +\rangle_x$	$(h/2)\vert -\rangle_y$	$(h/2)\vert +\rangle_y$
S_y	$i(h/2)\vert -\rangle$	$-i(h/2)\vert +\rangle$	$-i(h/2)\vert -\rangle_x$	$i(h/2)\vert +\rangle_x$	$(h/2)\vert +\rangle_y$	$-(h/2)\vert -\rangle_y$
S_x	$-(h/2)\vert -\rangle$	$(h/2)\vert +\rangle$	$(h/2)\vert +\rangle_x$	$-(h/2)\vert -\rangle_x$	$i(h/2)\vert -\rangle_y$	$-i(h/2)\vert +\rangle_y$

Red: huh, those are the only ones that ‘stayed’ in the same state!
Green: huh, look, all those ‘flopped’ states! (a few highlighted)
Magnitude of coefficient the same for all… interesting!
… anything else? (makes you curious, doesn’t it??)

Possible questions to ask:

1. What patterns do you observe in the table?
2. Which results stand out to you and how do they stand out?
3. What about the data is surprising to you?
4. What do you think it means to be in a particular ‘basis’?
5. Based on your observations, what happens when you multiply an operator with a state vector? Devise a rule to describe this.
6. How could you follow up this activity to test your rule?

Why is this worth doing?

- They learned the week before this course that a matrix acting on a vector linearly transforms it
- HOWEVER: Previous spins courses found students conflate operators with measurements!
- *Students need to make meaning for it themselves! I found no confusion on this topic among my students doing this activity*
- Helps them understand eigenvectors, what it means to be in a basis…
- Practices meaningful scientific process!

General remarks

- Improved exam scores: 70% to 80% from 2009 to 2010, with 2010 exam extended to include vocabulary, sense-making, and explanations.
- More extensive lab reports including better reasoning shown and conclusions drawn
- Students asked 'better' questions and were more forthcoming with what they thought was difficult

- This is EASY to do
- This is HARD to do ☺