Solution method and error evolution of student responses to chain rule problems within a thermodynamics course

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SPECIAL REQUEST: If you are taking the thermodynamics paradigm (PH 423) next year, please leave the room!

INTRODUCTION

MOTIVATION:

- To better understand the evolution of errors and solution method choices in students' responses to chain rule problems in thermodynamics
- To improve student learning experiences with partial derivatives and chain rules in thermodynamics

METHODS:

1. Collected, scanned, and anonymized student responses
2. Sorted student responses by method, per assignment
3. Analyzed student migration between methods
4. Sorted student responses by contained errors, per assignment
5. Analyzed student error evolution
6. Analyzed correlations in methods and errors

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PROMPTS

Quiz 14 prompt:

Given the definitions below, evaluate the requested partial derivative.

$$U = x^2 + y^2 + z^2$$

$$z = \ln(y - x)$$

$$Find \left(\frac{\partial U}{\partial z}\right)_y$$

Final exam question 4.b prompt:

Given the definitions below, evaluate the requested partial derivative.

$$U = \frac{3}{2} N K_B T - \frac{aN^2}{V}$$

$$S = N K_B \left(\ln\left(\frac{\mathrm{N - Vb}}{NC} T^{\frac{3}{2}} \right) + \frac{5}{2} \right)$$

$$Find \left(\frac{\partial U}{\partial V}\right)_S$$

Prompts have similarities, but are not parallel:

- Context
- Number of variables
- Constant variable
- Complexity

PROMPTS

Quiz 14 prompt:

Given the definitions below, evaluate the requested partial derivative.

$$\boldsymbol{U} = \boldsymbol{x}^2 + \boldsymbol{y}^2 + \boldsymbol{z}^2$$

$$\boldsymbol{z} = \ln(\boldsymbol{y} - \boldsymbol{x})$$

$$Find\ \left(\frac{\partial U}{\partial z}\right)_y$$

Final exam question 4.b prompt:

Given the definitions below, evaluate the requested partial derivative.

$$\boldsymbol{U} = \frac{3}{2} N K_B \boldsymbol{T} - \frac{aN^2}{\boldsymbol{V}}$$

$$\boldsymbol{S} = N K_B \left(\ln\left(\frac{\mathrm{N} - \boldsymbol{V}\mathrm{b}}{NC} \boldsymbol{T}^{\frac{3}{2}}\right) + \frac{5}{2}\right)$$

$$Find\ \left(\frac{\partial U}{\partial V}\right)_S$$

Prompts have similarities, but are not parallel:

- Context
- Number of variables
- Constant variable
- Complexity

POSSIBLE SOLUTION METHODS

$$Given: \quad \begin{gathered} U = x^2 + y^2 + z^2 \\ z = \ln(y - x) \end{gathered} \qquad Find: \left(\frac{\partial U}{\partial z}\right)_y$$

- Variable Re-Expression (Var RE)
  - –Replacing the undesired variable
- Differential Re-Expression (Diff RE)
  - –Replacing the undesired differential
- Implicit Differentiation (Imp Diff)
  - –Implicitly differentiating the initial function
- Chain Rule Diagram (CRD)
  - –Using a chain rule diagram to create a chain rule
- Differential Division (Diff Div)
  - –Mindfully dividing by a differential

POSSIBLE SOLUTION METHODS

$$Given: \quad \begin{matrix} U = x^2 + y^2 + z^2 \\ z = \ln(y - x) \end{matrix} \qquad Find: \left(\frac{\partial U}{\partial z}\right)_y$$

- Variable Re-Expression (Var RE)
  - –Replacing the undesired variable
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$$x = y - e^z$$

$$U = (y - e^z)^2 + y^2 + z^2$$

Only works if the undesired variable can be isolated in the secondary equation!

POSSIBLE SOLUTION METHODS

$$Given: \quad \begin{matrix} U = x^2 + y^2 + z^2 \\ z = \ln(y - x) \end{matrix} \qquad Find: \left(\frac{\partial U}{\partial z}\right)_y$$

- Variable Re-Expression (Var RE)
  –Replacing the undesired variable
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$$dx = 1dy - e^z dz$$

$$dU = 2xdx + 2ydy + 2zdz$$

$$dU = 2x(1dy - e^z dz) + 2ydy + 2zdz$$

$$dU = [2x + 2y]dy + [-2xe^z + 2z]dz$$

$$dU = \left(\frac{\partial U}{\partial y}\right)_z dy + \left(\frac{\partial U}{\partial z}\right)_y dz$$

POSSIBLE SOLUTION METHODS

$$Given: \quad \begin{matrix} U = x^2 + y^2 + z^2 \\ z = \ln(y - x) \end{matrix} \qquad Find: \left(\frac{\partial U}{\partial z}\right)_y$$

- Variable Re-Expression (Var RE)
  –Replacing the undesired variable
- Differential Re-Expression (Diff RE)
  –Replacing the undesired differential
- Implicit Differentiation (Imp Diff)
  –Implicitly differentiating the initial function

$$\left(\frac{\partial U}{\partial z}\right)_y = 2x\left(\frac{\partial x}{\partial z}\right)_y + 2y\left(\frac{\partial y}{\partial z}\right)_y + 2z\left(\frac{\partial z}{\partial z}\right)_y$$

- Chain Rule Diagram (CRD)
  –Using a chain rule diagram to create a chain rule
- Differential Division (Diff Div)
  –Mindfully dividing by a differential

POSSIBLE SOLUTION METHODS

$$Given: \quad \begin{array}{c} U = x^2 + y^2 + z^2 \\ z = \ln(y - x) \end{array} \qquad Find: \left(\frac{\partial U}{\partial z}\right)_y$$

- Variable Re-Expression (Var RE)
  –Replacing the undesired variable
- Differential Re-Expression (Diff RE)
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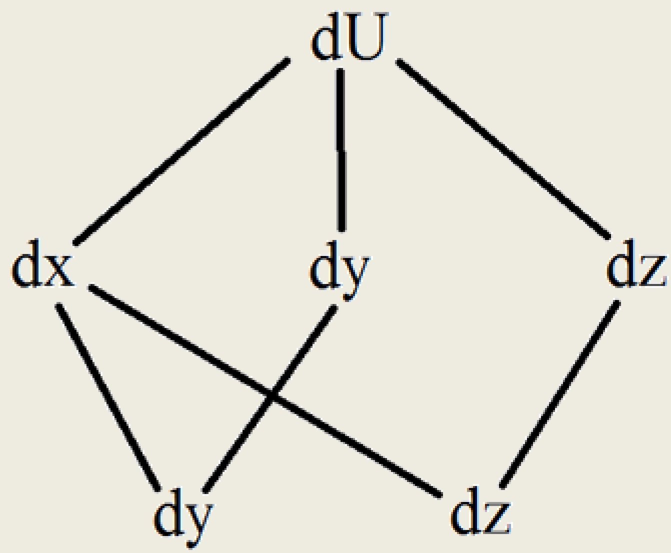

$$\left(\frac{\partial U}{\partial z}\right)_y = \left(\frac{\partial U}{\partial x}\right)_{y,z} \left(\frac{\partial x}{\partial z}\right)_y + \left(\frac{\partial U}{\partial z}\right)_{x,y}$$

POSSIBLE SOLUTION METHODS

$$Given: \quad \begin{gathered} U = x^2 + y^2 + z^2 \\ z = \ln(y - x) \end{gathered} \qquad Find: \left(\frac{\partial U}{\partial z}\right)_y$$

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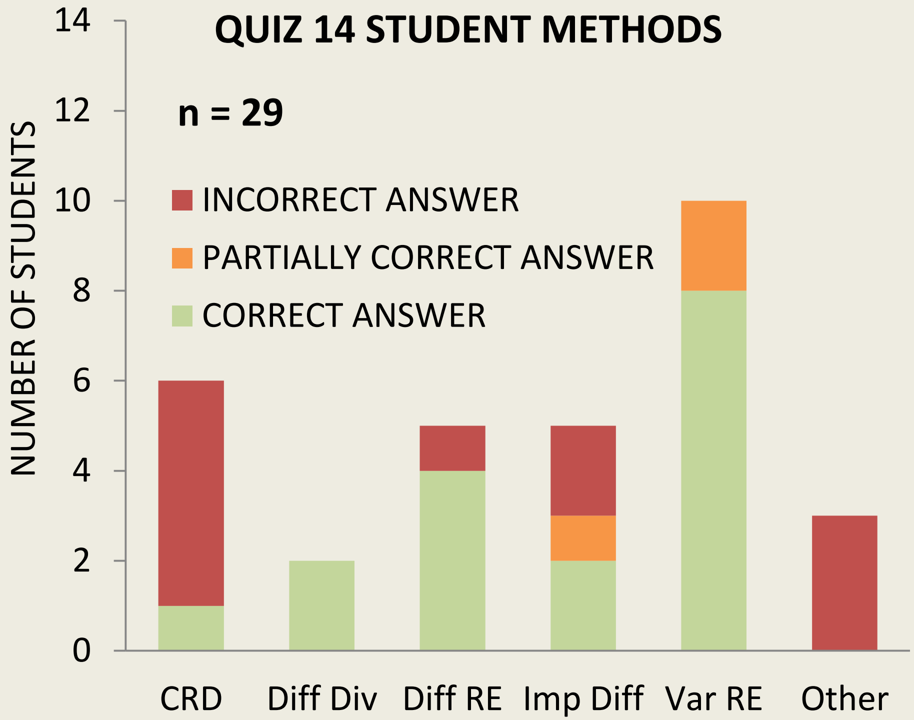
$$dU = 2xdx + 2ydy + 2zdz$$

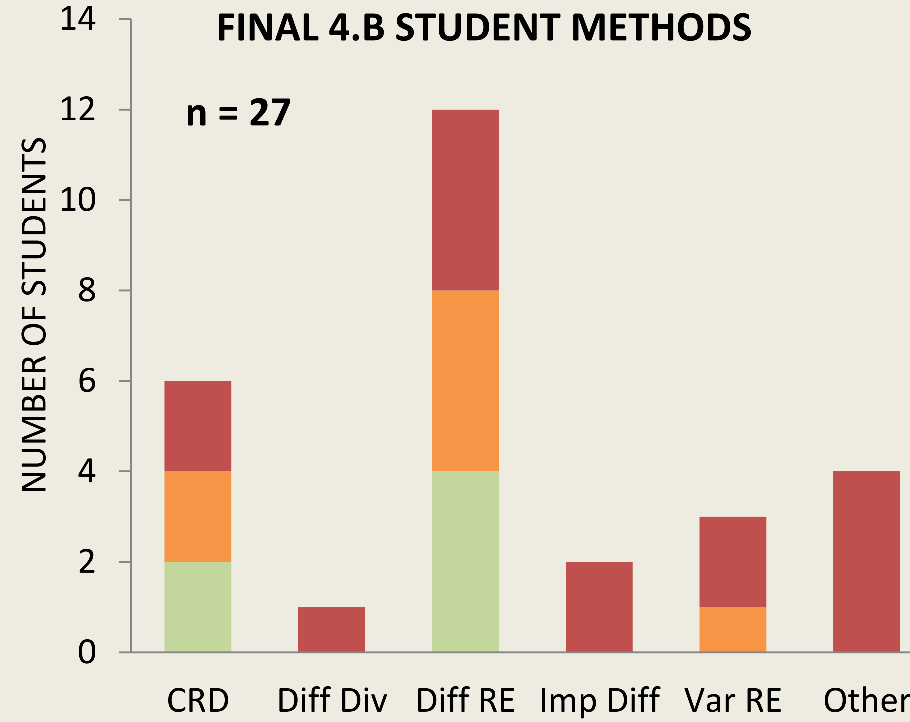
$$\frac{dU}{dz} = \frac{2xdx + 2ydy + 2zdz}{dz}$$

$$\frac{dU}{dz} = 2x\frac{dx}{dz} + 2y\frac{dy}{dz} + 2z\frac{dz}{dz}$$

$$\left(\frac{\partial U}{\partial z}\right)_y = 2x\left(\frac{\partial x}{\partial z}\right)_y + 2y\left(\frac{\partial y}{\partial z}\right)_y + 2z$$

SOLUTION METHOD DISTRIBUTION





QUIZ 14 PROMPT:

$$U = x^2 + y^2 + z^2$$

$$z = \ln(y - x) \quad Find \left(\frac{\partial U}{\partial z}\right)_y$$

FINAL 4.B PROMPT:

$$S = NK_B\left(\ln\left(\frac{\mathrm{N} - \mathrm{Vb}}{NC}T^{\frac{3}{2}}\right) + \frac{5}{2}\right)$$

$$U = \frac{3}{2}NK_BT - \frac{aN^2}{V} \quad Find \left(\frac{\partial U}{\partial V}\right)_S$$

CONCLUSIONS

Implications for Instruction:

- Use of Diff RE and CRD should be further encouraged
- Students need more practice with:
  - Constructing and reading chain rule diagrams
  - Holding variables constant while evaluating partial derivatives
  - Identifying partial derivatives

Future Work:

- Examination of a pre-test problem
- Better synonymy between prompts

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