Week 2 "Tutorial" – SLAC Lightning Strike

Goals:

1. Understand what delta functions are, how to integrate them, and use them to describe charge distributions.
2. Describe/interpret/represent a physical situation mathematically (course scale learning goal 1 and 2)
3. Check answer using dimensional analysis (course scale learning goal 7)
4. Invoke sophisticated symmetry arguments to predict an answer (course scale learning goals 5b and 7)
5. Learn how to apply Gauss's Law to a physical situation (course scale learning goals 1, 5c, and 9)
6. Communicate reasoning/thought process to group members/LA/Instructor (course scale learning goal 4)

This tutorial is based on:

- CU Boulder original

Materials needed: big white boards, dry erase markers

Tutorial Summary: Students start with basic delta function properties, then sketch a charge density described by a delta function. Next, they describe the charge density of a charged tube (SLAC) using a delta function. They check this answer in various ways. Then, symmetry is used to figure out which way the E-field points before using Gauss's Law to calculate it. As a challenge problem, they figure out where in space there is a divergence.

A few words about running this tutorial:

The room setup has several small tables, with four chairs each. Each group was given several markers, and a big white board that covered the whole table. This allows

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students to easily communicate their ideas to each other, and to myself. It also helps me communicate with the students more effectively.

Before each group begins, I let them know that this is completely voluntary, so they will not be required to turn in the tutorial. However, I advise them that it would be in their interest to fill things out as completely as possible, since many of the topics would also be covered on the homework. The group members are encouraged to work together, and I wander from group to group to see if there are any misconceptions, or if anyone is completely lost/stuck. In the words of Steve Pollock, "We will try our hardest not to give you any of the answers, but we will also try our hardest to make sure you figure everything out for yourself.

Reflection after administering the tutorial

Number of students: 14 (2 groups of 5, 1 group of 4)
Date: Friday, January 25. Second week of class.

My overall reaction after the tutorial was… that students really benefited by working through this. There were several comments made by students about how clarifying parts of this tutorial were for them. Our intention was to make this tutorial shorter than last week's (because so many student did not complete last week's). However, it seemed that this one was about the same length (perhaps even longer) than last week's tutorial. The only group that got through everything stayed 80-85 minutes. This tutorial introduced the *application* of Gauss's Law prior to being covered in lecture. I think it was really useful for students to use Gauss's Law themselves before seeing this done in lecture.

Part 1

i. Sketch: $f(x) = \delta(x-3)$

Everyone was able to sketch this right away. It was covered in lecture the same day as the tutorial.

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(can you *really* sketch this?)

ii. Integrate:

$$\int_{-10}^{10} c \cdot f(x) \cdot dx =$$

$$\int_{-\infty}^{\infty} c \cdot x \cdot f(x) \cdot dx =$$

They had no trouble doing this. It had been addressed in lecture. There was some discussion concerning whether or not the limits of integrations mattered.

iii. What is the physical situation represented by this volume charge density? Make a 3-D sketch of the charge distribution: $\rho(x, y, z) = c \cdot \delta(x - 3)$

Many students asked me if this charge density describe a point charge. I reminded them that x, y, and z all need to be considered. Then I asked if there are any constraints on y and z. It became clear to them at this point that this described a plane, not a point. One student commented while working on this part, "I feel like I understand delta functions so much better now, even though we've used them in other classes."

iv. What is the infinitesimal area, dA, of a small patch on a cylindrical shell centered on the z-axis?

I really expected this to be trivial. When I watched students working on this, they didn't seem to have any problems, but it did take a bit of thought.

Part 2

SLAC (Stanford Linear Accelerator Center) is where quarks (including the charm quark), and the z-boson were discovered. Particles are accelerated inside a long metal cylinder, which is 3 miles long and has a 70 cm diameter.

One afternoon, the beam line is struck by lightening, which gives it a surface charge density σ. Moments after the lightening strikes, Stanford physicists run an

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experiment and claim they've discovered a new particle! You will investigate whether the surface charge of the beam line could have affected the experiment.

After reading part 2, several students weren't sure if the beam line is hollow (it says "cylinder"). There was also a little bit of confusion if the charge density is static, or if an experiment is being run. So to clarify, all the questions are referring to a static surface charge density covering a hollow cylindrical surface.

i. Determine an expression for the volume charge density, ρ, of the beam line.

This was the most challenging part of the whole tutorial. Every group asked questions about this, and many were the same. The most common question was, "So the surface charge is kind of 2-dimensional, but volume charge density is 3-D. How do you change something from 2-D to 3-D?" The best way to guide students away from their confusion was to reference part 1, problem (iii.). I told them that on that problem, they were given a charge density, and they sketched the physical arrangement (math to physics)! Now, they are given the physical arrangement, and they need to describe the charge density mathematically. For students concerned with going from 2-D to 3-D, it might be helpful to point out that on part 1, problem (iii.), they were given a 3-D charge density, but sketched a 2-D surface of charge. One or two students wondered, "Would the charge distribute itself evenly over the beam line?" I pointed out that it was metal, and the student was still not completely convinced that it would be a uniform charge distribution. He seemed to not understand how charge behaves in a conductor. It was very surprising to see this type of confusion at this level.

(note: this is similar to HW3 Q2b)

ii. Check your answer by integrating to find the total charge. (are the units correct?)

This was extremely clarifying for many students. I did have to guide several students to set up the integral properly. But once they integrated, they regained confidence in their answer to question (i.) after seeing that the charge-per-area time the area equaled charge.

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iii. What are the units of your delta function in (i.)? (This is another way of checking your answer to (i.), so don't *only* use your answer to (i.) to check the units)

I noticed several students skip this. Either they already knew the units of the delta function (because it was covered in lecture), or they were completely confident in their answer after doing the check on (ii.).

iv. What direction does the E-field point at all points in space? Explain in detail how you know.

This was a very worth-while problem. At first, students just "knew" which way E pointed. But when questioned, they couldn't tell me why. They kept saying symmetry, "because it's pretty much infinite so there's the same amount of charge on one side as the other." But they lacked a convincing argument. Several students seemed very satisfied after we discussed the symmetry of the situation, and why the E-field could only point in the s-direction.

v. Use Gauss's Law to find the E-field at all points in space.

One of the three groups did not get to this problem in the hour. In one of the groups that did get to this problem, I saw one very interesting misconception. A student knew both forms of Gauss's Law, and thought they were equivalent. He set all four terms equal. We looked at it more closely, to see if the units were in agreement, and he saw that they were not. Many students did not know which form of Gauss's Law to use here. To help them decide, they were asked which form of Gauss's Law would lend itself to a closed imaginary surface to integrate over?

(note: This is the E-field produced by *just* the outer shell of the coax cable on HW3 Q7)

vi. Does the charge σ on the beam line affect the particles being accelerated inside it? Could it affect the electronic equipment outside the tunnel? Could the Stanford scientists have discovered a new particle while the beam tunnel was charged?

I did not see any of the groups discuss this.

Challenge Problem:

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You have found that when charged, SLAC's beam line produces an E-field. In all space, where is this E-field's divergence zero? Where is it non-zero?

One group debated this question. It was a very clarifying discussion, it seemed, once they distinguished between the "spreading" of field lines (no divergence) from the "adding/creating" or "subtracting/destroying" of field lines (positive or negative divergence).

Relevant homework problems:

Q2. δ FUNCTIONS AND CHARGE DISTRIBUTIONS

a) On the previous homework we had two point charges: +3q located at x=-D, and -q located at x=+D.

Write an expression for the *volume* charge density ρ(**r**) of this system of charges.

b) On the previous homework we had another problem with a spherical surface of radius *R* (Fig 2.11 in Griffiths) which carried a uniform surface charge density σ. Write an expression for the *volume* charge density ρ(**r**) of this charge distribution. (Hint: use spherical coordinates, and be sure that your total integrated charge comes out right.)

c) Suppose a linear charge density is given to you as $\lambda(x) = q_0\delta(x) + 4q_0\delta(x-1)$

Describe in words what this charge distribution looks like.

How much total charge do we have?

Assuming that q_0 is given in Colombs, what are the *units* of all other symbols in this equation (including, specifically the symbols λ, x, the delta function itself, the number written as "4" in front of the second term, and the number written as "1" inside the last δ function.

Q7. COAXIAL CABLE.

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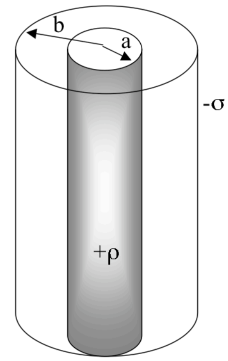

A long coaxial cable carries a uniform volume charge density ρ throughout an inner cylinder (radius *a*) and a uniform surface charge density σ on the outer cylinder (radius *b*). The cable is overall electrically neutral. Find **E** everywhere in space, and graph it.

b) Last week you considered how much charge a child's balloon might hold before sparking. Now let's consider the same question for a real life coax cable. Making any reasonable guess as to physical dimensions for a cable like one that might attach your stereo to your TVs - what would be the maximum static charge you could put onto that cable?
(This is *estimation* - I don't care if you're off by a factor of 2, or even 10, but would like to know the rough order of magnitude of the answer!) From where to where do you expect a spark to go, if it *does* break down?

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