Recorder: ______________________________

Task Master: ____________ **Cynic:** ____________ **Other:** ______________

THE WIRE

Working in small groups (3 or 4 people), solve as many of the problems below as possible. Try to resolve questions within the group before asking for help. The Recorder is responsible for writing up the group's results and turning it in. Show your work! Full credit will only be given if your answer is supported by calculations and/or explanations as appropriate.

Consider the vector field given by *(μ_0 and I are constants)*:

$$\vec{\boldsymbol{B}} = \frac{\mu_0 I}{2\pi}\left(\frac{-y\,\hat{\boldsymbol{\imath}} + x\,\hat{\boldsymbol{\jmath}}}{x^2+y^2}\right) = \frac{\mu_0 I}{2\pi}\,\frac{\hat{\boldsymbol{\phi}}}{r}$$

$\vec{\boldsymbol{B}}$ is the magnetic field around a wire along the z-axis carrying a constant current I in the z-direction.

1. **Ready:**

(a) Determine $\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}}$ on any radial line of the form $y = mx$, where m is a constant.

(b) Determine $\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}}$ on any circle of the form $x^2 + y^2 = a^2$, where a is a constant.

2. **Go:** For each of the following curves C_i, evaluate the line integral $\int_{C_i} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}}$.

(a) C_1, the *top* half of the circle $r = 5$, traversed in a *counterclockwise* direction.

(b) C_2, the *top* half of the circle $r = 2$, traversed in a *counterclockwise* direction.

(c) C_3, the *top* half of the circle $r = 2$, traversed in a *clockwise* direction.

(d) C_4, the *bottom* half of the circle $r = 2$, traversed in a *clockwise* direction.

(e) C_5, the radial line from $(2, 0)$ to $(5, 0)$.

(f) C_6, the radial line from $(-5, 0)$ to $(-2, 0)$.

3. ***FOOD FOR THOUGHT***

(a) Find ***closed*** curves C_7 and C_8 such that this integral is nonzero over C_7 and zero over C_8. *It is enough to draw your curves; you do **not** need to parameterize them.*

(b) Ampère's Law says that, for any closed curve C, this integral is (μ_0 times) the current flowing ***through*** C (in the z direction). Can you use this fact to explain your results to part (a)?

(c) Is $\vec{\boldsymbol{B}}$ conservative?