Students' and Experts' Ways of Reasoning about Partial Derivatives Across STEM Contexts

Eric Weber Tevian Dray Corinne Manogue David Roundy Mary Bridget Kustusch
Oregon State University DePaul University

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ABSTRACT

A common feature across STEM disciplines is the study of change, whether studying how changing a design parameter affects the operation of a prototype, or how pressure changes when we adiabatically compress a gas. Indeed, the nature of scientific measurement is to control some physical quantities while measuring others. Mathematically, we express the concept of changing one parameter while fixing others by using partial derivatives. However, how we use partial derivatives and how we talk about partial derivatives vary dramatically across STEM disciplines. The purpose of this poster is to share our preliminary results from student and expert problem-solving interviews about partial derivatives.

MOTIVATION FOR STUDY

Difficulty conceiving of meaning of partial derivative for students.

Experts employ a wide variety of reasoning strategies about partial derivatives, including the use of difference quotients constructed from numerical data, graphical reasoning about the slope at a single point, graphical reasoning about the shape of the graph, and symbolic computation.

Experts we interviewed adopted a a single reasoning strategy, and did not necessarily transition spontaneously to other strategies.

STRANDS OF WORK

First Strand

Survey across STEM disciplines of representations of partial derivatives used by experts

- -Identify normative practices that are common across many STEM disciplines
- -Identify practices which are discipline specific.

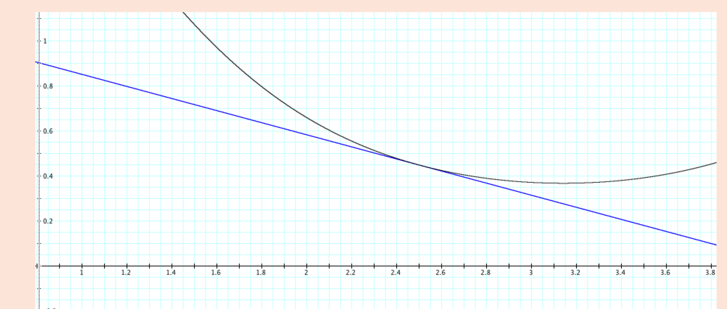
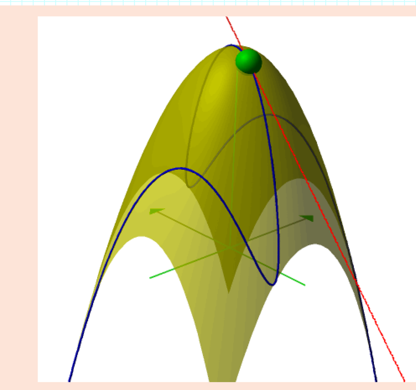
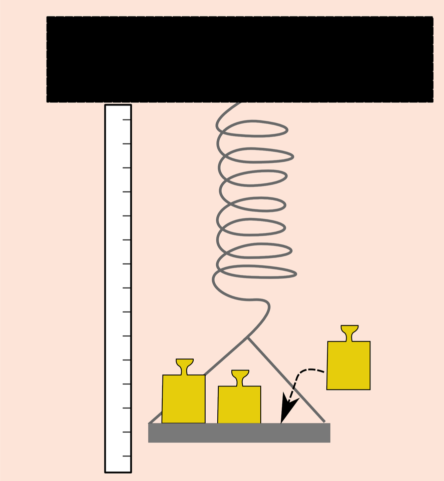
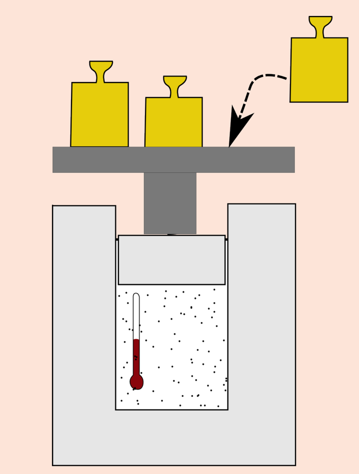
Second Strand

Student understanding of partial derivatives

- -Physics
- -Engineering
- -Mathematics

Third Strand

Create learning trajectories related to partial derivatives

	Operation on Expression	Ratio of Small Quantities	Tangent Line	Experiment
Representation	$x^n \to nx^{n-1}$ $x^n y^m \to nx^{n-1} y^m$	$\dfrac{\Delta T}{\Delta p}$ $\left.\dfrac{\Delta T}{\Delta p}\right\vert_{S_0} = \dfrac{T(p+\Delta p, S_0) - T(p, S_0)}{\Delta p}$	 	 
Reasoning	" I need the symbols then I can do the differentiation" " A derivative comes from applying a process to the algebra no matter how many variables I have"	" The derivative is a ratio of a small change in temperature to a small change in pressure" "A derivative compares how one physical quantity changes when another physical quantity is changed with 'everything else held fixed'"	"The derivative is the slope of the tangent line" "The derivative is the slant of the tangent line, but the direction of the line matters in this case"	"I can measure the derivative by measuring the change in one property that happens when I make an adjustment to a second property, while holding a third (and possibly fourth or fifth) property fixed." "A partial derivative is a way of describing a scientific experiment, in which you change one parameter, while holding other parameters fixed, and measure a change that results."
Subtleties/Issues	If $f = \dfrac{x+y}{r}$, What is $\left.\dfrac{\partial f}{\partial x}\right\vert_{y=y_0}$?	*How small is appropriately small in an experimental setting? *How is the ratio of small quantities related to the limit definition of derivative? *What does "everything else held fixed" mean?	* Some experts and students assume they can use curve-fitting software without any further knowledge about the functional form. * This approach presupposes that one has already figured out how to hold fixed the "other" independent variables.	* The analysis of the experiment would use one of the other approaches, most commonly either the ratio or tangent approach. * Changing parameters, holding them fixed, and measuring parameters can be a challenging task for some physical properties.

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