

The Partial Derivative Machine

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PERC 2013, Portland, OR

Abstract

Research has shown that students struggle to understand the use of partial derivatives in thermodynamics. We have designed an apparatus, which we have called a Partial Derivative Machine, that serves as a mechanical analogue of a thermodynamic system. Using this device, students have a tangible way to wrestle with issues related to partial derivatives and thermodynamics, such as which variables are held fixed, how many variables are independent, and how energy can be added to a system. In this paper, we present a description of the apparatus, an introduction to the associated activities, and an overview of how this apparatus can be connected to thermodynamic systems.

Acknowledgments

This work was supported by NSF DUE-1023120.



A Mechanical Analog for Thermodynamics

The Partial Derivative Machine can be used to teach various mathematical techniques such as:

- Total Differentials
- Maxwell Relations
- Partial Derivative Manipulations
- Cyclic Chain Rule
- Legendre Transforms

What is a Partial Derivative Machine?

The Partial Derivative Machine is an apparatus consisting of a central spring system that can be stretched via four strings extending outward. This central system is on a large piece of particle board which features a pulley on two adjacent corners (Corners C and D), and a knob on all four corners. The system is held in place using the knobs at A and B. Adding weight to the strings at C and D causes the system to stretch, and this stretching can be measured using the flags on the strings (Example labeled E).

Manipulating the System:

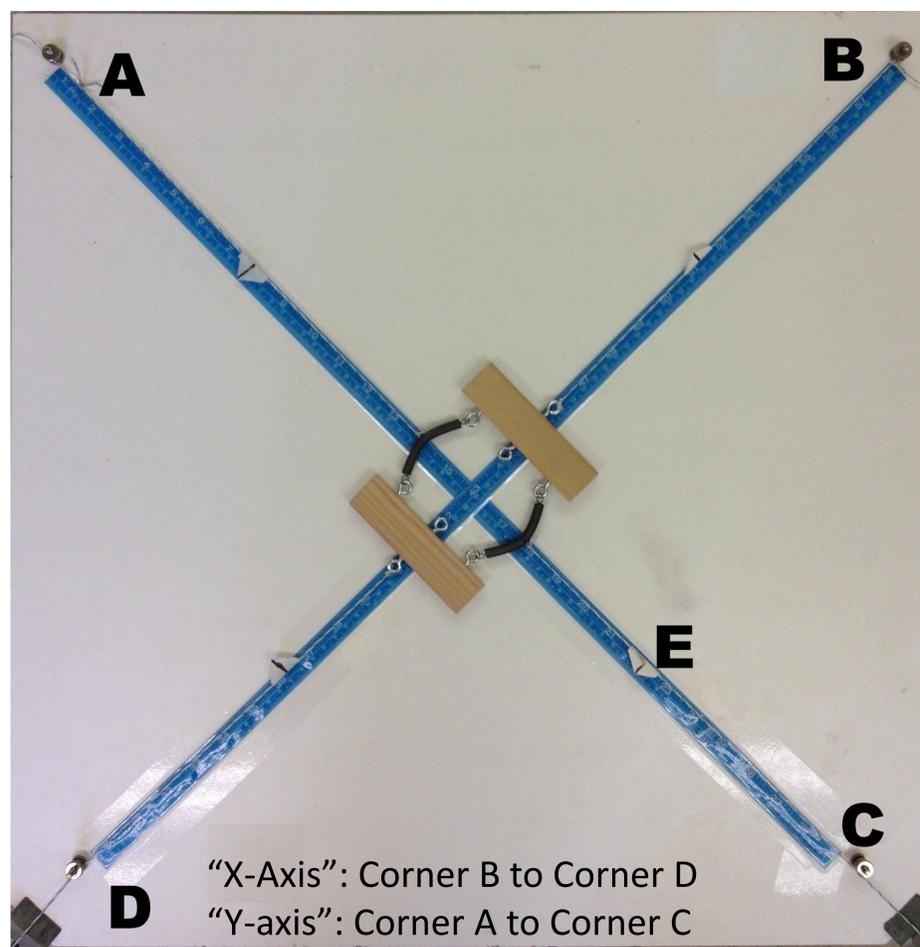
There are two conditions under which the system can be manipulated.

Method 1:

1. Tighten knob on corner C or D to fix y or x (respectively).
2. Add weight to the free string.
3. Measure changes to the other variables.

Method 2:

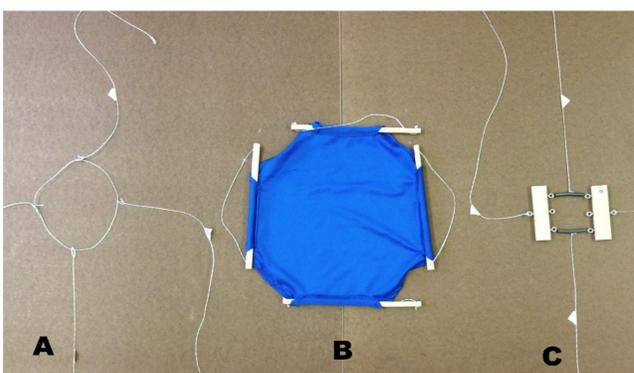
1. Leave knobs on both corner C and D free.
2. Add weight to either string at C or D.
3. Measure changes to other variables.



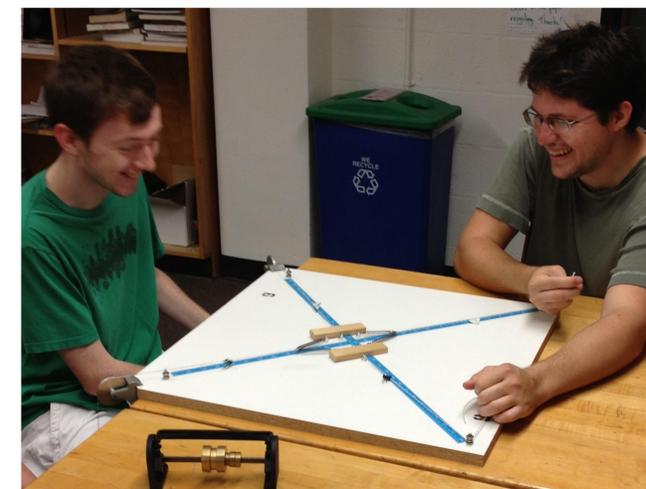
Physical Variables:

- x : distance between the flags on the X strings
- y : distance between the flags on the Y strings
- F_x : tension in the X oriented strings
- F_y : tension in the Y oriented strings

Alternative Central Systems:



By using central systems such as a loop of string (A), a piece of spandex (B), and different spring systems, students can experience various relationships between the properties of the system.



Classroom Activities:

Playing with the Machine (≈15 minutes):

Determine:

- The properties of the system that can be controlled.
- The properties of the system that can be measured.
- The number of independent properties of the system.

Finding a Simple Derivative (≈20 minutes):

Find the derivative $\frac{\partial x}{\partial F_x}$ keeping in mind that this derivative may differ between the situation where y is constant compared with the situation where F_y is held constant.

Integrated Lab (≈50 minutes):

Determine the potential energy of the system by measuring the relationships between x , y , F_x , and F_y .