Navigating a Thermo Maze

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Paradigms in Physics
www.physics.oregonstate.edu/portfolioswiki

In recent years, there have been several studies suggesting that upper-division students often get lost in the maze of partial derivatives and complicated chain rules ubiquitous in thermodynamics. As part of a project researching how to help students learn to think like practicing physicists, we are interviewing experts (primarily faculty who teach thermodynamics) to understand how they navigate through this maze. We asked each expert to solve a challenging and novel thermodynamics problem using the van der Waals equations of state and to reflect upon their path(s) through the problem. To date, we have found a tremendous variety in both solution strategies and sense-making tools. Additionally, we gave the same problem to several junior-level physics majors who had just completed the Paradigms in Physics: Energy & Entropy course, which will allow us to analyze how they dealt with the problem compared to the experts. This poster will present an overview of the project, some initial observations, and future directions.

The Maze

The following are the equations of state for a van der Waals gas, which modifies the ideal gas law by considering particles with non-zero volume that have an attractive force between them (i.e. a van der Waals force).

$$p = \frac{NkT}{V - N\,b} - \frac{a\,N^2}{V^2}$$
$$S = Nk\left\{\ln\left[\frac{(V-N\,b)T^{3/2}}{N\Phi}\right] + \frac{5}{2}\right\}$$
$$U = \frac{3}{2}NkT - \frac{a\,N^2}{V}$$

Find the following quantity for a van der Waals gas:

$$\left(\frac{\partial U}{\partial p}\right)_S$$

Current & Expected Data

- 6-8 Physics faculty (incl. two co-authors, one of whom is the *Energy & Entropy* instructor)
- 2 Math faculty (incl. one co-author)
- 1 Physics grad student (graduate TA for *Energy & Entropy* course)
- 1 Math Ed grad student (former Paradigms student)
- 6 undergraduate physics majors (just completed *Energy & Entropy*)
- Classroom video data (2009-2012) with groups solving similar problems

PERC Paper/Poster

An Expert Path Through a Thermo Maze

This case study will present one expert's path to a solution (using 1st law), including the his sense-making, detours, and dead-ends. It will also present his outline of two other approaches (using energy equation of state and using differentials), as well as his reflections on these methods and their utility in teaching undergraduates.

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Interviewed faculty from other institutions

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Key Branch Points

Energy Equation

1st Law of Thermodynamics

$$dU = T\,dS - p\,dV$$

Other

$$H = U + pV$$
$$F = U - TS$$
$$G = H - TS$$

Equation of State

$$U = \frac{3}{2}NkT - \frac{a\,N^2}{V}$$

Differentials *v.* Partials

Differentials

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$
$$dp = \left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT$$
$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_V dT$$

Partial Derivatives

$$\left(\frac{\partial V}{\partial p}\right)_S = \left(\frac{\partial V}{\partial T}\right)_S \left(\frac{\partial T}{\partial p}\right)_S$$
$$\left(\frac{\partial V}{\partial T}\right)_S = -\left(\frac{\partial V}{\partial S}\right)_T \left(\frac{\partial S}{\partial T}\right)_V$$
$$\left(\frac{\partial T}{\partial p}\right)_S = \left[\left(\frac{\partial p}{\partial T}\right)_S\right]^{-1}$$
$$\left(\frac{\partial p}{\partial T}\right)_S = \left(\frac{\partial p}{\partial T}\right)_V + \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_S$$

Explicit *v.* Implicit Derivatives

Explicit Derivatives

$$dU = \frac{3}{2}Nk\,dT + \frac{aN^2}{V^2}dV$$

Implicit Derivatives

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$

Solving a System of Equations

Substitution Method

1. solve dS equation for dV
2. substitute dV into dU equation
3. substitute dV into dp equation
4. solve dp equation for dT
5. substitute dT into dU equation
6. set $dS = 0$
7. divide dU equation by dp

Addition/Subtraction Method

1. multiply dp and dS equations by dV coefficients and subtract.
2. multiply dp and dS equations by dT coefficients and subtract.
3. write dV and dT in terms of dp and dS
4. substitute dV and dT into dU equation
5. set $dS = 0$
6. divide dU equation by dp

Navigational Tools

Dimensional Analysis

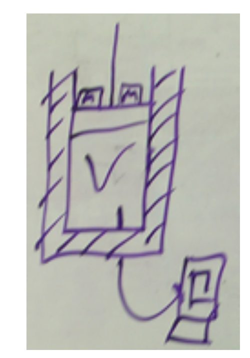
$$p = \frac{NkT}{V - N\,b} - \frac{a\,N^2}{V^2}$$

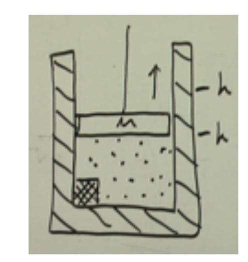
"Ok, so, as I'm writing this, I'm thinking that these [V, Nb] are both extensive, that's [N] extensive, this [p] is intensive, that's [T] intensive, this [aN^2/V^2] is intensive, all is good... b has dimensions of volume, a has dimensions of, it has dimensions of energy times volume, huh, interesting."

Name the Experiment

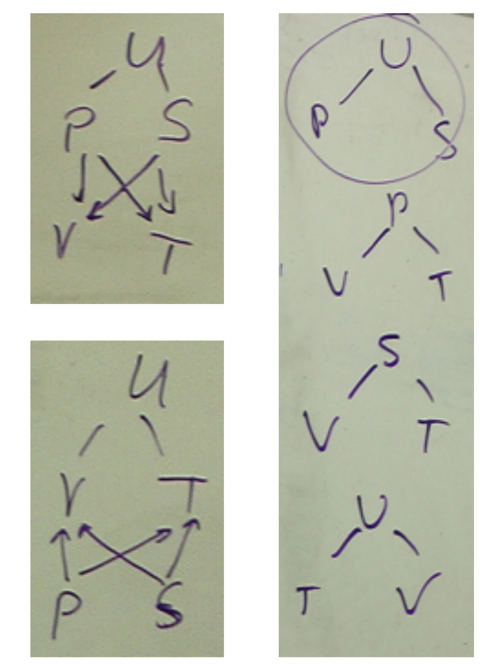
Draw/Sketch an experiment that would measure

$$\left(\frac{\partial U}{\partial p}\right)_S$$



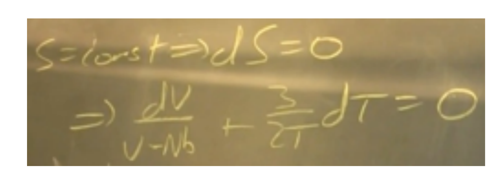


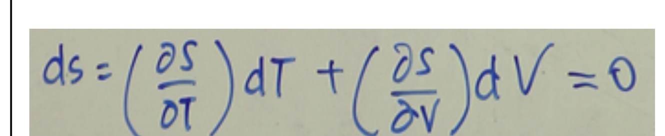
Chain Rule Diagrams

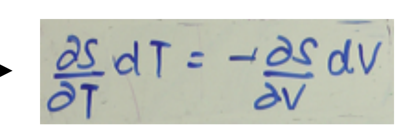


Shortcuts

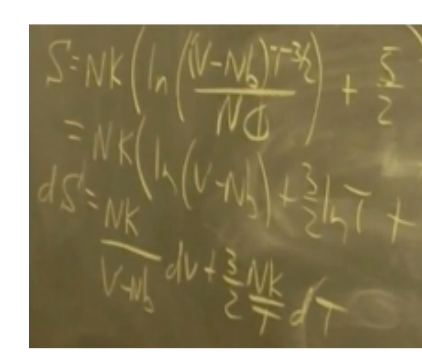
Variables held constant







Logarithm rules



Name the thing you don't know