

Algebra Worksheet

Knowing how to solve both linear and quadratic equations is a key step to knowing how to solve problems that you will encounter in both math and physics classes. This worksheet will give some practice in one variable, then move on to solving equations in two variables, then solving quadratics. We will begin with some review of key algebraic ideas and strategies. Then look at some examples that you will practice on. The final section of this worksheet will allow you to practice your algebra skills on a real physics problem.

Review Problems

Simplify:

1. $7x + 5x$

2. $9x^2 - x^2$

3. $9y + 5y^2 + 3y + 4y^2$

4. $7(4t - 5) - 8t$

5. $7 - 4[3 - (4s - 5)]$

6. $14n^2 + 5 - [7(n^2 - 2) + 4]$

7. $(-3v^2k^{-5})^3(2v^4k^7)^2$

8. $(5 + \sqrt{7})(3 - \sqrt{2})$

1 Linear Equations in one variable

Disclaimer: If any of the terminology used in this worksheet is unfamiliar (or forgotten) that is a sign that you should review some math.

Definition A *linear equation* is an equation that can be written in the form

$$ax + b = 0$$

where a and b are constants.

Here, x is the variable and this equation is considered *linear* because the power of x is equal to one ($x = x^1$).

Strategy

Step 1: Simplify each side of the equation.

This would involve things like removing parentheses, fractions, decimals, and combining like terms.

- To remove parentheses, use the distributive property.
- To remove fractions, multiply each side of the equality by the least common denominator of all the fractions.

Step 2: Use addition and subtraction properties to move all of the variable terms to one side of the equality and all other terms to the other side.

Step 3: Use multiplication and division properties to remove coefficients from in front of the variable.

Step 4: Check your answer by plugging it into the variable.

Example: Solve the equation $2(t + 5) - 7 = 3(t - 2)$

$2t + 10 - 7 = 3t - 6$ Remove the parentheses by using the distributive property.

$$2t + 3 = 3t - 6$$

$2t - 3t + 3 = 3t - 3t - 6$ Move the variable terms to one side.

$$-t + 3 = -6$$

$-t + 3 - 3 = -6 - 3$ Move the non-variable term to the other side.

$$-t = -9$$

$\frac{-t}{-1} = \frac{-9}{-1}$ Divide by -1 to remove the coefficient from the variable. Notice that the t has -1 in front of it.

$$t = 9$$

Now to check our answer:

$$2(9 + 5) - 7 = 3(9 - 2) \quad \text{Plug value in for } t.$$

$$2(14) - 7 = 3(7)$$

$$28 - 7 = 21$$

$$21 = 21 \quad \text{Check!}$$

Practice Problems

1. $\frac{5}{4}s + \frac{1}{2} = 2s - \frac{1}{2}$

2. $.35y - .2 = .15y + 1$

3. A student on a scooter is initially traveling at 23 m/s. (Yes. She's bookin' it.) Find how long it takes her (in seconds) to reach a velocity of 31 m/s if her acceleration is 2 m/s^2 . (Hint: Use the equation $V_f = V_i + at$ where V_f is her final velocity, V_i is her initial velocity, a is her acceleration, and t is time.)

2 Linear Equations in Two Variables

Now we will look at systems that have two linear equations and two unknowns (variables).

Definition: A *system of linear equations* is two or more linear equations that are being solved

simultaneously.

Here, we will be looking at systems that have only two linear equations and two unknowns. In general, a solution of a system in two variables is an ordered pair that makes BOTH equations true.

There are two ways to solve systems of linear equations in two variables:

1. The Substitution Method
2. The Elimination Method

Which method you choose is entirely up to you. Use the one that makes the most sense or works best for the problem. First, we will look at the strategy for using the Substitution Method.

Strategy

Step 1: Simplify if needed.

This step uses the techniques used when solving a linear equation in one variable. Simplify each of the equations in the system before solving.

Step 2: Solve one equation for either variable.

It doesn't matter which equation you use or which variable you choose to solve for, the goal is to make it as simple as possible. If one of the equations is already solved for one of the variables, that is a quick and easy way to go. If you need to solve for a variable, then try to pick one that has a 1 as a coefficient. That way when you go to solve for it, you won't have to divide by a number and run the risk of having to work with a fraction (yuck!!).

Step 3: Substitute what you get for step 2 into the other equation.

This is why it is called the substitution method. Make sure that you substitute the expression into the OTHER equation, the one you didn't use in step 2. This will give you one equation with one unknown.

Step 4: Solve for the remaining variable.

Solve the equation set up in step 3 for the variable that is left.

Step 5: Solve for the second variable.

Plug the value found in step 4 into any of the equations in the problem and solve for the other variable.

Step 6: Check the proposed ordered pair solution in BOTH original equations.

Example: Solve the following:

$$\begin{cases} 3x - 2y = 6 \\ x - y = 1 \end{cases}$$

Step 1: Simplify if needed.

Both of these equations are simplified. Move on to step 2.

Step 2: Solve one equation for either variable.

It does not matter which equation or which variable you choose to solve for. One choice may be wiser than the other. If solving for one variable is proving too complicated, go back and start with the other. The easiest route here is to solve the second equation for x .

Solving the second equation for x we get:

$$x - y = 1$$

$$x - y + y = 1 + y \quad \text{Add } y \text{ to both sides of the equation.}$$

$$x = y + 1$$

Step 3: Substitute what you get for step 2 into the other equation.

$$3x - 2y = 6$$

$$3(y + 1) - 2y = 6 \quad \text{Substitute the value you found for } x \text{ into the other equation.}$$

Step 4: Solve for the remaining variable.

$$3(y + 1) - 2y = 6$$

$$3y + 3 - 2y = 6 \quad \text{Distribute 3 over } (y + 1).$$

$$y + 3 = 6 \quad \text{Combine like terms.}$$

$$y + 3 - 3 = 6 - 3 \quad \text{Subtract 3 from both sides.}$$

$$y = 3$$

Step 5: Solve for the second variable.

Plug the result for y into the equation in step 2 to find x .

$$x = y + 1$$

$$x = 3 + 1$$

$$x = 4$$

Step 6: Check the solutions in BOTH original equations.

$$\begin{cases} 3x - 2y = 6 \\ x - y = 1 \end{cases}$$

$$\begin{cases} 3(4) - 2(3) = 6 \\ 4 - 3 = 1 \end{cases}$$

$$\begin{cases} 12 - 6 = 6 \\ 1 = 1 \end{cases}$$

$$\begin{cases} 6 = 6 \\ 1 = 1 \end{cases}$$

Thus, (4,3) is a solution to the system.

Now for the **Elimination Method**.

Strategy

← not picking, I know.

Step 1: Simplify and put both equations into $ax + by + c$ form, if necessary.

Simplify just like you would in the substitution method, step 1.

Step 2: Multiply one or both equations by a number that will create opposite coefficients for either x or y if needed.

Looking ahead, we will be adding (or subtracting) these equations. In that process, we need to make sure that one of the variables drops out, leaving us with one equation and one unknown. The only way we can guarantee that is if we are adding opposites (subtracting). It doesn't matter which variable you choose to drop out, you just want to keep it as simple as possible. For example, if you had a $2x$ in one equation and a $3x$ in another equation, we could multiply the first equation by 3 and get $6x$ and the second equation by -2 to get a $-6x$. So when you go to add these two together they will drop out.

Step 3: Add the equations. If it is easier to think of this as subtracting the equations, that is fine. Adding opposite signs and subtracting is really the same thing. For example, $3+(-2)$ is the same thing as $3-2$. The variable that has the opposite coefficients will drop out in this step and you will be left with one equation with one unknown.

Step 4: Solve for the remaining variable.

Solve the equation found in step 3 for the variable that is left.

Step 5: Solve for the second variable.

Plug the value found in step 4 into any of the equations in the problem and solve for the other variable.

Step 6: Check the solutions in BOTH original equations.

You can plug the proposed solution into both equations. If it makes both equations true, then

you have your solution to the system.

Example:

$$\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 13 \\ \frac{1}{5}x + \frac{1}{8}y = 5 \end{cases}$$

Step 1: Simplify and put both equations into $ax + by - c$ form, if necessary.

We can simplify both equations by multiplying each separate one by its least common denominator, just like you can do when you are working with one equation. As long as you do the same thing to both sides of an equation, you keep the two sides equal to each other. Here, we will multiply the first equation by 6, and the second by 40.

$$\begin{cases} (6)(\frac{1}{2}x + \frac{1}{3}y) = (6)(13) \\ (40)(\frac{1}{5}x + \frac{1}{8}y) = (40)(5) \end{cases}$$
$$\begin{cases} 3x + 2y = 78 \\ 8x + 5y = 200 \end{cases}$$

Step 2: Multiply one or both equations by a number that will create opposite coefficients for either x or y if needed.

Again, you want to make this as simple as possible. Note how the coefficient on y in the first equation is 2 and in the second equation it is 5. We need to have opposites, so if one of them is 10 and the other is -10, they would cancel each other out when we go to add them. So, lets multiply the first equation by 5 and the second equation by -2, this would create a 10 and a -10 in front of the y variables and we will have our opposites.

$$\begin{cases} (5)(3x + 2y) = (5)(78) \\ (-2)(8x + 5y) = (-2)(200) \end{cases}$$
$$\begin{cases} 15x + 10y = 390 \\ -16x - 10y = -400 \end{cases}$$

Step 3: Add the equations.

$$\begin{cases} 15x + 10y = 390 \\ -16x - 10y = -400 \end{cases}$$
$$-x = -10 \quad \text{Note that the } y\text{'s dropped out.}$$

Step 4: Solve for the remaining variable.

$$-x = -10$$
$$x = 10$$

Step 5: Solve for the second variable.

You can choose any equation used in this problem to plug in the found x value. Lets plug in 10 for x into the first simplified equation (found in step 1) to find y 's value.

$$3x + 2y = 78$$

$$3(10) + 2y = 78 \quad \text{Plug 10 in for } x.$$

$$30 + 2y = 78$$

$$30 + 2y - 30 = 78 - 30 \quad \text{Subtract 30 from both sides.}$$

$$2y = 48$$

$$\frac{2y}{2} = \frac{48}{2} \quad \text{Divide each side by 2.}$$

$$y = 24$$

Step 6: Check the solutions in BOTH original equations.

$$\begin{cases} \frac{1}{2}(10) + \frac{1}{3}(24) = 13 \\ \frac{1}{5}(10) + \frac{1}{8}(24) = 5 \end{cases}$$

$$\begin{cases} 5 + 8 = 13 \\ 2 + 3 = 5 \end{cases}$$

$$\begin{cases} 13 = 13 \\ 5 = 5 \end{cases}$$

Thus, $(10,24)$ is a solution to the system.

Practice Problems

Solve this system using the substitution method:

$$1. \begin{cases} 4x + y = 5 \\ 2x - 3y = 13 \end{cases}$$

Solve this system using the elimination method:

$$2. \begin{cases} 2x - 3y = 4 \\ 4x + 5y = 3 \end{cases}$$

3. The sum of Orion and Sagan's age is 24, and the difference between their ages is 6. Find their ages given that Orion is older than Sagan.

4. A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees, and totalled \$487. The second order was for 6 bushes and 2 trees, and totalled \$232. The bills do not list the per-item price. What were the costs of one bush and of one tree?

3 Quadratic Equations

The *standard form* for a quadratic equation is

$$ax^2 + bx + c = 0$$

where a does not equal 0.

Introduction

We will be looking at solving a specific type of equation called the quadratic equation. Two methods of solving these types of equations are solving by factoring, and by using the quadratic equation. Sometimes one method won't work or another is just faster, depending on the quadratic equation given. Note that the difference between linear equations and quadratic equations is that the highest exponent on the variable on the quadratic equation is 2.

Solving Quadratic Equations by Factoring

You can solve a quadratic equation by factoring if, after writing it in standard form, the quadratic expression factors.

Strategy

Step 1: Simplify each side if needed.

Like linear equations, this would involve things like removing parentheses, removing fractions, adding like terms, etc.

Step 2: Write in standard form, if needed.

If it is not in standard form, move any term(s) to the appropriate side by using the addition/subtraction property of equality. Also, just for clarity, make sure that the squared term is written first, the x term is second and the constant is third and it is set equal to 0.

Step 3: Factor.

Step 4: Use the Zero-Product Principle: If $ab = 0$, then $a = 0$ or $b = 0$.

0 is our magic number because the only way a product can become 0 is if at least one of its factors is 0.

Step 5: Solve for the linear equation(s) set up in step 4.

If a quadratic equation factors, it will factor into either one linear factor squared or two distinct linear factors. So, the equations found in step 4 will be linear equations. You can solve them using the techniques discussed above.

Example:

Solve $x^2 - 10 = 3x$ by factoring.

Step 1: Simplify each side if needed.

This quadratic equation is already simplified.

Step 2: Write in standard form, if needed.

$$x^2 - 10 = -3x$$

$$x^2 - 10 + 3x = -3x + 3x \quad \text{Add } 3x \text{ to both sides to set equation equal to zero.}$$

$$x^2 + 3x - 10 = 0 \quad \text{Rearrange into standard form.}$$

Step 3: Factor.

$$x^2 + 3x - 10 = 0 \quad \text{Equation in standard form.}$$

$$(x + 5)(x - 2) \quad \text{The equation factored.}$$

Step 4: Use the Zero-Product Principle.

We know

$$x^2 + 3x - 10 = (x + 5)(x - 2) = 0$$

So, by the Zero-Product principle, either $(x + 5) = 0$ or $(x - 2) = 0$. These are linear equations, and can be solved as such.

Step 5: Solve for the linear equation(s) set up in step 4.

$$x + 5 = 0$$

$$x = -5$$

$$x - 2 = 0$$

$$x = 2$$

There are two solutions to this quadratic equation: $x = -5$ and $x = 2$.

Now on to our other technique for solving quadratic equations, using the **Quadratic Formula**.

Theorem: When $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You can solve any quadratic equation by using the quadratic formula. This comes in handy when a quadratic equation does not factor or is difficult to factor.

Strategy

Step 1: Simplify each side if needed.

Step 2: Write in standard form, if needed.

Step 3: Identify a, b, and c.

When the quadratic equation is in standard form, $ax^2 + bx + c = 0$, then a is the coefficient in front of the x^2 term, b is the coefficient in front of the x term, and c is the constant term.

Step 4: Plug the values found in step 3 into the quadratic formula.

Step 5: Simplify if possible.

Example:

Solve $3x^2 = 7x + 20$ by using the Quadratic Formula.

Step 1: Simplify each side if needed.

This quadratic is already simplified.

Step 2: Write in standard form, if needed.

$$3x^2 = 7x + 20$$

$3x^2 - 7x - 20 = 7x - 7x + 20 - 20$ Subtract and add terms from both sides to get one side equal to zero.

$$3x^2 - 7x - 20 = 0 \quad \text{Standard form.}$$

Step 3: Identify a, b, and c.

Here, $a = 3$, $b = -7$, and $c = -20$.

Step 4: Plug the values found in step 3 into the quadratic formula.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-20)}}{2(3)}$$

Step 5: Simplify if possible.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-20)}}{2(3)}$$

$$x = \frac{7 \pm \sqrt{49 + 240}}{6} \quad \text{Simplify under the radical first.}$$

$$x = \frac{7 \pm \sqrt{289}}{6}$$

$$x = \frac{7 \pm 17}{6} \quad \text{Take the square root of 289.}$$

$$x = \frac{7+17}{6} \text{ or } x = \frac{7-17}{6} \quad \text{Split the equation into its two parts.}$$

$$x = \frac{24}{6} \text{ or } x = \frac{-10}{6} \quad \text{Continue to simplify.}$$

$$x = 4 \text{ or } x = \frac{-5}{3}$$

Thus, $x = (4, \frac{-5}{3})$

Here is a quick word about the solutions that are possible when using the Quadratic Formula.

Discriminant

When a quadratic equation is in standard form, the expression, $b^2 - 4ac$ that is found under the square root part of the quadratic formula is called the discriminant. The discriminant can tell you how many solutions there are going to be and if the solutions are real numbers, the type of numbers that represent real world measurables, or complex imaginary numbers, which, since they have no "real world" use, won't be worried about here.

$b^2 - 4ac > 0$	Two distinct real solutions
$b^2 - 4ac = 0$	One real solution
$b^2 - 4ac < 0$	Two distinct complex imaginary solutions

Practice Problems

1. Solve by factoring: $x^2 + x = 42$

2. Solve by using the Quadratic Formula: $x^2 + 10x + 25 = 0$

3. An object is launched at 19.6 m/s from a 58.8 meter tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where s is in meters. When does the object strike the ground? Note that you will get two answers. (Hint: You are looking for the time when the object hits the ground so set the equation equal to zero and solve for t .)

Did you notice something odd about the solutions to the last problem? Was one of your answers negative? This negative result is, indeed, a solution to the problem but it doesn't seem to make any sense. How can we have a negative unit of time? This negative answer actually means that the ball also hits the ground x seconds before it was launched. If we could rewind time, that's when the ball would have last been at ground level. This is a correct mathematical answer, but in physics, we only accept answers that have real-world meaning. Thus, in this case, we can ignore the negative result and conclude that the object hits the ground at precisely the time that is the positive result.

All right. Now that you have had some practice working with equations algebraically, let's put it to use in the real world and derive a couple of important physics equations. We will be working completely generally here, so there will be no numerical coefficients, but don't freak out. The algebra is still the same and these general solutions give equations that can be used in every case, so they are much more useful.

Perfectly Elastic Collisions in One Dimension

When two particles collide, they exert forces on one another that are much larger than any external acting forces. Thus, we may assume that external forces are negligible, with the consequence that the momentum of the system remains constant. In other words, the sum of the momentums of each particle before a collision is the same as the sum of the momentums of each particle after the collision (conserved). In elastic collisions, the loss in kinetic energy is negligible, and so the kinetic energy of the system is also conserved. Here are the constants, variables, and equations we will be working with:

Constants (Knowns)	
m_1	mass of particle one
m_2	mass of particle 2
v_1	velocity of particle one before the collision
v_2	velocity of particle two before the collision
Variables (Unknowns)	
V_1	velocity of particle one after the collision
V_2	velocity of particle two after the collision

Equations	
$\mathbf{p} = m \mathbf{v}$	The linear momentum, \mathbf{p} , of a particle
$K = \frac{1}{2}mv^2$	The kinetic energy, K , of a particle

Since momentum is conserved, we can say

$$m_1v_1 + m_2v_2 = m_1V_1 + m_2V_2 \quad (3.1)$$

But we also know, since this is a perfectly elastic collision, that kinetic energy is conserved, thus

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 \quad (3.2)$$

I know this looks a little scary but is just algebra. The main thing is to stay calm and do all of the algebra steps above. We will walk through this together and it will be *great* practice.

Let's say that we know what the masses of the two objects are and their initial velocities. What we want to find is the final velocities of the two objects. Right away, we can see that we have two equations, the momentum equation and the kinetic energy equation, and two unknowns, V_1 and V_2 . This means that we can use the same techniques we learned in solving systems of equations to figure out this problem.

Let's make our task slightly less confusing by renaming the constants and variables in the table above to get rid of those subscripts. We'll let:

$$\begin{aligned} m_1 &= s \\ m_2 &= t \\ v_1 &= x \\ v_2 &= y \\ V_1 &= w \\ V_2 &= z \end{aligned}$$

Then, by substituting these into equations (3.1) and (3.2), our equations become:

$$sx + ty = sw + tz \quad (3.3)$$

and

$$\frac{1}{2}sx^2 + \frac{1}{2}ty^2 = \frac{1}{2}sw^2 + \frac{1}{2}tz^2 \quad (3.4)$$

We can multiply equation (3.4) by 2 to simplify to:

$$sx^2 + ty^2 = sw^2 + tz^2 \quad (3.5)$$

Now we will work together. I will tell you what to do, and you will carry out the calculations in the spaces. Along the way, I will give you checkpoints so you can be sure that you are on the right track. Ready? Remember, be brave! It's just algebra.

First, solve the kinetic energy equation (3.5) for z^2 .

Now solve the momentum equation (3.3) for z .

We can square what you found in the step above to find another expression for z^2 .

$$\left(\frac{sx + ty - sw}{t}\right)^2 = z^2$$

Is this what you got? Good! Let's proceed. Why did we square that last equation? We did that so we could set both expressions for z^2 equal to one another and thereby eliminate one of our variables z . Set the two expressions we found for z^2 equal to one another and you'll see what I mean. (Just write it in the space. Don't solve anything yet.)

See what I mean? We have used the technique of elimination cleverly to rid ourselves of one

of our variables! Now we are only left with one variable, w , which means that even though the rest of this is going to get a bit messy, we are just solving an equation in one variable. This one will be a quadratic so it will nicely tie all of the concepts we talked about at the beginning of the unit together. Now let's work with this equation and eventually solve for w .

You should now have something that looks like this:

$$\left(\frac{sx + ty - sw}{t}\right)^2 = \frac{sx^2 + ty^2 - sw^2}{t}$$

Go ahead and square the left side.

Did you get

$$\frac{s^2x^2 + t^2y^2 + s^2w^2 + 2stxy - 2s^2xw - 2styw}{t^2} = \frac{sx^2 + ty^2 - sw^2}{t} \quad (3.6)$$

If not, check what you got against what you see in equation (3.6). Maybe your variables or terms are in a different order than in equation (3.6). That's O.K. It is still the same as what you see in equation (3.6). However, if the problem is more than just a matter of order, go back and check your calculations. Remember the process for squaring trinomials. Next, multiply both sides by t^2 to clear the fractions.

We can simplify a little by subtracting the t^2y^2 terms from both sides.

Now, move both terms from the right side of the equation to the left side by adding one term to both sides and subtracting the other from both sides. You should end up with zero on the right side.

You should have something that looks like

$$s^2x^2 + s^2w^2 + 2stxy - 2s^2xw - 2styw - stx^2 + stw^2 = 0$$

Since we are solving for w , and w is squared in some of our terms, we are going to have to resort to solving this as if it were a quadratic equation (because it is one). So let's put this beast into standard form, $aw^2 + bw + c = 0$ by gathering some terms and factoring.

$$(s^2 + st)w^2 + (-2s^2x - 2sty)w + (s^2x^2 + 2stxy - stx^2) = 0$$

We are going to need to use the Quadratic Formula (QF), as this thing is not so easily factorable. So, next, we identify our a, b and c .

$$\begin{aligned}a &= (s^2 + st) \\b &= (-2s^2x - 2sty) \\c &= (s^2x^2 + 2stxy - stx^2)\end{aligned}$$

Recall that we are going to be plugging all of this into the QF, $w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, but that is going to be a huge mess to try to solve if we just plug in right now. Instead, let's find $-b$, b^2 , $4ac$, and $b^2 - 4ac$ separately, then plug their simplified forms into the QF.

First, find $-b$.

Now, b^2

Next, $4ac$

And finally, $b^2 - 4ac$. (Use the b^2 and the $4ac$ that you just found.)

So, the QF, $w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, becomes

$$\begin{aligned} w &= \frac{2s^2x + 2sty \pm \sqrt{s^2t^2(2y - 2x)^2}}{2(s^2 + st)} \\ &= \frac{(2s^2x + 2sty) \pm (st(2y - 2x))}{2(s^2 + st)} \\ &= \frac{(2s^2x + 2sty) \pm (st(2y - 2x))}{2(s(s + t))} \\ &= \frac{(2s^2x + 2sty) \pm (st(2y - 2x))}{2s(s + t)} \end{aligned}$$

Here is an example of when using the negative part of a solution doesn't make sense, as discussed above, so we will only proceed with the positive part.

So, we have

$$\begin{aligned} w &= \frac{(2s^2x + 2sty) + (st(2y - 2x))}{2s(s + t)} \\ &= \frac{(2s^2x + 2sty) + 2st(y - x)}{2s(s + t)} \\ &= \frac{(sx + ty) + t(y - x)}{(s + t)} \end{aligned}$$

That's it! We have found our solution! Now, when we plug back in the constants and variables that we had at the very beginning, we get an expression for the final velocity of particle one. Here it is:

$$V_1 = \frac{(m_1v_1 + m_2v_2) + m_2(v_2 - v_1)}{(m_1 + m_2)}$$

Since this is a perfectly elastic collision, we don't have to do any more calculating to find V_2 (thanks goodness) since it is just a mirror image of V_1 . Thus its equation is

$$V_2 = \frac{(m_2v_2 + m_1v_1) + m_1(v_1 - v_2)}{(m_2 + m_1)}$$

You did it! That is one of the most involved algebra problems you will encounter, so, pat your self on the back, then go do something a little more fun...like your other physics homework.

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