Exploring Concepts of Euclidean Geometry Through Comparison with Spherical and Taxicab Geometries

Dave Damcke, Tevian Dray, Maria Fung, Dianne Hart, and Lyn Riverstone

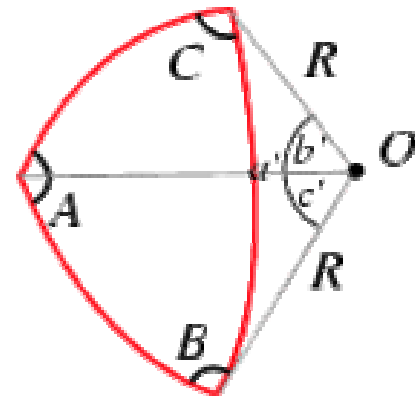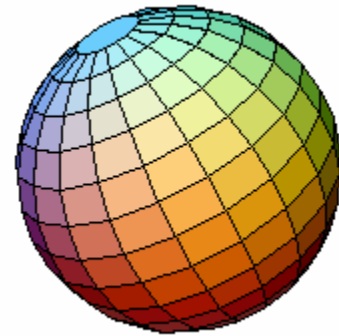January 6th, 2008, Joint Mathematics Meetings, San Diego, CA

Comparing Different Geometries

- Originally designed for use in the Oregon Mathematics Leadership Institute (NSF-funded partnership project— OSU/PSU/TDG/10 OR school districts-NSF/EHR–0412553; ODE/Oregon ESEA Title II-B MSP)

- Aimed at deepening geometry content knowledge in K-12 teachers

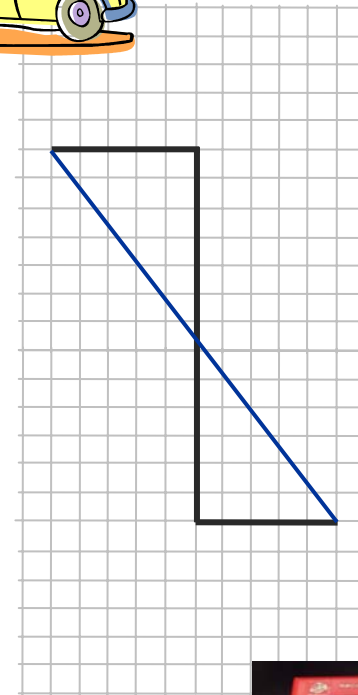- Appropriate as a focus course for middle school teachers

Table of Contents for Course

- Basic concepts in Spherical Geometry:
  - Lines
  - Parallel and perpendicular lines
  - Set of points equidistant from 2 given points
  - Common perpendiculars
  - Polygons: bi-angles, triangles, squares
  - Circles

Table of Contents Continued

- Introduction to Taxicab geometry
  - Distance
  - Midpoints
  - Sets of points equidistant from two given points
  - Squares
  - Circles and π
  - Triangles and congruence

Group Projects

- Capstone experience for course
- Outside of class group explorations
- Investigations of a problem or idea from either Spherical or Taxicab geometries
- Comparison to Euclidean geometry
- Class presentations and posters

Spherical Geometry Project Topics

- Explore the relationship between angle sum and area

- Investigate the geometric properties of the globe that are preserved by a cylindrical or conical projection onto the plane

- Investigate the Platonic solids and their resulting tessellations of the sphere

- Investigate reflections, translations, and rotations on the sphere

Taxicab Geometry Project Topics

- Investigate the result of adding one-way streets in Taxicab geometry
- Investigate the result of adding a mass transit route in Taxicab geometry
- Investigate Taxicab geometry if streets are laid out in a triangular grid
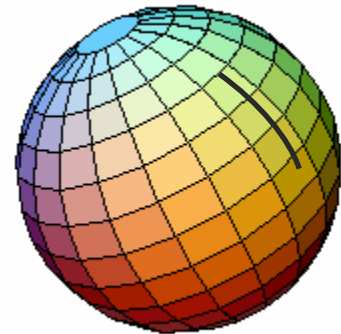- Investigate conic sections in Taxicab geometry

Example of a Project Poster

Structure of Course

- Daily group explorations of rich mathematical tasks in cooperative learning groups
- Carefully orchestrated group work procedures and whole class discussions
- Sequencing presentations with focus on fundamental ideas and misconceptions
- Focus on precise explanations, questioning, challenging, relating, conjecturing, justifying and generalizing

A Spherical Geometry Task

- Distance Measurement
  - Describe how to measure distance on the plane
  - Describe how to measure distance on the sphere
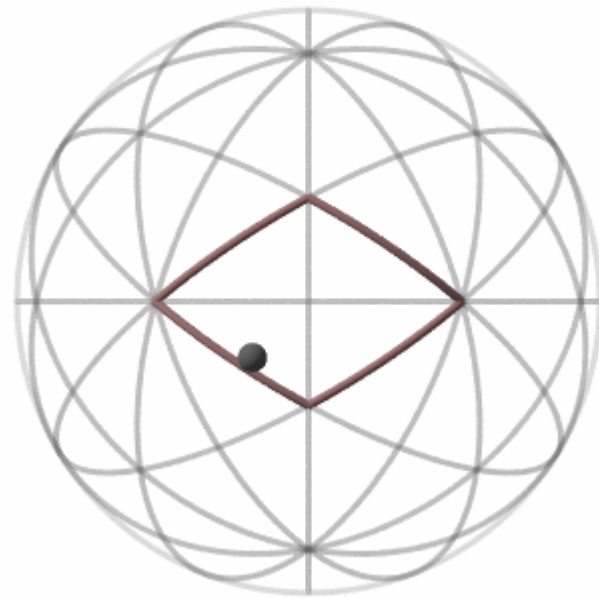 - Explain what units you can use

Example Continued

- Manipulatives used: Lénárt Spheres with spherical tool kits, string, rulers, compasses
- Group roles: group captain, explainer 1, explainer 2, and recorder
- Protocol used: private think time, followed by go-around protocol, group work with roles as above
- Discussion: start with Euclidean plane case (rulers, linear units) and move to Spherical case (spherical rulers or string, degrees)

Example Continued

- Fundamental math ideas: distance between two distinct points is measured along the straight line segment connecting them and its analog on sphere (focus on "shorter" arc of the great circle)
- Questions to steer the discussion
  - Why are great circles "lines" on sphere?
  - Is there always a unique great circle connecting two distinct points?
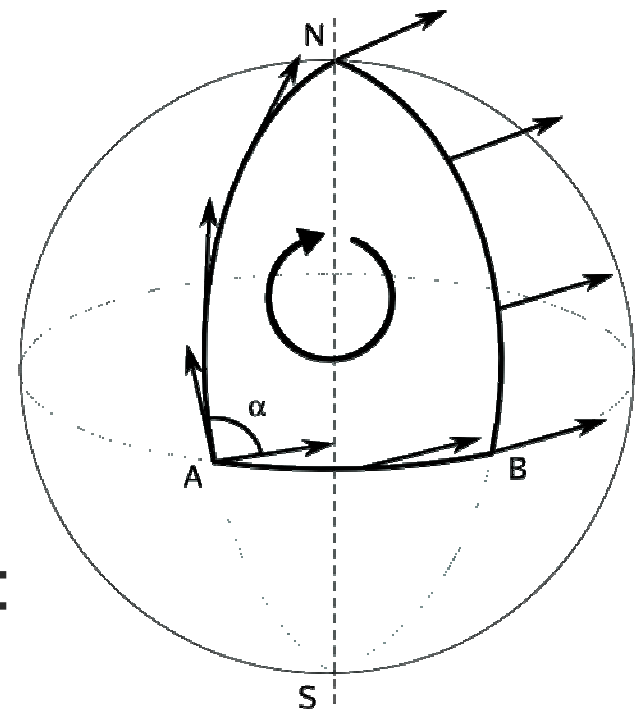 - How is it possible to use degrees as units on the spherical compass?

Interest-Based Spherical Tasks

- Task 1: investigate "squares" on the sphere; possible extension: think about other special quadrilaterals in the plane and their analogs on the sphere
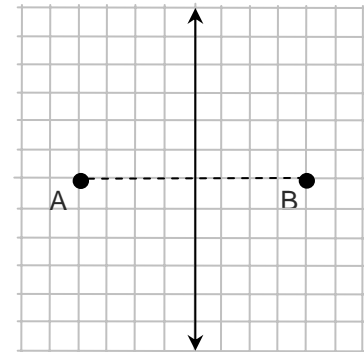
Interest-Based Spherical Tasks

- Task 2: suppose you walk around a closed triangular path made up of straight line segments without turning—do you face the same way when you returned as when you started; possible extension: other closed polygonal paths
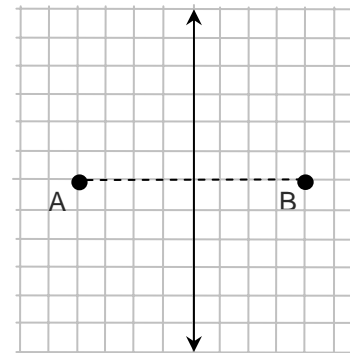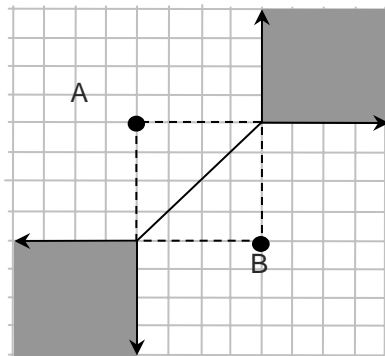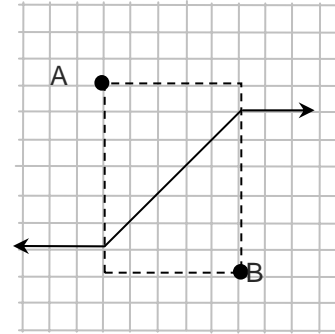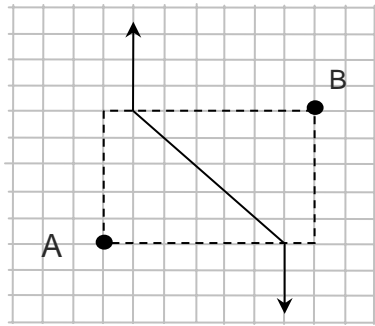
A Taxicab Geometry Task

- **General set-up:**
  - Lois and Clark live in Smallville and are looking for an apartment. The street map shows each person's workplace. If they would like to live the same Taxicab distance from their jobs, where could they live?

Taxicab Task Continued

Four Different Point Configurations (A, B, C, D)

Jigsaw Puzzle

- First, each student thinks independently about her own configuration

- Second, all A's get together and discuss in a go-around protocol; analogously for all B's, C's, and D's

- Consensus is reached and 3 advancing/clarifying questions are generated
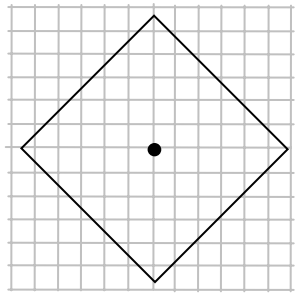
Jigsaw Puzzle Continued

- Students return to their home groups; A and C switch problems; B and D switch problems

- Partners help each other with the prepared questions

- Whole group discussion concludes this activity
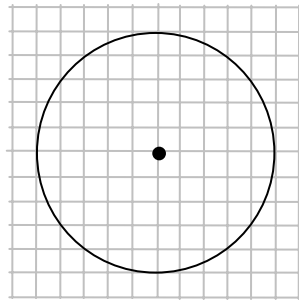
Comparison Charts

- Generated at the end of each of the units on Spherical and Taxicab geometries

- Two construction methods
  - Students generate the comparison objects and conclusions
  - Instructors assign comparison objects
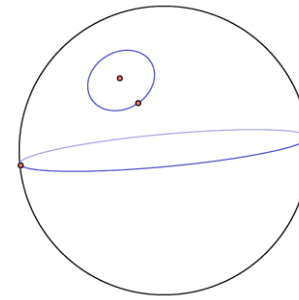
Comparison Charts Continued
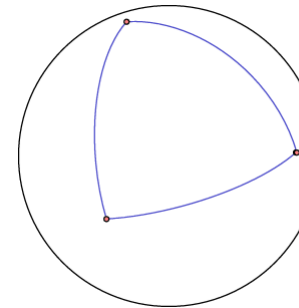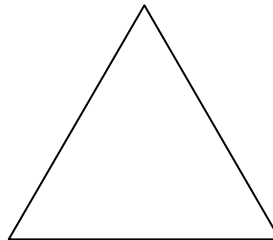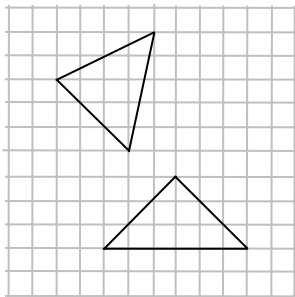
- Circles and equilateral triangles

$\pi=4$ $\pi\sim3.14$ $\pi\in[2,3.14]$

Dissemination Plans

- Writing retreat in July of 2008 for entire geometry team to prepare text book materials

- Articles in preparation for The Montana Mathematics Enthusiasts (pedagogy) and Journal of Mathematics Teacher Education (content)

- "Dare to Compare? Introduction to Using Comparison Charts in the Mathematics Classroom" to appear in May/June 2008 TOMT