Using Non-Euclidean Geometry to Teach Euclidean Geometry to K–12 Teachers

David Damcke, Univ. of Portland
Tevian Dray, Oregon State Univ.
Maria Fung, Western Oregon Univ.
Dianne Hart, Oregon State Univ.
Lyn Riverstone, Oregon State Univ.

Oregon Mathematics Leadership Institute (OMLI)

- NSF-funded partnership project—OSU/PSU/TDG/10 OR school districts
  (NSF/EHR–0412553; ODE/Oregon ESEA Title II-B MSP)
- Aimed at increasing mathematics achievement of K–12 students
- 3-week intensive summer institutes in 6 different mathematics content areas and in leadership skills

Mathematics Content Courses at OMLI

- 15 two-hour sessions for groups of K–12 teachers with 4-member instructional teams
- Content areas represented:
  - Number and Operation
  - Geometry
  - Abstract Algebra
  - Probability and Statistics
  - Measurement and Change
  - Discrete Mathematics

The Geometry Team

- One faculty member from OSU with geometry expertise and interests in mathematics education (planning);
- One faculty member from WOU with experience teaching in-service and pre-service K–12 teachers;
- Two instructors from OSU with varied teaching experiences;
- One master teacher, currently working with pre-service teachers at Univ. of Portland.

Comparing Different Geometries Course at OMLI

Course goals and objectives:

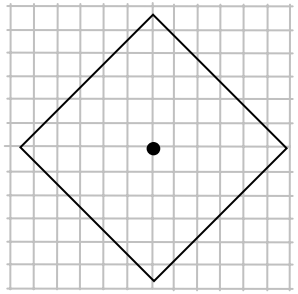
- Develop geometry content knowledge in K–12 teachers
  - Parallel and perpendicular lines, midpoints, perpendicular bisectors, circles
  - Squares, triangles, perimeter, tessellations
- Allow K–12 teachers to make connections between Euclidean and non-Euclidean geometries by examining similarities and differences

Comparing Different Geometries Course at OMLI

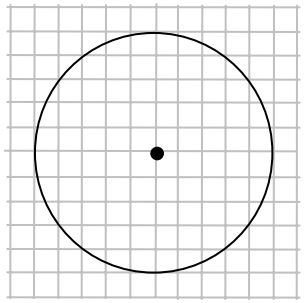
- Taxicab Geometry compared with Euclidean Geometry (EG) (1 week)
- Spherical Geometry compared with Euclidean geometry (1 week)
- Independent project and presentations (1 week)
- Comparison charts between EG and Taxicab, and EG and spherical

Comparison

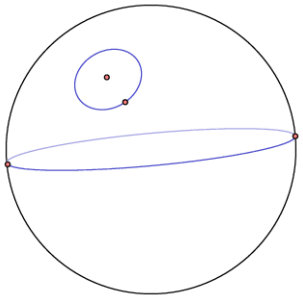
Circles



$\pi=4$

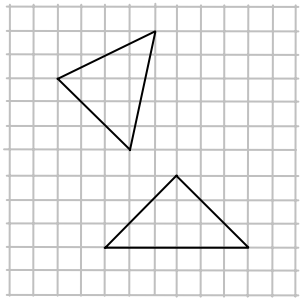


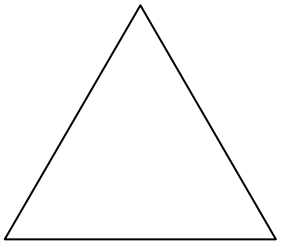
$\pi\sim3.14$

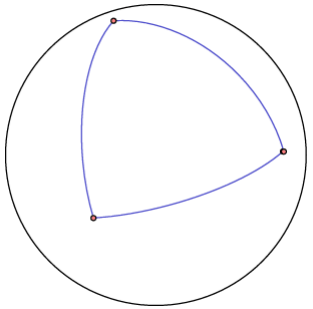


$\pi\in[2,3.14]$

Equilateral Triangles







Taxicab Geometry Explorations

- Taxicab Distance
- Taxicab Midpoints
- Taxicab Set of Points Equidistant from Two Given Points
- Taxicab Circles and Taxicab-π
- Taxicab Triangles
- Project Topics

Spherical Geometry Explorations

- Lines on the sphere
- Perpendicular lines
- Spherical triangles
- Spherical reflections
- Analogues of squares on the sphere

Successful Projects

- Taxicab Geometry on hexagonal grids
- Taxicab Geometry with one-way streets
- Taxicab Geometry with a subway line
- Taxicab Conic Sections
- Spherical Triangles Area Formula
- Spherical Tessellations

Pedagogical Techniques

- Hands-on explorations
- Cooperative Groups
- Group norms and protocols
- Mathematical discourse
- Justification of mathematical ideas

Hands-on Explorations

Manipulatives used:

- Lénárt Spheres with spherical tool kits
- Etch-a-Sketch for Taxicab distance
- Grid paper, colored pencils and markers
- String, rulers, compasses

Cooperative Groups

- Heterogeneous groups reassigned every couple of days
- Roles for maximum accountability
  - Team Captain
  - Facilitator
  - Resource Manager
  - Recorder/Reporter

Norms and Protocols

- Classroom norms ensuring everyone's participation and encouraging risk-taking
- Protocols for group work including private think time, pair-and-share, go-around protocols, jigsaw puzzle
- Protocols for class discussion and presentations including group roles

Mathematical Discourse

- Explaining
- Questioning
- Challenging
- Relating
- Conjecturing
- Justifying
- Generalizing

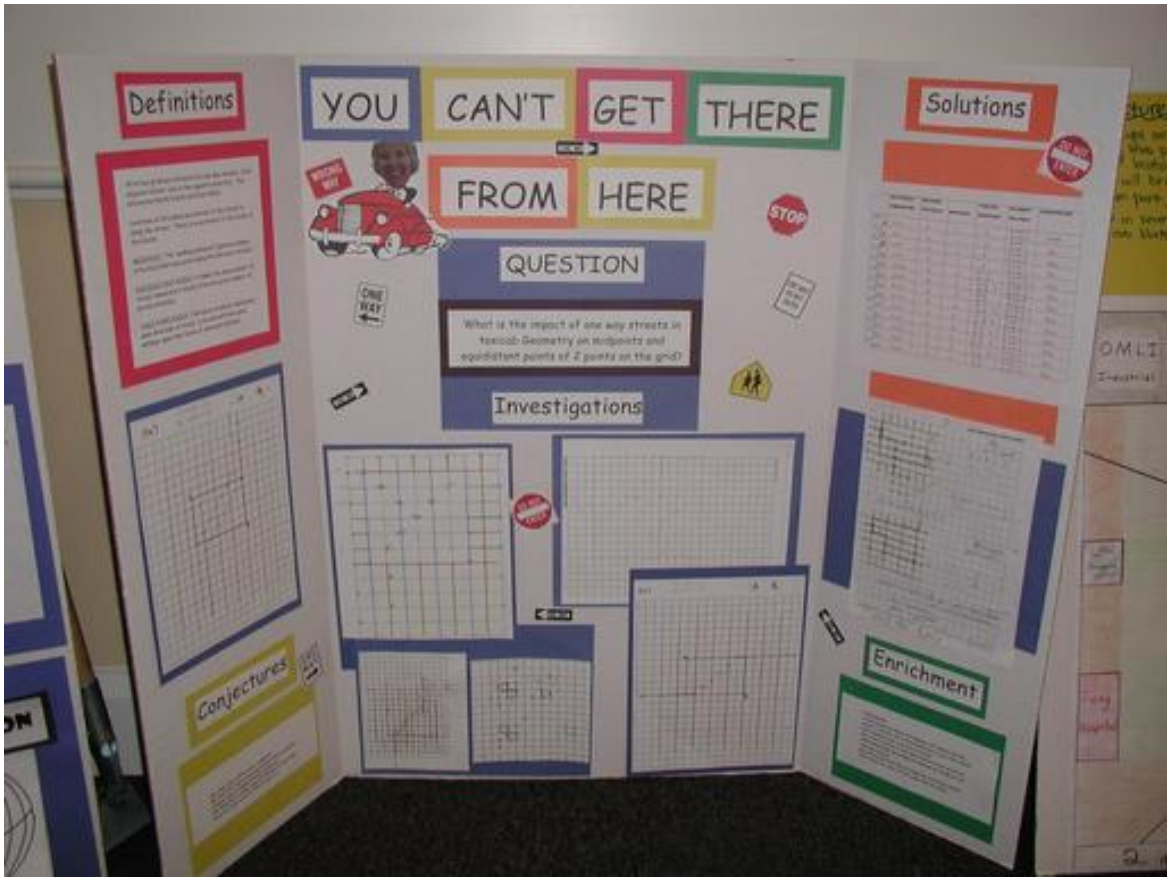
Justification of Mathematical Ideas

- Attention to undefined terms, definitions, axioms and theorems
- From explanations and generalizations of observed math ideas to answering “why?”
- Oral and written presentation of elementary proofs, from informal to formal

Successes and Changes

- Reported improved understanding of the role of definitions and undefined terms, parallel lines, distance and circles
- More challenges needed for 9–12 group-level-based explorations
- Changing the order of topics
- Keeping activities and tasks as open-ended as possible

Projects!


Definitions
YOU CAN'T GET THERE
FROM HERE
QUESTION
Investigations
Solutions
Conjectures
Enrichment
STOP