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Von Klitzing Wins Nobel Physics Prize for Quantum Hall Effect

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Von Klitzing wins Nobel Physics Prize for quantum Hall effect

The Royal Swedish Academy of Sciences has awarded the 1985 Nobel Prize in Physics to Klaus von Klitzing, a director of the Max Planck Institute for Solid-State Research in Stuttgart. The 1.8-million-kroner ($230 000) prize was awarded for his discovery of the quantized Hall effect in 1980. Von Klitzing, who was at the time a Heisenberg fellow at the University of Würzburg, discovered the effect at the high-field magnet laboratory in Grenoble, a joint facility of the Max Planck Institute and the CNRS.

The quantized Hall effect, a remarkable macroscopic quantum phenomenon occurring at low temperature in two-dimensional electron systems subjected to high magnetic fields, has in the past five years initiated considerable experimental and theoretical activity. Its most striking manifestation is the appearance of Hall-conductivity plateaus at integral multiples of e^2/h with an astonishing precision quite unanticipated by the theorists—and quite oblivious to the imperfections or geometric details of the semiconductor interfaces at which the effect is measured (PHYSICS TODAY, June 1981, page 17).

Von Klitzing's accidental observation of conductivity plateaus while measuring semiconductor carrier densities and the elementary theory of Landau-level filling suggested to him that one might exploit the Hall effect to achieve a high-precision measurement of e^2/h, a fundamental constant of nature, and hence ultimately a better determination of the fine-structure constant, independent of the electron's gyromagnetic ratio. Not only would this provide pure physics with a stringent test of quantum electrodynamics, it would also serve the applied-physics and engineering communities by furnishing a fundamental, easily reproducible standard of electrical resistance. To the nearest ohm, h/e^2 has a value of 25 813 Ω. The International Bureau of Weights and Measures is considering a redefinition of the standard ohm in terms of the quantum Hall effect.

"In the theory, e^2/h is the fundamental unit of [two-dimensional] conductivity," von Klitzing told us, "but nobody

VON KLITZING

recognized that you can measure it directly from the Hall effect—probably because theorists don't know that one can measure the Hall conductivity independently of the geometry of the system. My idea was to make sample-independent measurements at filled Landau levels."

Even if the theorists had thought about how to do such a measurement, they would not have anticipated the existence of broad conductivity plateaus yielding (by now) e^2/h to almost a part in ten million. After the first quantum-Hall-effect results were published[1] five years ago by von Klitzing and his colleagues Gerhardt Dorda (Siemens) and Michael Pepper (Cambridge), the scramble was on to find a theoretical explanation for why measurements at a crystal interface full of imperfections yield such astonishingly precise agreement with a naive theory that lightly waves these imperfections aside.

Von Klitzing saw the first indications of the Hall-conductivity plateaus as he was measuring carrier densities in 1978. At first he thought this an uninteresting, sample-dependent effect. But when he noticed that the plateaus were always occurring at the same values, he began to make the connection to e^2/h. When he moved his experiment from Grenoble to Würzburg to investigate the precision of the effect with a carefully calibrated resistance standard, von Klitzing told us, he assumed the plateaus would agree with ne^2/h only within a few percent. He foresaw nothing like the part-per-million precision reported in the 1980 paper.

Having thoroughly digested the quantized-Hall-effect data, the theorists have reached something of a consensus that, far from being a disturbing complication, the localized electron states due to lattice imperfections are *essential* to the observation of the effect. In a perfect crystal interface, they tell us, one would see no Hall-conductivity plateaus at all. From the 1974 theory of Tsuneya Ando (University of Tokyo), which largely ignored localization, one would expect the Hall conductivity to measure the electron densities of filled Landau levels. "But," points out Livermore theorist Robert Laughlin, "with lots of electrons immobilized by lattice imperfections, you'd never get that kind of precision with a quantized density measurement. In any case, the broad plateaus tell us that whatever we're measuring is unaffected by adding lots of electrons. I'm now convinced that what you're measuring in the quantized Hall effect is the charge of the electron itself."

The experimenters have allowed the theorists little respite. In the spring of 1982, Daniel Tsui, Horst Störmer and Arthur Gossard at Bell Labs, having already replicated[2] von Klitzing's result in a somewhat different setting, discovered[3] something quite new—the *fractional* quantum Hall effect (PHYSICS TODAY, July 1983, page 19). This extension of the effect to fractional multiples of e^2/h "knocked our socks off," remembers Laughlin.

The fractional quantum Hall effect presented a much subtler theoretical puzzle than did von Klitzing's integral quantum Hall effect, as we must now call it. "Theoretical progress has been much slower here," Laughlin told us, "and nightmarishly difficult experiments are still needed to confirm the theory many of us think is the right one." Just as in the case of the integral Hall effect, Laughlin's theory asserts, one is measuring the charge of the

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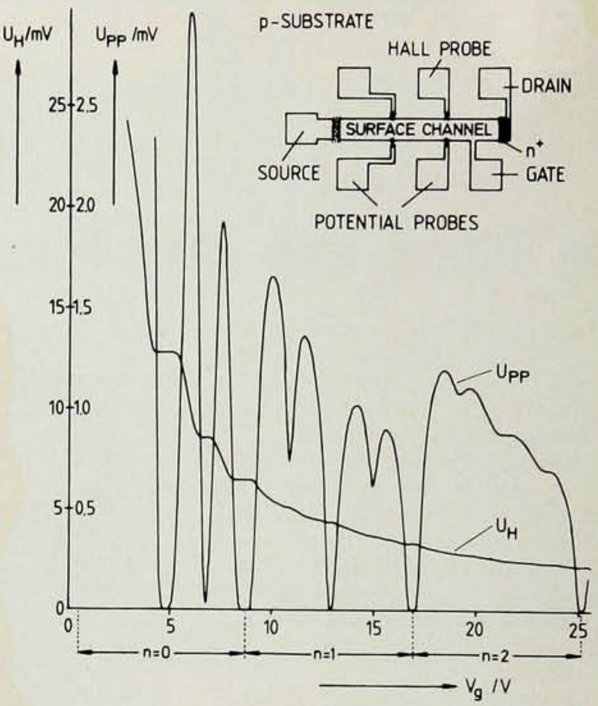
carrier—in this case the fractionally charged excitations of a novel quantum liquid[4] (see page 89).

The classical Hall effect is essentially the tendency of charged particles in crossed magnetic and electric fields to drift sideways, that is, in the direction orthogonal to both fields. If, for example, one imposes a perpendicular magnetic field on a current-carrying conducting strip, the Lorentz force will tend to pile up the moving electrons near one edge of the strip, producing a transverse "Hall voltage" across its surface. In the extreme, dissipation-free case of noninteracting electrons in a vacuum, the only net motion is in the $\mathbf{E}\times\mathbf{B}$ direction. The electrons will in general execute cyclotron orbits around the magnetic-field lines with the cyclotron frequency $\omega_c = eB/m$, but the net motion is a cycloid sideways drift at the "Hall velocity," E/B. Averaged over times large compared with $1/\omega_c$, the electric field does no net work on the electrons in the absence of dissipative collisions.

In the quantized Hall effect, just such a dissipation-free situation can occur even in a semiconductor crystal—when all the conduction electrons find themselves in fully occupied Landau levels. All the quantum-Hall experiments involve a two-dimensional system of electrons trapped at an interface—in von Klitzing's case, the inversion layer in the silicon just below the oxide surface of a metal-oxide-semiconductor field-effect transistor (MOSFET) at temperatures below 2 K. The low temperature assures that all the electrons are in the ground state of the trapping potential well at the silicon surface; they are free to move only in the plane of the inversion layer. If one now imposes a strong magnetic field normal to the inversion layer, this ground state breaks up into Landau levels as the electrons execute tiny cyclotron orbits in the plane. An electron in the nth Landau level will gain a cyclotron-orbit energy of $(n - ½)\hbar\omega_c$.

There is a strict upper limit on the density of electrons in any one Landau level. For a given magnetic field strength imposed on such a two-dimensional sea of conduction electrons, a Landau level is fully occupied when the two-dimensional density is $1/\pi r_c^2$, where $r_c = (2\hbar/eB)^{1/2}$, the radius of a cyclotron orbit of energy $\hbar\omega_c$. One can think of this as the tightest possible packing of cyclotron orbits in the plane—or alternatively, one electron per magnetic-flux quantum per Landau level.

For an arbitarily chosen population density and magnetic field at low temperature, some highest Landau level will in general be partially filled, with all lower-lying Landau levels fully occupied. If, in this general case, one starts a current flowing in the inversion layer by imposing an electric field in the plane, one will see an unspectacular manifestation of the Hall effect. The current flow will have components both parallel and perpendicular to the electric field. The conductivity is generalized to a two-by-two matrix. Its off-diagonal element σ_{xy}, the "Hall conductivity," is the current density divided by the electric-field component transverse to it. Although σ_{xy} may be technologically interesting as a measure of the carrier density available in the semiconducting sample under scrutiny, its complicated dependence on the nature and geometry of the sample renders it—in the general case—something less than fundamental.



The first report of the quantized Hall effect, as published by von Klitzing, Dorda and Pepper in 1980. As the population of the MOSFET inversion layer increases with increasing gate voltage (V_g), the Hall resistance, as measured by the voltage (U_H) transverse to the current, falls by quantized, stepwise plateaus. The longitudinal voltage (U_{pp}) falls almost to zero at each plateau, indicating lossless current flow when the Landau levels are full.

What von Klitzing demonstrated is that σ_{xy} does indeed take on a very fundamental and sample-independent character in the special case where the highest occupied Landau level is full. In that case—assuming a perfect interface—the current flow must be completely nondissipative. The only state into which a moving electron could be scattered would be the first unoccupied Landau level. But that would involve the electron's jumping an energy gap of $\hbar\omega_c$, an essentially impossible event at the low temperatures and high magnetic fields of these experiments. The situation is then like that of the vacuum. The current flows only perpendicular to any electric field in the plane, drifting at the Hall velocity, E/B. The current density is then simply the combined charge density of the n fully occupied Landau levels times the Hall velocity. Dividing by the electric field to get the Hall conductivity, one cancels all dependence on field strengths and gets simply $\sigma_{xy} = ne^2/h$, independent of everything except the fundamental constants. The question that remains is: What does one see at a real, imperfect lattice interface, where the Fermi level that determines the energy surfaces of the two-dimensional sea of conduction electrons has local irregularities?

Measuring the Hall conductivity to high precision is relatively straightforward. Von Klitzing's MOSFETS employed oxide-covered high-mobility silicon strips of various geometries supplied by Dorda and Pepper. At temperatures below 2 K a fixed current was made to flow along the surface inversion layer of the silicon strip (typically a few hundred microns long) from source to drain, while probes measured the longitudinal voltage drop along the current direction and the transverse Hall voltage across the strip. In the two-dimensional case with lossless current flow, σ_{xy} is simply the reciprocal of the measured Hall resistance. In the Würzburg experiments, von Klitzing measured this Hall resistance very accurately by calibrating the 10-kΩ reference resistor in the device against a standard maintained at the Physikalisch Technische Bundesanstalt in Braunschweig. In this way he could measure the Hall conductivity to about a part in a million. The original discovery was made with a 20-tesla field at the Grenoble magnet laboratory, but the 15-tesla magnets available at Würzburg were sufficient for the subsequent high-precision measurements.

The conductivity plateaus that characterize the quantum Hall effect appear in von Klitzing's experiment as one fills successive Landau levels at a fixed magnetic field by steadily raising the population of conduction electrons in the inversion layer. One accomplishes this by increasing the MOSFET gate voltage that provides the trapping electric field normal to the interface, and thus continuously raising the population, and hence the Fermi level, of the two-dimensional electron sea. As the Fermi level rises to fill a particular Landau level, the Hall conductivity, as plotted against the gate voltage, increases steadily and unspectacularly. But then, presumably when the nth Landau level is completely filled, the Hall conductivity suddenly levels off to form a wide, spectacularly flat plateau at a value of ne^2/h, accurate to about a part in ten million, until the next Landau level begins filling.

Repeating this exercise with MOSFETS of various geometries, von Klitzing found no observable difference from one sample to the next. When he monitored the longitudinal voltage drop along the current, he found, as expected from the naive theory, that the longitudinal resistance drops almost to zero at each Hall-conductivity plateau; the current flow is essentially lossless when all the occupied Landau levels are full.

Shortly thereafter, Tsui and Gossard at Bell Labs achieved[2] the same result with a modulation-doped GaAs/AlGaAs heterojunction in place of von Klitzing's silicon MOSFET. In this case, however, the mobile electron population at the interface is fixed by the doping of the AlGaAs layer and the bending of the conduction band at the interface. Therefore, instead of varying a gate voltage, the Bell Labs group varied the filling of the Landau levels by varying the imposed magnet field. As B increases for fixed population, the cyclotron orbits get tighter and the maximum allowed density of each Landau layer becomes correspondingly larger.

The GaAs heterostructure offers several advantages over silicon MOSFETS for further quantum-Hall-effect research. Because the effective electron mass is significantly smaller in GaAs, the energy separation $\hbar\omega_c$ between Landau levels is correspondingly larger. Therefore one can see the effect at lower magnetic fields (8 T) and higher temperatures (4 K). Even more important, one can grow much cleaner interfaces in a GaAs heterostructure. The fractional quantum Hall effect, discovered the following year by Tsui and Störmer, has been seen at Bell Labs only in the very cleanest heterostructures grown by Gossard and James Hwang.[3] For this completely unanticipated discovery, Tsui, Störmer and Gossard were awarded the 1984 Oliver Buckley Prize for Condensed Matter Physics (PHYSICS TODAY, March 1984, page 107).

In the naive theory, the steplike Hall plateaus found by von Klitzing present a puzzle. The electrons in each Landau level are delocalized along equipotentials of the Hall field. In the absence of localized electron states, all the electrons would be ensconced in Landau levels, the Fermi level would jump directly from one Landau level to the next and the Hall conductivity would equal ne^2/h only at singular points. Perhaps the plateaus arise because the Fermi level becomes temporarily pinned between Landau levels by localized electron states that are due to lattice imperfections. But if dirt is doing the trick, how can one explain the extraordinary part-per-ten-million accuracy of the plateau conductivity levels?

After von Klitzing's discovery, University of Maryland theorist Richard Prange undertook[5] to explain the unexpected accuracy of the quantized Hall effect by treating lattice imperfections as strong delta-function scatterers in a simple, calculable model. He concluded that whereas localized impurity states broken off from the Landau levels carry no current, the extended electron states in the Landau levels compensate precisely for this loss of carriers by carrying just enough extra current to preserve the precision of the quantized Hall effect.

His calculation was, however, highly model dependent, unrealistically neglecting electron–electron interactions and Landau-level mixing. Arguing that so general a phenomenon must come from general principles, not details, Laughlin has produced[6] a model-independent derivation of the quantum Hall effect. Laughlin's argument, based essentially on gauge invariance, seems now to express something of a consensus among theorists working in this area. Considering a *Gedankenexperiment* in which a two-dimensional Hall current flows around a strip that is closed upon itself to form a loop and threaded by a magnetic flux, he asks what happens when that flux is adiabatically increased, one flux quantum at a time. Extended electron states that persist coherently around the loop will contribute to the current induced by the increasing flux. But because these states are pinned in phase by closing upon themselves, they can respond to the increasing flux only by transferring an integral number of electrons per flux quantum from one edge of the strip to the other. The resulting transverse Hall voltage gives precisely von Klitzing's result. The Hall conductivity is essentially measuring the charge of the electron. Localized states, on the other hand, can have arbitrary phase under gauge transformation and thus do not contribute to the Hall current. "Adding electrons after a Landau level is filled does nothing," Laughlin explains. "They're stuck in localized states. Without localization by lattice imperfections, you'd see no effect. The quantum Hall effect is in fact the most important consequence of localization that's ever been discovered."

Serge Luryi and Rudolf Kazarinov at Bell Labs have produced[7] a heuristically appealing percolation-model derivation in which the distinction between extended and localized electron states is purely topological. Both states are regarded as wavefunction fibers extending along equipotentials, but it is the topologically "global" states that take on the entire applied voltage. As the Fermi level rises gradually over a potential surface made uneven by lattice imperfections, the conductivity plateaus result from topological transformations of the landscape "from lakes on a landmass to islands in a sea." A virtue of this model, Luryi told us, is that it explains the plateaus without requiring a high density of imperfections and truly localized states.

None of the theoretical work on the integral quantum Hall effect offered the slightest inkling or explanation of the fractional quantum Hall effect that was soon to follow.

Von Klitzing believes that "we have yet to find a complete microscopic theory of the integral quantum Hall effect." At the Stuttgart Max Planck Institute he is now working primarily on thin-layer and quantum-well phenomena, mostly in GaAs heterostructures. Although he himself is not involved in growing these structures, his move last winter from the Technical University of Munich was in part impelled by the molecular-beam-epitaxy facilities at Stuttgart. He had joined the Munich physics faculty shortly after his historic 1980 experiment.

Von Klitzing received his PhD at Würzburg in 1972. In 1981 the German Physical Society awarded him its Walter Schottky Prize for Solid State Research for his discovery of the quantum Hall effect. "I am happy that semiconductor physics has once again been recognized with a Nobel Prize," he told us. "I hope that this will lead—at least in Germany—to more support of solid-state physics in this area. In Germany, no company does the kind of basic research one sees at Bell Labs or

IBM; only the Max Planck Institute and a few university groups." As for the prospect that the Stuttgart area might become a new Silicon Valley, he told us, "they're trying, but there's no decision as yet."

—BERTRAM SCHWARZSCHILD

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Nobel Prize in Chemistry to Hauptman and Karle

Although the Nobel Prize has been frequently awarded to researchers who have applied the techniques of x-ray crystallography to the determination of molecular structure, x-ray crystallography as a mature science has seldom been recognized by the Nobel committee. This oversight was redressed when the 1985 Nobel Prize in Chemistry was awarded to Herbert A. Hauptman of the Medical Foundation of Buffalo, New York, and Jerome Karle of the US Naval Research Laboratory in Washington, D.C., "for their outstanding achievements in the development of direct methods for the determination of crystal structure." Many chemists and biologists who rely heavily on the methods developed by Hauptman and Karle feel that the prize was long overdue.

HAUPTMAN

KARLE

Before the pioneering work[1] of Hauptman and Karle in the early 1950s, the determination of the three-dimensional structure of a moderately large molecule proceeded by indirect trickery. Molecules that did not contain heavy atoms took months or years to solve—if, indeed, they were solved at all. Crystals that did not have a center of symmetry were generally ignored. But the direct method of Hauptman and Karle, run on modern computers, allows the structures of mid-sized molecules to be routinely determined in about two days. "Nowadays the method is used everywhere," says Hauptman, who estimates that between 40 000 and 50 000 structures have been determined using their method—with 5000 new structures a year being added to the list.

Karle's wife, Isabella, made a vital contribution to the further development of the direct methods invented by Karle and Hauptman by tackling the general problem of determining the structure of crystals that do not have centers of symmetry. Her work led to the so-called symbolic addition procedure, which was used to determine[2] the first noncentrosymmetric crystal structure, L-arginine dihydrate. The symbolic addition procedure has since been applied to molecules that contain as many as 200 independent atoms. Jerome Karle says that "the major accomplishments in the development and application of the symbolic addition procedure are hers."

The phase problem. Diffraction of x rays is the most convincing demonstration of the existence of the regular arrangement of atoms in a crystal. The variously oriented atomic planes in a crystal each reflect x rays toward unique spots on an x-ray diffraction photograph. The pattern of diffraction spots can therefore be used to identify the repetitious crystal structure. But information about the molecules that form a crystal is also contained in the diffraction pattern, encoded in the intensities and phases of the diffraction spots. Working backward from this information to a three-dimensional molecular structure is simple: The rule says that each spot is a term in the Fourier transform of the molecular structure. Unfortunately, reality is rarely as kind as theory.

The trouble is that photographic film records intensity, but is insensitive to phase. Crystallographers have invented a host of ingenious dodges to get around the phase problem (PHYSICS TODAY, November 1982, page 17). One standard method is to substitute heavy atoms selectively at various positions in the molecule being studied. Because heavy atoms reflect x rays more strongly than light ones, they act as markers. By comparing diffraction patterns of the modified and unmodified molecules, one can often infer enough information to solve the structure. For smaller structures the heavy atom gives enough information that one can solve the structure completely, but it may also severely distort the structure. To verify that the structure is correct the crystallographers must first make sure it makes good chemical sense (proper bond distances and angles, and good packing distances) and that it reproduces the measured diffraction pattern. Most of the triumphs of x-ray crystallography before Hauptman and Karle's work were based on substitution techniques and inspired guesswork. A typical thesis project in those bad old days was the determination of the structure of one or two mid-sized molecules.

Bridging. Just after the Second World War Karle and his wife Isabella, now working at the Naval Research Laboratory, became interested in developing the quantitative aspects of electron diffraction of gas molecules. In a gas the molecules are oriented randomly, so the resulting diffraction patterns are not spots, but diffuse rings. The Karles found that the analysis was simplified if they placed physical restraints on the problem, such as requiring that the Fourier transform be nonnegative, because it represents a probability density. "This nonnegativity worked so well, it was so exciting," recalls Karle, "that I began to think of other applications."

Around this time, in 1947, Hauptman came to the Naval Research Laboratory and joined the Karles. "The war was over, I got out of the Navy and about all I knew was that I wanted to