has the property that, whereas in most of space if we normalize ψ to one particle per unit volume, there exists a volume in which it diverges as $(L/a)^3$. Moreover, if the proportion of localized states is significant (say a finite fraction η of the whole), then in any volume ξ^3/η, if we pick the right energy, we should find that a solution of the Schrödinger equation exists which decays exponentially to zero and yet, when $|\psi|^2$ is integrated over all space, the integral is comparable with that within a few multiples of ξ from the centre of the localized region.

While we have no formal proof that such states cannot exist, it seems so improbable as to raise doubts about the possibility of coexistence.

3.5. Hopping conduction

If the Fermi energy E_F lies below the mobility edge E_c, we have seen that conduction may be of two kinds.

1. By excitation to the mobility edge. We may then give σ_0 in eqn (3.14) the value

$$\sigma_0 \simeq 0.03e^2/\hbar L_i.$$

The inelastic diffusion length may then be the result of collisions with phonons, or Auger processes in which an electron loses energy to another which has energy below E_F.

2. By thermally activated hopping, if $N(E_F)$ is finite. This is a process in which an electron in an occupied state with energy below E_F receives energy from a phonon, which enables it to move to a nearby state above E_F. A process of this kind was first described by Miller and Abrahams (1960) as an explanation of impurity conduction in doped and compensated semiconductors (Chapter 4). In this work, the electron was supposed always to move to the nearest empty centre. Their analysis resulted in an expression for the conductivity

$$\sigma = \sigma_3 \exp(-\varepsilon_3/k_B T).$$

ε_3 is expected to be of the form

$$\varepsilon_3 \sim 1/N(E_F)a^3$$

where a is the distance between nearest neighbours. This is discussed further in Chapter 4.

Mott (1968) first pointed out that at low temperatures the most frequent hopping process would *not* be to a nearest neighbour. The argument in its simplest form is the following. Within a range R of a given site the density

of states per unit energy range is, near the Fermi energy,

$$(4\pi/3)R^3N(E_F).$$

Thus for the hopping process through a distance R with lowest activation energy, this energy ΔE will be the reciprocal of this,

$$\Delta E = 1/(4\pi/3)R^3N(E_F).$$

Thus, so far as the activation energy is concerned, the further the electron hops the smaller will be ΔE. But hopping over a large distance involves tunnelling and the probability will contain a factor

$$\exp(-2\alpha R)$$

where $1/\alpha$ is the decay length of the localized wave function. So there will be an optimum hopping distance R, for which

$$\exp(-2\alpha R)\exp(-\Delta E/k_BT)$$

is a maximum. This will occur when

$$2\alpha R + 1/\{(4\pi/3)R^3N(E)k_BT\} \tag{3.22}$$

has its minimum value, that is when

$$R = \{1/8\pi N(E)\alpha k_BT\}^{1/4} \tag{3.23}$$

Substituting for R in (3.22), we see that the hopping probability and thus the conductivity is of the form

$$A\exp(-B/T^{1/4}), \tag{3.24a}$$

where

$$B = 2\left(\frac{3}{2\pi}\right)^{1/4}\left(\frac{\alpha^3}{k_BN(E_F)}\right)^{1/4}. \tag{3.24b}$$

For other methods of deriving this equation, giving somewhat different values of B, see Mott and Davis (1979, p. 32) or Shklovskii and Efros (1984). In two-dimensional problems, 1/3 replaces 1/4 (Hamilton 1972).

This form of conduction is called 'variable-range hopping'. On the experimental side, both in doped crystalline semiconductors and amorphous materials it has frequently been observed, and the form

$$\sigma = A\exp(-B/T^{\nu}) \tag{3.25}$$

often represents the behaviour. Experimentally, however, it is difficult to determine the value of ν.

There is an extensive literature on the value of the constant A. A review giving values for single and multiphonon hopping is given by Emin (1975).

For a recent discussion see Su
(1991).

Efros and Shklovskii (19
interaction between the electr
should be given by $\nu = \frac{1}{2}$, the
J. H. Davies *et al.* (1982). Th
sidered an empty and an occ
with energies ε_a, ε_b above and
to move an electron from one

where $\varepsilon = \varepsilon_a - \varepsilon_b$. Thus around
there is a sphere of volume

$(4\pi/3)$

in which the other electron can

To proceed further we must
density of states. $N(E)$ is the de
field (Hartree or Hartree–Fock
(Thouless 1970) is finite at E_c, a
of states for non-interacting p
density of states seen by an elect
if the surrounding electrons are
single-particle density of states,
duction, as for tunnelling, this is
have the thermodynamic density
allowed to relax, which must be
magnetism.

Efros and Shklovskii argued tha
range of energies distant ε from th

$(4\pi/3$

where

$n(\varepsilon) =$

This must not tend to infinity as ε
power of ε and $N_0(E)$ as $(E - E_F)^s$
in most cases. The density of state
minimum. So $N(E)$ in (3.24b) shou
$\alpha = 1/\xi$, yielding

$\sigma = \sigma_0 \exp$