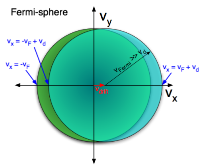
PH575 Spring 2014

Lecture #14
Free electron metals: electrical & thermal conductivity Sutton Ch. 8 pp 158-164; Kittel Ch. 6.



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w3.msi.vxu.se/~pku/ FyB702/Fermi_sphere.html

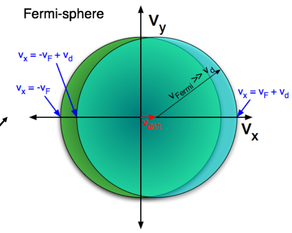
Electrical conductivity

$$q\vec{\varepsilon} = \hbar \frac{d\vec{k}}{dt}$$

$$\vec{k}(t) = \vec{k}(0) - \frac{e\vec{\varepsilon}}{\hbar} t$$

$$d\vec{k}(t) = -\frac{e\vec{\varepsilon}}{\hbar} dt$$

independent of $\vec{k}$



Collisions with thermal vibrations and defects (not stationary ions or other electrons as Drude envisaged) stop the Bloch oscillations and cause electron to settle to a drift velocity.

Average time between collisions τ

$$\langle \delta\vec{k} \rangle = -\frac{e\vec{\varepsilon}}{\hbar}\tau$$

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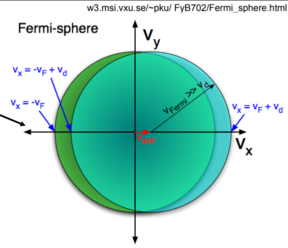
Electrical conductivity

$$\vec{v}_{drift} = \frac{\hbar\delta\vec{k}}{m^*} = -\frac{e\vec{\varepsilon}}{m^*}\tau$$

Only a small fraction nv_{drift}/v_F contribute to conductivity.
They all have velocity about v_F



$$\vec{j} = n\left(\frac{v_{drift}}{v_F}\right) q\vec{v}_F = nq\vec{v}_{drift}$$

$$= \frac{ne^2\tau}{m^*}\vec{\varepsilon} \equiv \sigma\vec{\varepsilon} \equiv \frac{\vec{\varepsilon}}{\rho}$$

Carrier density n; charge q; conductivity σ; resistivity ρ. This is Ohm's law (I α V).

$$\frac{ne^2\tau}{m^*} = \sigma$$

This is the free electron model prediction for the conductivity

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Electrical conductivity

$$\sigma = \frac{ne^2\tau}{m^*} = ne\left(\frac{e\tau}{m^*}\right)$$

New quantity called MOBILITY (μ) is defined by

$$\sigma \equiv ne\mu$$

$$\mu = \frac{e\tau}{m^*}$$

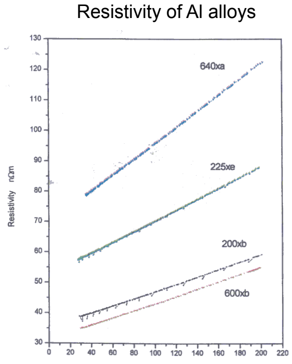
What are units of μ?
Often written as cm^2/Vs … do these units make sense as a response to an electric field?

Cu at RT has $\rho \approx 20$ nΩ m.

What is σ_{300K}?
What is τ_{300K}?
What is **mean free path**?
How many lattice constants?

These quantities become T-independent at low T -> scattering by fixed impurities. At higher T, scattering by thermal vibrations.
If mfp ≈ 30 nm, then what is σ?

Resistivity of Al alloys



http://midas.npl.co.uk/midas/content/images/CMMT_069_fig5.gif

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Thermal Conductivity

Classical: $K = \frac{1}{3}Cv\ell$ ← Mean free path; v: velocity; C: Specific heat per unit volume

Fermi electron gas:

$$K_{el} = \underbrace{\frac{1}{3}\frac{\pi^2}{3}D(E_F)k_B^2 T}_{C_{el}}\, v_F \underbrace{v_F\tau}_{\ell}$$

$$= \frac{\pi^2 nk_B^2\tau}{3m}T$$

The electronic contribution dominates in pure metals (τ large).
In disordered alloys, reduction of mfp may mean phonon contribution is significant.

Wiedemann- Franz Law:
At not-too-low temperatures, the ratio of the thermal to electrical conductivity of metals is proportional to temperature and the constant of proportionality is material independent!

$$\frac{K_{el}}{\sigma T} = \frac{\dfrac{\pi^2 n k_B^2 \tau}{3m^*}}{\dfrac{ne^2\tau}{m^*}} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 \equiv L = 2.45\times 10^{-8} W\Omega / K^2$$

Metal	0°C	100°C	Metal	0°C	100°C
Ag	2.31	2.37	Pb	2.47	2.56
Au	2.35	2.40	Pt	2.51	2.6
Cu	2.23	2.33	W	3.04	3.20
Mo	2.61	2.79	Zn	2.31	2.33

Seebeck Effect: Electrical voltage from temperature gradient

$V = S\Delta T$

S = Seebeck coefficient
- measured in V/K
- few μV/K metals; 100s μV/K semicond
- k_B/e is 82 μV/K – natural unit of measure

What happens when we heat?

Many carriers Few carriers

Seebeck Effect: Electrical voltage from temperature gradient

$V = S\Delta T$

S = Seebeck coefficient
- measured in V/K
- few μV/K metals; 100s μV/K semicond
- k_B/e is also V/K measure

What happens when we heat?

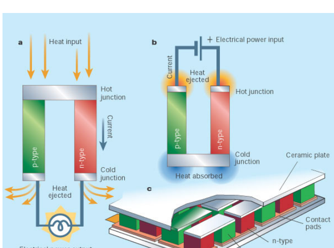
Diffusion Electric field results

Seebeck Effect: Electrical voltage from temperature gradient

Can show that S for free electron gas is: $S = -\frac{c_v}{3ne}$

With correct form for C_v (accounting for Fermi statistics) we get correct size, temperature dependence, but it is always negative! Need the concept of a hole.

Seebeck Effect: Electrical voltage from temperature gradient

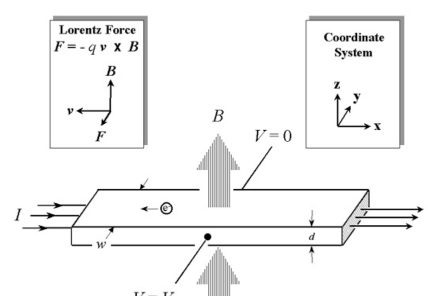


- solid state devices
- Bi_2Te_3 is used

$$ZT = \frac{S^2\sigma}{\kappa_{thermal}} = \frac{S^2\sigma}{\kappa_{elec}+\kappa_{phonon}}$$

Magnetic Response: Hall Effect

$$\vec{F} = m^*\left(\frac{d}{dt}+\frac{1}{\tau}\right)\vec{v} = q\left(\vec{\varepsilon}+\vec{v}\times\vec{B}\right)$$



www.eeel.nist.gov/ 812/effe.htm

Figure 1

Hall Effect: field perpendicular to current results in transverse voltage

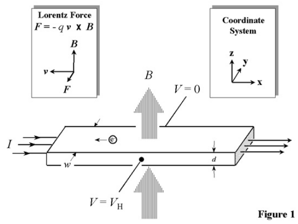
$$m^*\frac{d\vec{v}}{dt} + \frac{m^*\vec{v}}{\tau} = q\left(\vec{\varepsilon} + \vec{v}\times\vec{B}\right)$$

Static case: *d/dt = 0* $\vec{B} = B\hat{z} = (0,0,B)$

$$\frac{m^*}{\tau}v_x = q\left(\varepsilon_x + v_y B\right)$$

$$\frac{m^*}{\tau}v_y = q\left(\varepsilon_y - v_x B\right)$$

$$\frac{m^*}{\tau}v_z = q\varepsilon_z$$



Magnetic Response: Hall Effect

Special case: $v_y = 0$

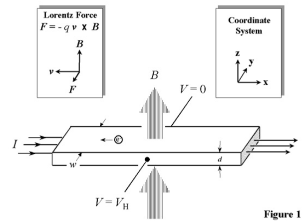
$$v_x = \frac{q\tau}{m^*}\varepsilon_x;\ j_x = \frac{nq^2\tau}{m^*}\varepsilon_x$$

$$\varepsilon_y = v_x B = \frac{q\tau B}{m^*}\varepsilon_x$$

Hall Coefficient R_H:

$$R_H \equiv \frac{\varepsilon_y}{\left|\vec{j}\times\vec{B}\right|} = \frac{\varepsilon_y}{j_x B} = \frac{1}{nq}$$

$R_H < 0$ for electrons
$R_H > 0$ for holes



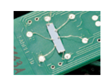
Magnetic Response: Hall Effect

Routinely used to measure carrier type and concentration in semiconductors
This derivation is for simple one-band model; more complex if 2 or more bands involved
R_H large if *n* small
Related concept is mobility μ of carriers:

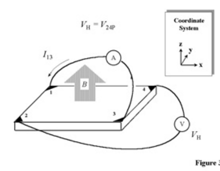
$$\sigma = \frac{ne^2\tau}{m^*} \equiv ne\mu \qquad \mu = \frac{e\tau}{m^*} = \sigma R_H$$

Mobility μ of carriers measured in cm^2/Vs (usually), and is more easily understood as [cm/s]/[V/cm] or velocity per field.
$\mu_{GaAs} \approx 8000\ cm^2/Vs$, $\mu_{Si} \approx 100\ cm^2/Vs$, $\mu_{p\text{-}TCO} \approx 1\ cm^2/Vs$

LakeShore Hall System



http://www.eeel.nist.gov/812/hall.html



Van der Pauw configuration --->