

# The Harmonic Oscillator UP & Time evolution

Read McIntyre 9.7 & 9.8

PH451/551

# Reading Quiz

1. Write the time dependence of a general quantum state of the HO.
2. What is the “selection” rule for dipole transitions between states of the HO?
3. What is Ehrenfest’s theorem?

# Reading Quiz

1. Write the time dependence of a general quantum state of the HO.

$$|\psi(t)\rangle = e^{-i\omega t/2} \sum_n e^{-in\omega t} c_n |n\rangle$$

2. What is the “selection” rule for dipole transitions between states of the HO?

$$\Delta n = \pm 1$$

3. What is Ehrenfest’s theorem?

QM expectation values obey classical laws. e.g.

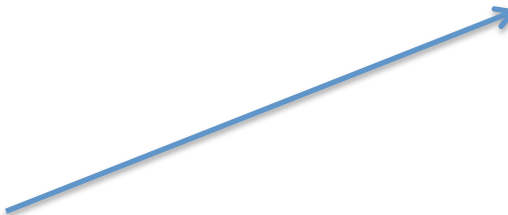
$$\frac{d}{dt} \langle x \rangle = \frac{\langle p \rangle}{m}$$

# Recap - matrices

1. Matrix element:  $A_{nm} \equiv \langle n | A | m \rangle$

2. Hamiltonian:  $H \doteq \hbar\omega \begin{pmatrix} \frac{1}{2} & 0 & 0 & \dots \\ 0 & \frac{3}{2} & 0 & \dots \\ 0 & 0 & \frac{5}{2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

$|\psi\rangle \doteq \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \end{pmatrix}$



3. General quantum state:

4. Ladder operators:

$$a \doteq \begin{pmatrix} 0 & \sqrt{1} & 0 & \dots \\ 0 & 0 & \sqrt{2} & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad a^\dagger \doteq \begin{pmatrix} 0 & 0 & 0 & \dots \\ \sqrt{1} & 0 & 0 & \dots \\ 0 & \sqrt{2} & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

# Uncertainty principle

- How to calculate?

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \equiv \sqrt{\langle (x - \langle x \rangle)^2 \rangle} = ?$$

$$\Delta p \equiv \sqrt{\langle (p - \langle p \rangle)^2 \rangle} = ?$$

# Superposition

- Electric dipole interaction:  $\langle n_f | qx | n_i \rangle$
- Calculate time dependence for different eigenstates ->
- And for general state ...

# Coherent state

- Gaussian ground state of HO doesn't propagate (eigenstate)
- Displaced Gaussian changes shape as it propagates in free space, but not in HO (coherent state)
- Coherent state models classical particle
- State of minimum uncertainty ( $\Delta x = \Delta p = 1/\sqrt{2}$ )
- See animations on book web page & class page