The Harmonic Oscillator UP & Time evolution

Read McIntyre 9.7 & 9.8

PH451/551

Reading Quiz

1. Write the time dependence of a general quantum state of the HO.
2. What is the “selection” rule for dipole transitions between states of the HO?
3. What is Ehrenfest’s theorem?

Reading Quiz

1. Write the time dependence of a general quantum state of the HO.

$$\left|\psi(t)\right\rangle = e^{-i\omega t/2}\sum_n e^{-in\omega t} c_n \left|n\right\rangle$$

2. What is the “selection” rule for dipole transitions between states of the HO?

$$\Delta n = \pm 1$$

3. What is Ehrenfest’s theorem?

QM expectation values obey classical laws. e.g.

$$\frac{d}{dt}\left\langle x\right\rangle = \frac{\left\langle p\right\rangle}{m}$$

Recap - matrices

1. Matrix element: $A_{nm} \equiv \langle n | A | m \rangle$

2. Hamiltonian: $H \doteq \hbar\omega \begin{pmatrix} \frac{1}{2} & 0 & 0 & \cdots \\ 0 & \frac{3}{2} & 0 & \cdots \\ 0 & 0 & \frac{5}{2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

3. General quantum state: $|\psi\rangle \doteq \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \end{pmatrix}$

4. Ladder operators: $a \doteq \begin{pmatrix} 0 & \sqrt{1} & 0 & \cdots \\ 0 & 0 & \sqrt{2} & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$ $a^\dagger \doteq \begin{pmatrix} 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

Uncertainty principle

- How to calculate?

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \equiv \sqrt{\left\langle \left(x - \langle x \rangle \right)^2 \right\rangle} = ?$$

$$\Delta p \equiv \sqrt{\left\langle \left(p - \langle p \rangle \right)^2 \right\rangle} = ?$$

Superposition

- Electric dipole interaction: $\left\langle n_f \middle| qx \middle| n_i \right\rangle$

- Calculate time dependence for different eigenstates ->

- And for general state …

Coherent state

- Gaussian ground state of HO doesn't propagate (eigenstate)
- Displaced Gaussian changes shape as it propagates in free space, but not in HO (coherent state)
- Coherent state models classical particle
- State of minimum uncertainty (Dx=Dp=1/root2)
- See animations on book web page & class page