The Harmonic Oscillator
Matrix notation

Read McIntyre 9.5

PH451/551

Reading Quiz

1. Write the Hamiltonian *H* of the HO in matrix notation:

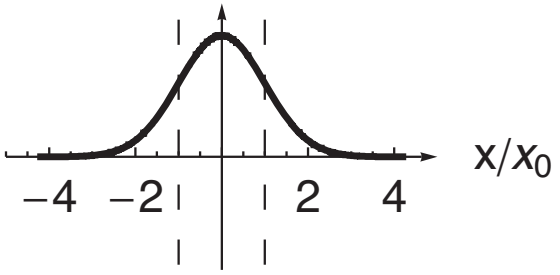
2. Write a general quantum state of the HO in matrix notation:

3. Write the form of the ladder operators in matrix notation.

Recap

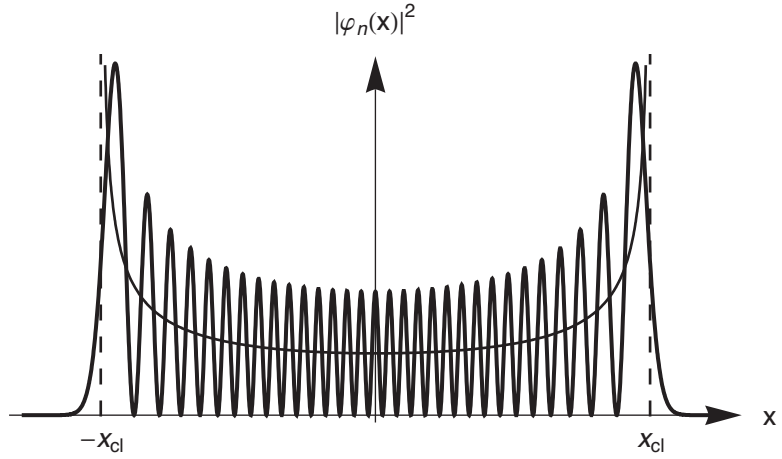
1. Eigenstates: kets & wf $\varphi_n(x) \doteq |n\rangle$ $\varphi_n(x) = \langle x|n\rangle$
2. Expectation values, projections, probabilities, normalization, time dependence

   $|\psi\rangle = ?\left[|0\rangle + i|1\rangle + e^{i\delta}|3\rangle - |4\rangle\right]$
3. <x> and <p> are easy in HO, because of ladder operators! (No spatial integrals necessary!)



Classical limit

- Probability density: $P(x) = \psi^*(x)\psi(x)$
- *n*-> large value:
  How is this classical?
  Other systems?



Matrix representation

- x

$$H_{ij} = \langle i | H | j \rangle$$

$$H \doteq \hbar\omega \begin{pmatrix} \frac{1}{2} & 0 & 0 & \cdots \\ 0 & \frac{3}{2} & 0 & \cdots \\ 0 & 0 & \frac{5}{2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Matrix representation

$$Q_{ij} = \langle i | Q | j \rangle$$

- A, a+, x, p?

$$a \doteq \begin{pmatrix} 0 & \sqrt{1} & 0 & \cdots \\ 0 & 0 & \sqrt{2} & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad a^\dagger \doteq \begin{pmatrix} 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$