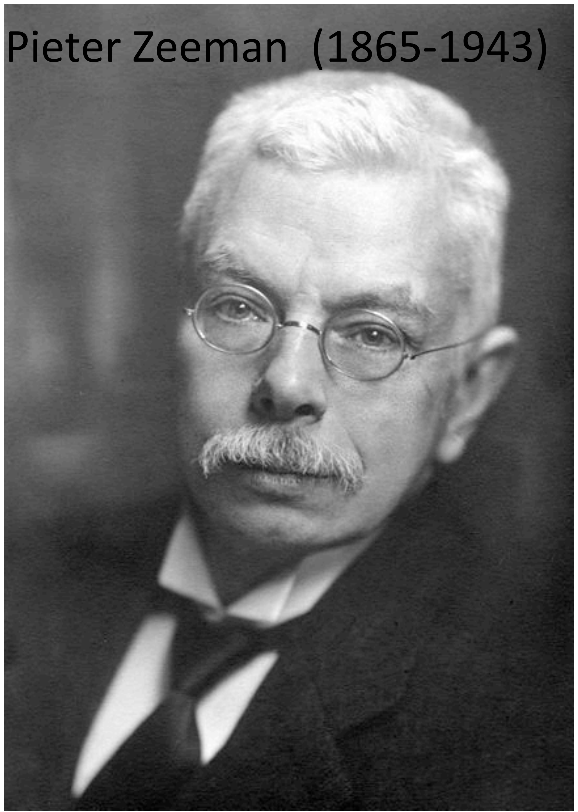
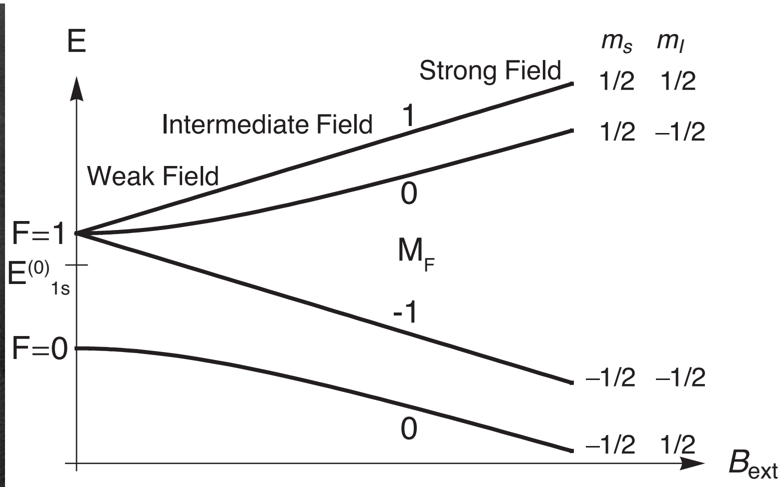


Pieter Zeeman (1865-1943)



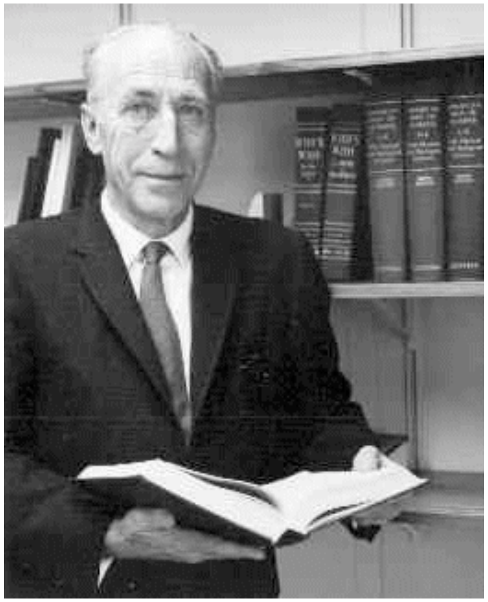
Zeeman Effect

Eugene Wigner (1902-1995)

Read McIntyre 12.3

PH451/551

Carl Eckhart (1902-1973)



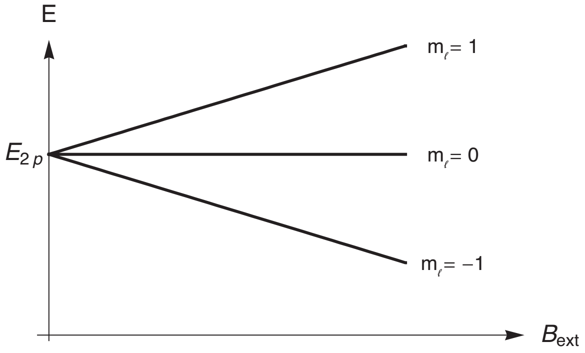
Alfred Landé (1888-1976)

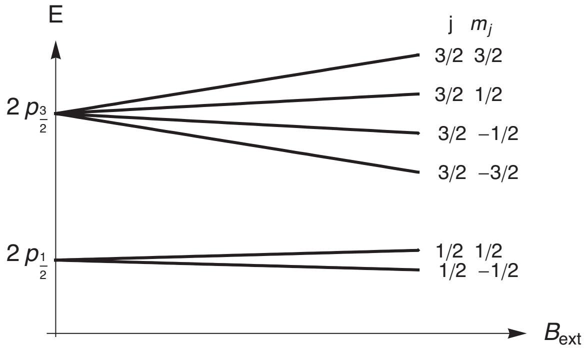
Reading Quiz

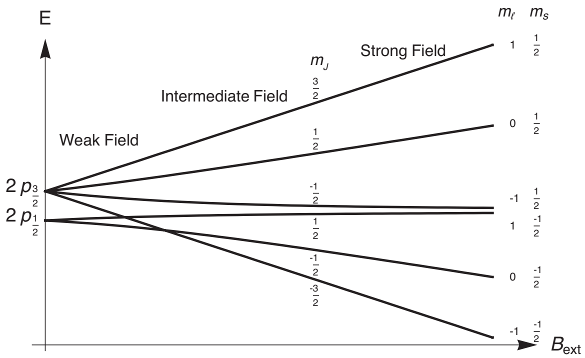
1. What is the perturbation Hamiltonian for the Zeeman effect?
2. Draw a qualitative sketch of the effect of an external magnetic field on a set of energy levels

Reading Quiz

1. What is the perturbation Hamiltonian for the Zeeman effect? $H' = -\mu . B_{ext}$
2. Draw a qualitative sketch of the effect of an external magnetic field on a set of energy levels

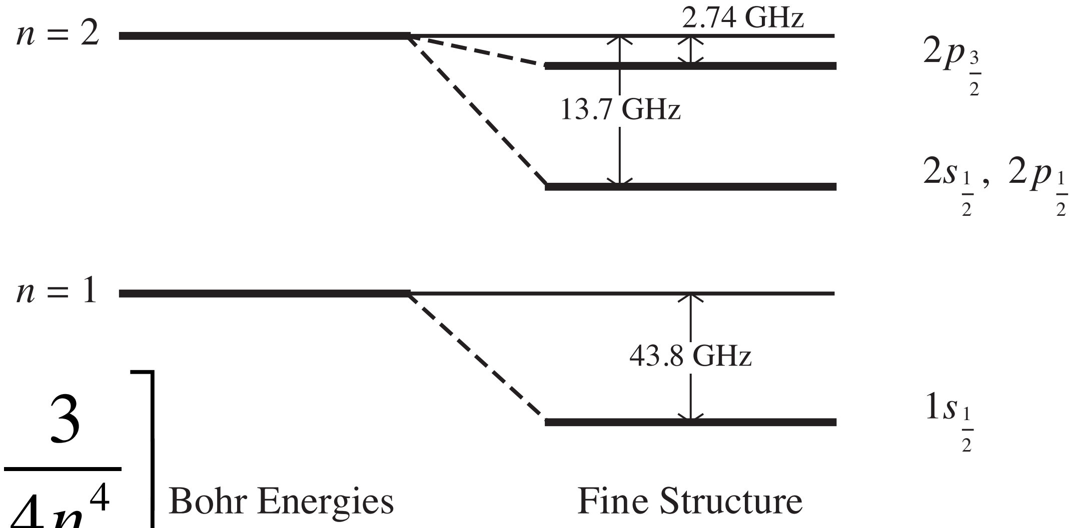






Recap: Fine structure correction

1. When we put E_{SO} and E_{rel} together, we get:



$$E_{rel}^{(1)} = -\frac{1}{2}\alpha^4 mc^2 \left[\frac{1}{n^3\left(\ell + \frac{1}{2}\right)} - \frac{3}{4n^4} \right]$$

$$E_{SO}^{(1)} = \frac{1}{4}\alpha^4 mc^2 \frac{j(j+1) - \ell(\ell+1) - \frac{3}{4}}{n^3 \ell\left(\ell + \frac{1}{2}\right)(\ell+1)}$$

$$E_{fs}^{(1)} = E_{rel}^{(1)} + E_{SO}^{(1)} = -\frac{1}{2}\alpha^4 mc^2 \frac{1}{n^3}\left[\frac{1}{j + \frac{1}{2}} - \frac{3}{4n} \right]$$

Magnetic moments, *g*, etc

1. Classical magnetic moment of a charged point particle with angular momentum *L*:

$$\mu = IA = \frac{q\pi r^2}{T} = \frac{q2m\pi f r^2}{2m} = \frac{q}{2m} m\omega r^2 = \frac{q}{2m} L$$

2. Bohr magneton: $\mu_B = \frac{|e|\hbar}{2m_e}$

   Nuclear magneton: $\mu_N = \frac{|e|\hbar}{2m_p}$

3. Classical particle (no spin)

$$\mu_e = -\frac{\mu_B}{\hbar} L;\ \mu_p = +\frac{\mu_N}{\hbar} L;\ \mu_n = 0\frac{\mu_N}{\hbar} L;$$

Magnetic moment, *g*, *etc.*

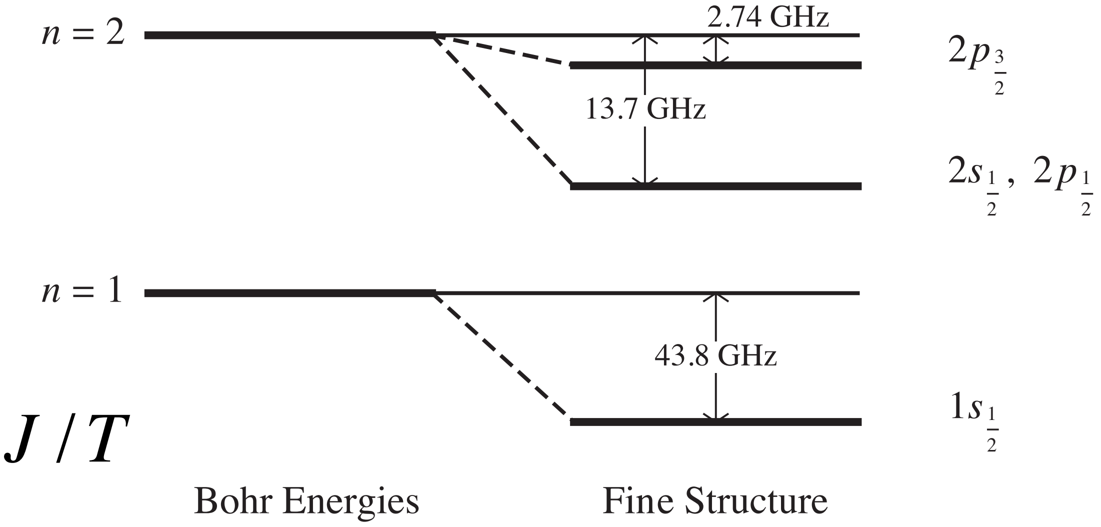
Caution: different sign conventions for g in literature

4. Spin is a **non-classical** angular momentum. Not all particles are point like. g-factor takes account of non-point-like structure, non-classical effects:
5. Electron: $\mu_e = -g_{e,L}\frac{\mu_B}{\hbar}L - g_{e,S}\frac{\mu_B}{\hbar}S$ $g_{e,S} = 2$
6. Proton: $\mu_p = +g_{p,L}\frac{\mu_N}{\hbar}L + g_{p,S}\frac{\mu_N}{\hbar}S$ $g_{p,S} = 5.58$
7. The neutron also has a magnetic moment, even though it is neutral. The moment comes from the internal quark and gluon structure.

$$\mu_p = -g_{n,S}\frac{\mu_N}{\hbar}S \qquad g_{n,S} = 3.82$$

Bohr magneton

4. Size of Bohr magneton:

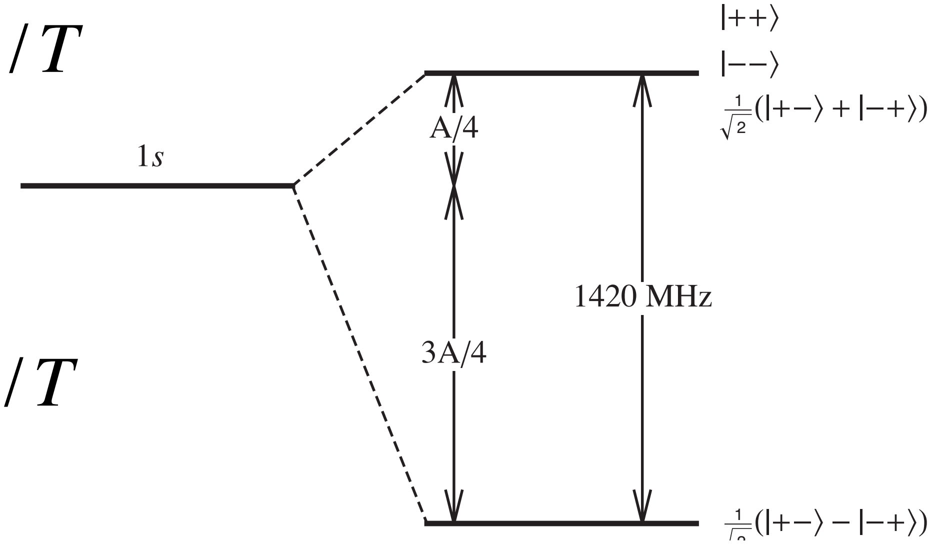


$$\mu_B = \frac{|e|\hbar}{2m_e}$$

$$\mu_B = 9.27400968(20)\times 10^{-24}\,J/T$$

$$\mu_B = 5.7883818066(38)\times 10^{-5}\,eV/T$$
$$= 60\mu eV/T$$

$$\mu_B = 13.996\ 245\ 55(31)\times 10^{9}\,Hz/T$$
$$= 14GHz/T$$



Zeeman, with spin

1. Apply external field:

$$H'_Z = -\mu \cdot \mathbf{B}_{ext} = g_{e,L}\frac{\mu_B}{\hbar}L_z B + g_{e,S}\frac{\mu_B}{\hbar}S_z B$$

$$H'_Z = \frac{\mu_B}{\hbar}L_z B + 2\frac{\mu_B}{\hbar}S_z B$$

2. What is H_0?

   If *B* is strong

$$H = \underbrace{H_{Coulomb}}_{H_0} + H'_Z + \underbrace{H'_{SO}}_{SMALL}$$

If *B* is weak

$$H = \underbrace{H_{Coulomb} + H'_{SO}}_{H_0} + H'_Z$$

Zeeman (with spin), strong field

$$H'_Z = \frac{\mu_B B}{\hbar}\left(L_z + 2S_z\right)$$

1. EXERCISE

In strong field, Find the energy corrections to the *n* = 3 states.

Hint, what are possible a.m. numbers? Spin?

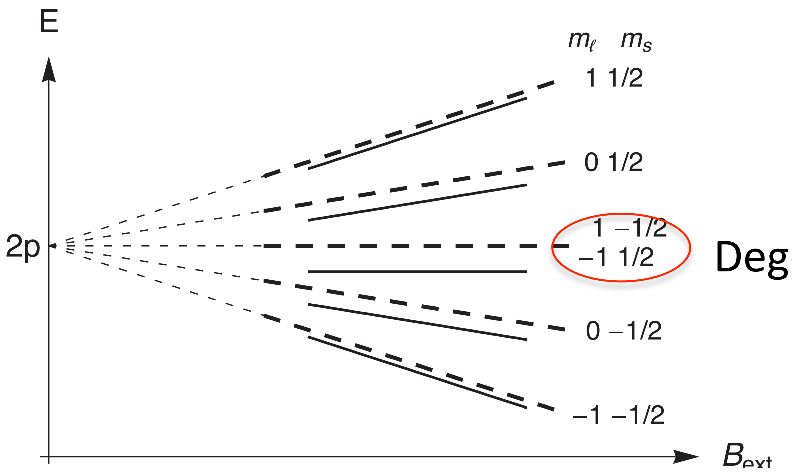
Zeeman (with spin), strong field

$$H = \underbrace{H_{Coulomb}}_{H_0} + \frac{\mu_B B}{\hbar}\left(L_z + 2S_z\right)$$

1. SO + rel (fine structure) smaller than Zeeman energy; so Zeeman is a perturbation on the usual H_0, which is diagonal in the UNCOUPLED basis!

2. H'_Z is diagonal in uncoupled basis! (dotted line)

$$E^{(1)}_{Zeeman} = \mu_B B\left(m_\ell + 2m_s\right)$$



Degenerate!

Zeeman (with spin), weak field

$$H = \underbrace{H_{Coulomb} + H'_{SO}}_{H_0} + H'_Z$$

$$H = \underbrace{H_0}_{H_{Coulomb}+H'_{SO}} + \frac{\mu_B B}{\hbar}\left(L_z + 2S_z\right)$$

Weak field: H$_0$ includes SO & is diagonal in coupled basis. Is *H*'$_Z$ diagonal?

$$\text{matrix elts} = \left\langle nj'm'_j\,\ell's \middle| H'_Z \middle| njm_j\ell s\right\rangle$$

Zeeman, with spin, weak field

1. C-G coeffs (table 11.3)

$$\text{matrix elts} = \left\langle n j' m'_j \ell' s \middle| H'_Z \middle| n j m_j \ell s \right\rangle$$

$$\left| j m_j \ell s \right\rangle = \sum_{m_\ell m_s} \underbrace{\left\langle \ell s m_\ell m_s \middle| j m_j \ell s \right\rangle}_{C-G\quad coeffs} \left| \ell s m_\ell m_s \right\rangle$$

$j_1=1$		j	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	$j_2=\frac{1}{2}$	m	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
m1	m2							
1	$\frac{1}{2}$		1	0	0	0	0	0
1	$-\frac{1}{2}$		0	$\frac{1}{\sqrt{3}}$	0	0	$\sqrt{\frac{2}{3}}$	0
0	$\frac{1}{2}$		0	$\sqrt{\frac{2}{3}}$	0	0	$-\frac{1}{\sqrt{3}}$	0
0	$-\frac{1}{2}$		0	0	$\sqrt{\frac{2}{3}}$	0	0	$\frac{1}{\sqrt{3}}$
-1	$\frac{1}{2}$		0	0	$\frac{1}{\sqrt{3}}$	0	0	$-\sqrt{\frac{2}{3}}$
-1	$-\frac{1}{2}$		0	0	0	1	0	0

Zeeman, coupled basis, S_z and L_z

error in Eq. 12.63 for L_z

1. 2p states: why are the boxes degenerate subspaces?

$$H'_Z = \frac{\mu_B B}{\hbar}\left(L_z + 2S_z\right)$$

$$S_z \doteq \hbar \left(\begin{array}{cccccc} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{-\sqrt{2}}{3} & 0 \\ 0 & 0 & \frac{-1}{6} & 0 & 0 & \frac{-\sqrt{2}}{3} \\ 0 & 0 & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{-1}{6} & 0 \\ 0 & 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{1}{6} \end{array}\right) \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

$$L_z \doteq \hbar \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{\sqrt{2}}{3} & 0 \\ 0 & 0 & \frac{-1}{3} & 0 & 0 & \frac{\sqrt{2}}{3} \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{3} & 0 & 0 & \frac{-2}{3} \end{array}\right) \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

Zeeman, coupled basis, H'_Z

error in Eq. 12.63 for L_z

1. 2p states

$$E_{n,j}^{(0)} = E_n^{(0)} + E_{fs}^{(1)} = -\frac{Ry}{n^2} - \frac{1}{2}\alpha^4 mc^2 \frac{1}{n^3}\left[\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right]$$

$$L_z \doteq \hbar \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{\sqrt{2}}{3} & 0 \\ 0 & 0 & \frac{-1}{3} & 0 & 0 & \frac{\sqrt{2}}{3} \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{3} & 0 & 0 & \frac{-2}{3} \end{pmatrix} \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

note that degenerate subspaces are diagonal!!

$$H'_Z = \frac{\mu_B B}{\hbar}\left(L_z + 2S_z\right)$$

$$S_z \doteq \hbar \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{-\sqrt{2}}{3} & 0 \\ 0 & 0 & \frac{-1}{6} & 0 & 0 & \frac{-\sqrt{2}}{3} \\ 0 & 0 & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{-1}{6} & 0 \\ 0 & 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{1}{6} \end{pmatrix} \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

$$H'_z \doteq \mu_B B \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{-\sqrt{2}}{3} & 0 \\ 0 & 0 & \frac{-2}{3} & 0 & 0 & \frac{-\sqrt{2}}{3} \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{-1}{3} \end{pmatrix} \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

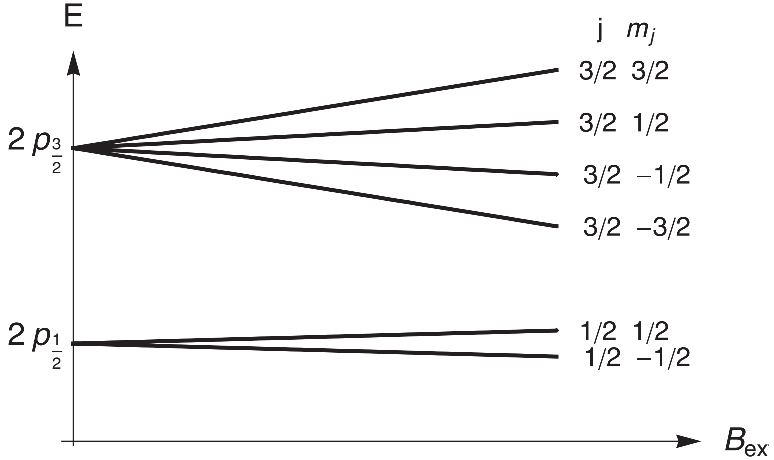
Zeeman, weak field, final result

1. 2p states

$$E_{n,j}^{(0)} = E_n^{(0)} + E_{fs}^{(1)} = -\frac{Ry}{n^2} - \frac{1}{2}\alpha^4 mc^2 \frac{1}{n^3}\left[\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right]$$

$$H'_z \doteq \mu_B B \left(\begin{array}{cccccc|c} 2 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2},\frac{3}{2} \\ 0 & \frac{2}{3} & 0 & 0 & \frac{-\sqrt{2}}{3} & 0 & \frac{3}{2},\frac{1}{2} \\ 0 & 0 & \frac{-2}{3} & 0 & 0 & \frac{-\sqrt{2}}{3} & \frac{3}{2},\frac{-1}{2} \\ 0 & 0 & 0 & -2 & 0 & 0 & \frac{3}{2},\frac{-3}{2} \\ 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2},\frac{1}{2} \\ 0 & 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{-1}{3} & \frac{1}{2},\frac{-1}{2} \end{array}\right)$$

??
Second
order!



$$E_Z^{(1)}\left(j=\tfrac{3}{2},\ell=1,m_j\right) = 2\mu_B B, \frac{2}{3}\mu_B B, \frac{-2}{3}\mu_B B, -2\mu_B B$$

$$E_Z^{(1)}\left(j=\tfrac{1}{2},\ell=1,m_j\right) = \frac{1}{3}\mu_B B, \frac{-1}{3}\mu_B B$$

Zeeman, final result

1. 2p states

$$H'_z \doteq \mu_B B \left(\begin{array}{cccccc} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{-\sqrt{2}}{3} & 0 \\ 0 & 0 & \frac{-2}{3} & 0 & 0 & \frac{-\sqrt{2}}{3} \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{-\sqrt{2}}{3} & 0 & 0 & \frac{-1}{3} \end{array}\right) \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

$$E_Z^{(1)}\left(j=\tfrac{3}{2},\ell=1,m_j\right)=2\mu_B B,\frac{2}{3}\mu_B B,\frac{-2}{3}\mu_B B,-2\mu_B B$$

$$E_Z^{(1)}\left(j=\tfrac{1}{2},\ell=1,m_j\right)=\frac{1}{3}\mu_B B,\frac{-1}{3}\mu_B B$$

2. Here's a general result (without proof ...)

$$E_Z^{(1)}=\mu_B B m_j\left(1\pm\frac{1}{2\ell+1}\right)$$

$$j=\ell+\tfrac{1}{2}$$

$$j=\ell-\tfrac{1}{2}$$

Landé g-factor

$$E_Z^{(1)}\left(j=\tfrac{3}{2},\ell=1,m_j\right)=\frac{4}{3}\mu_B B m_j$$

$$E_Z^{(1)}\left(j=\tfrac{1}{2},\ell=1,m_j\right)=\frac{2}{3}\mu_B B m_j$$

Zeeman, intermediate field

1. If fine structure H' and Zeeman H' are the same size, there is no preferred basis, so use coupled.

$$H' = \underbrace{H'_{SO} + H'_{rel}}_{\text{fine structure}} + H'_Z$$

2. Already worked out fine structure in coupled basis (diagonal) (Eq 12.47)

$$E_{fs}^{(1)} = E_{rel}^{(1)} + E_{SO}^{(1)} = -\frac{1}{2}\alpha^4 mc^2 \frac{1}{n^3}\left[\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right]$$

$$H'_{fs}(n=2) \doteq -\frac{1}{2(64)}\alpha^4 mc^2 \begin{pmatrix} \boxed{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \boxed{\begin{matrix} 5 & 0 \\ 0 & 5 \end{matrix}} \end{pmatrix} \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

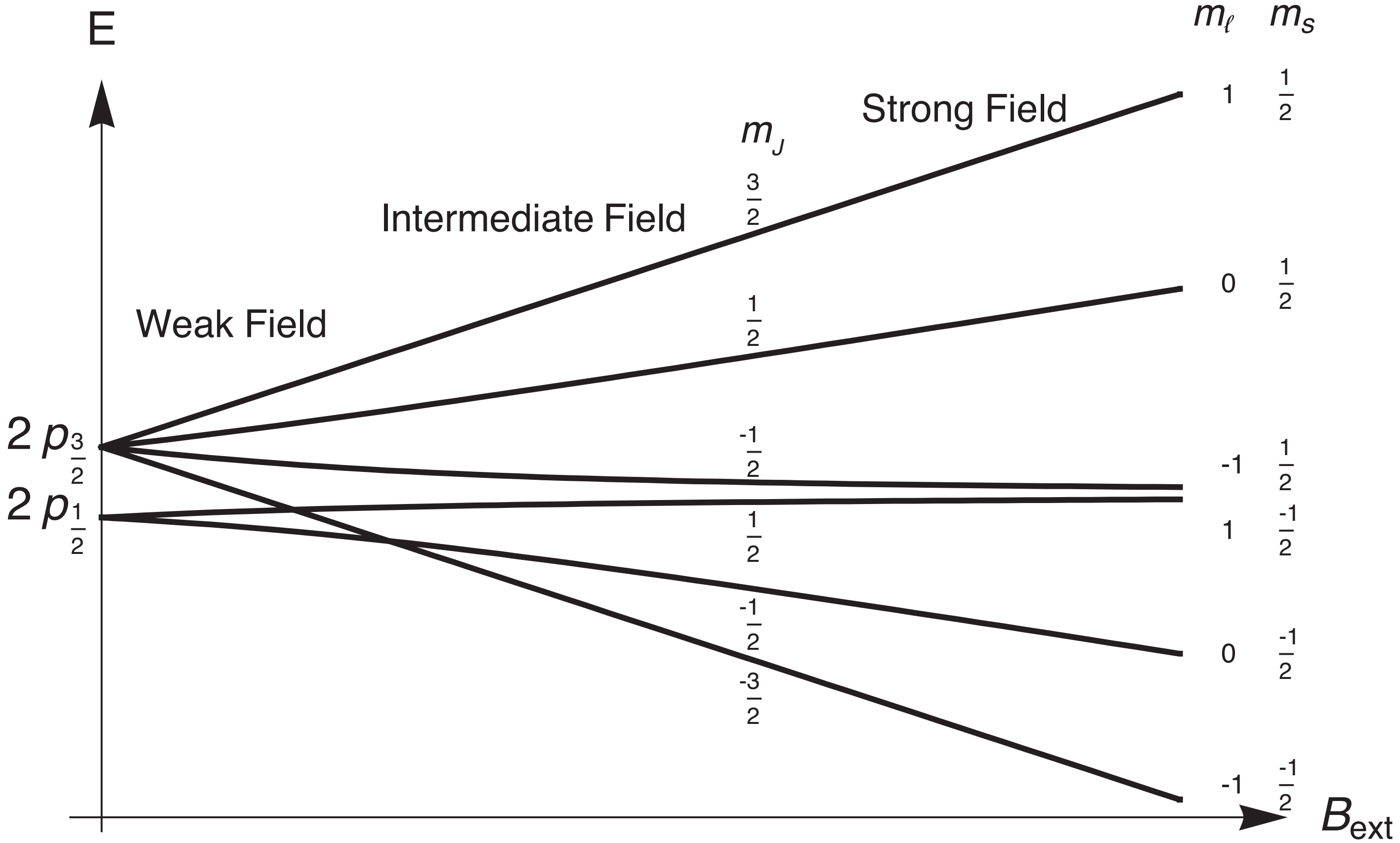
Zeeman, intermediate field

1. H'_Z is non-diagonal in coupled basis, and we’ve done this, too, for weak field (12.63 and 12.64)!

$$H'_z \doteq \mu_B B \left(\begin{array}{cc} \boxed{\begin{matrix} 2 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{-2}{3} & 0 \\ 0 & 0 & 0 & -2 \end{matrix}} & \begin{matrix} 0 & 0 \\ \frac{-\sqrt{2}}{3} & 0 \\ 0 & \frac{-\sqrt{2}}{3} \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & \frac{-\sqrt{2}}{3} & 0 & 0 \\ 0 & 0 & \frac{-\sqrt{2}}{3} & 0 \end{matrix} & \boxed{\begin{matrix} \frac{1}{3} & 0 \\ 0 & \frac{-1}{3} \end{matrix}} \end{array} \right) \begin{array}{c} \frac{3}{2},\frac{3}{2} \\ \frac{3}{2},\frac{1}{2} \\ \frac{3}{2},\frac{-1}{2} \\ \frac{3}{2},\frac{-3}{2} \\ \frac{1}{2},\frac{1}{2} \\ \frac{1}{2},\frac{-1}{2} \end{array}$$

Zeeman, intermediate field

1. Add together (n=2) Fig 12.9:



Non degenerate PT

$$H_0 \doteq \begin{pmatrix} A & 0 & 0 & 0 & 0 & 0 \\ 0 & B & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 \\ 0 & 0 & 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 & E & 0 \\ 0 & 0 & 0 & 0 & 0 & F \end{pmatrix}$$

$$H' \doteq \begin{pmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa \end{pmatrix}$$

$$H' \doteq \begin{pmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & \theta & 0 & 0 & 0 \\ 0 & \theta & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon & \phi \\ 0 & 0 & 0 & 0 & \phi & \kappa \end{pmatrix}$$

$$H' \doteq \begin{pmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & \theta & 0 \\ 0 & 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta & 0 & 0 \\ 0 & \theta & 0 & 0 & \varepsilon & \phi \\ 0 & 0 & 0 & 0 & \phi & \kappa \end{pmatrix}$$

Degenerate PT

$$H_0 \doteq \begin{pmatrix} \boxed{\begin{matrix} A & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \boxed{\begin{matrix} B & 0 \\ 0 & B \end{matrix}} \end{pmatrix}$$

$$H' \doteq \begin{pmatrix} \boxed{\begin{matrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \delta \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \boxed{\begin{matrix} \varepsilon & 0 \\ 0 & \kappa \end{matrix}} \end{pmatrix}$$

$$H' \doteq \begin{pmatrix} \boxed{\begin{matrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & \theta & 0 \\ 0 & \theta & \gamma & 0 \\ 0 & 0 & 0 & \delta \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \boxed{\begin{matrix} \varepsilon & \phi \\ \phi & \kappa \end{matrix}} \end{pmatrix}$$

$$H' \doteq \begin{pmatrix} \boxed{\begin{matrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \delta \end{matrix}} & \begin{matrix} 0 & 0 \\ \theta & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & \theta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \boxed{\begin{matrix} \varepsilon & 0 \\ 0 & \kappa \end{matrix}} \end{pmatrix}$$