Review of paradigms QM: 1-D potentials

Read McIntyre Ch. 5.2-5.5 (review) and 5.10.2
and
"Shooting method" numerical solutions pp. 1-3.5

QM Postulates

⑥ The time evolution of a quantum system is determined by the Hamiltonian or total energy operator *H*(*t*) through the Schrödinger equation

$$i\hbar \frac{d}{dt}\left|\psi(t)\right\rangle = H(t)\left|\psi(t)\right\rangle$$

QM Postulates

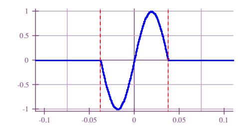
H(*t*) is an operator and mostly it is **not** explicitly time dependent. The "hat" notation (below) is to remind you it is an operator. Often, we leave off the hat. We therefore spend much time finding the energy eigenstates of a system from the **energy eigenvalue equation**:

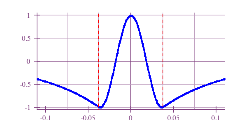
$$\hat{H}\left|\varphi_n\right\rangle = E_n\left|\varphi_n\right\rangle$$

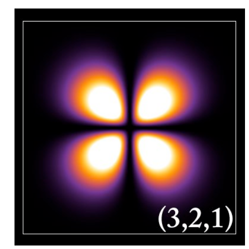
E_n is a constant number, not an operator. It is the energy eigenvalue.

A few systems can be solved analytically:

- Spin-1/2 and spin-1
- Infinite square well potential
- Finite square well potential
- H-atom
- Free particle
- Harmonic Oscillator (this course; next topic)







Many more can be solved approximately:

- Perturbation theory (this course, later)
- Computation e.g. shooting method (today)

Shooting method:

- This equation has solutions $\varphi(x)$ for any E, but the correct $\varphi(x)$ have the appropriate BOUNDARY CONDITIONS, and those correspond the E that we call the energy eigenvalues.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi_n(x)+U(x)\varphi_n(x)=E_n\varphi_n(x)$$

- What are the correct BCs?

Mathematica Exercise:

- $U(x)$ = Finite well potential energy

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi_n(x)+U(x)\varphi_n(x)=E_n\varphi_n(x)$$

- What is $U(x)$?
- Find the correct energy eigenvalues
- Use notebook on class site

What are the characteristics of wave functions in 1-D potentials?

- Oscillatory where E>U (nodes increase with E)
- Decaying where E<U (penetration depth increases with distance from "top of well")
- Wave functions are continuous
- Derivatives are continuous except if U=infinite

(Start delta function problem on hwk)