Perturbation theory non degen (& degen?)

Read McIntyre 10.3-10.4

PH451/551

Reading Quiz

If $H = H_0 + H'$

1. Equation for 1st order correction to energy of *n*th eigenstate?

2. Equation for 1st order correction to *n*th state?

3. Main features of 2nd order energy correction?

Recap

1. Energy: $$E_n = E_n^{(0)} + E_n^{(1)} + E_n^{(2)}$$

$$E_n^{(1)} = \left\langle n^{(0)} \middle| H' \middle| n^{(0)} \right\rangle$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{\left| \left\langle m^{(0)} \middle| H' \middle| n^{(0)} \right\rangle \right|^2}{\left(E_n^{(0)} - E_m^{(0)} \right)}$$

Recap (state)

1. State

$$\left|n\right\rangle = \left|n^{(0)}\right\rangle + \left|n^{(1)}\right\rangle$$

$$\left|n\right\rangle = \left|n^{(0)}\right\rangle + \sum_{m\neq n} \frac{\left\langle m^{(0)}\right| H' \left|n^{(0)}\right\rangle}{\left(E_n^{(0)} - E_m^{(0)}\right)} \left|m^{(0)}\right\rangle$$

Example HO

1. Add *E* field to 1-D HO Hamiltonian
2. *H'*=bx -> calculate first order correction to energy

$$E_n^{(1)} = \left\langle n^{(0)} \middle| H' \middle| n^{(0)} \right\rangle = 0$$

Example - HO

1. Add *E* field to 1-D HO Hamiltonian
2. Now calculate first order correction to state

$$\left|n^{(1)}\right\rangle = \sum_{m \neq n} \frac{\left\langle m^{(0)} \middle| H' \middle| n^{(0)} \right\rangle}{\left(E_n^{(0)} - E_m^{(0)}\right)} \left|m^{(0)}\right\rangle$$

Derivation – 2nd order energy

1. Second order equation

$$\left(H_0 - E_n^{(0)}\right)\left|n^{(2)}\right\rangle = \left(E_n^{(1)} - H'\right)\left|n^{(1)}\right\rangle + E_n^{(2)}\left|n^{(0)}\right\rangle$$

$$\left\langle n^{(0)}\right|\left(H_0 - E_n^{(0)}\right)\left|n^{(2)}\right\rangle =$$

$$\left\langle n^{(0)}\right|\left(E_n^{(1)} - H'\right)\left|n^{(1)}\right\rangle + \left\langle n^{(0)}\right| E_n^{(2)}\left|n^{(0)}\right\rangle$$

Derivation – 2nd order energy

1. Second order equation

$$\langle n^{(0)} | \left(E_n^{(0)} - E_n^{(0)} \right) | n^{(2)} \rangle =$$

$$\langle n^{(0)} | \left(H'_{nn} - H' \right) \sum_{m \neq n} \frac{H'_{mn}}{\left(E_n^{(0)} - E_m^{(0)} \right)} | m^{(0)} \rangle + E_n^{(2)}$$

Derivation – 2nd order energy

1. Second order equation

$$E_n^{(2)} = \sum_{m \neq n} \langle n^{(0)} | (H' - H'_{nn}) \frac{H'_{mn}}{\left(E_n^{(0)} - E_m^{(0)}\right)} | m^{(0)} \rangle$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{H'_{mn}}{\left(E_n^{(0)} - E_m^{(0)}\right)} \langle n^{(0)} | H' | m^{(0)} \rangle$$

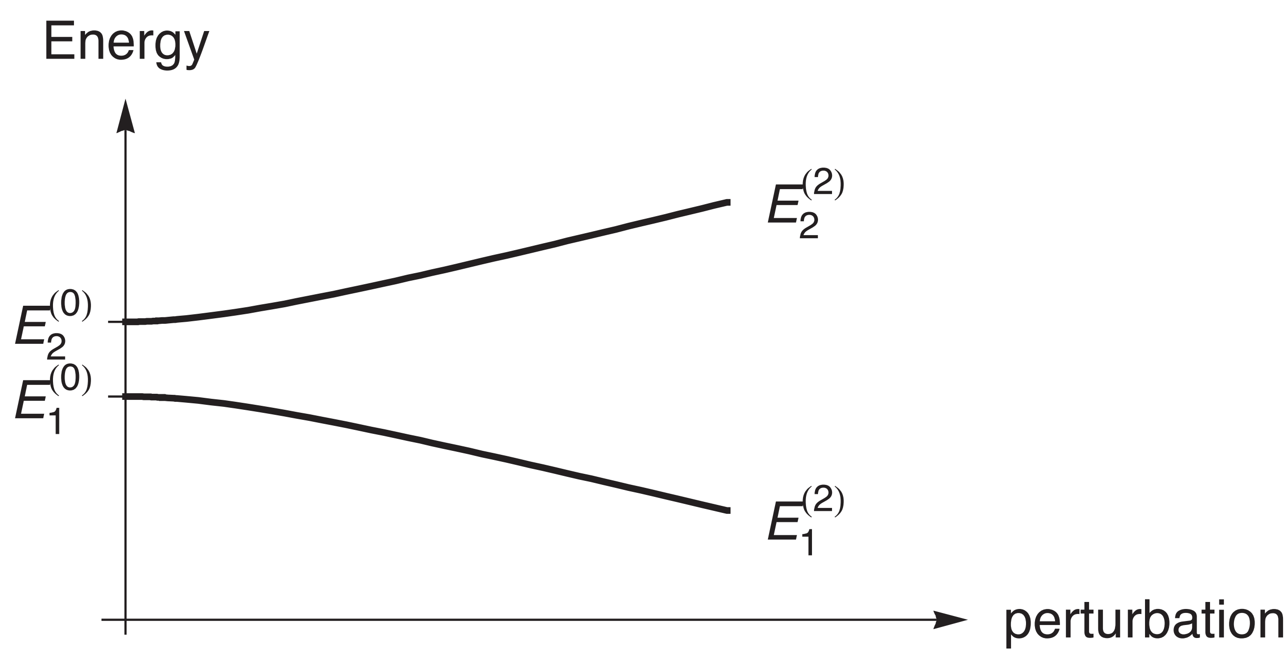
$$E_n^{(2)} = \sum_{m \neq n} \frac{H'_{mn} H'_{nm}}{\left(E_n^{(0)} - E_m^{(0)}\right)} = \sum_{m \neq n} \frac{H'_{mn} H'^{*}_{mn}}{\left(E_n^{(0)} - E_m^{(0)}\right)}$$

Derivation – 2nd order energy

1. Second order energy correction

$$E_n^{(2)} = \sum_{m \neq n} \frac{\left|H'_{mn}\right|^2}{\left(E_n^{(0)} - E_m^{(0)}\right)}$$

2. "States repel"



Matrix approach

1. Do you see how the matrix approach is simply writing out the bra-ket approach in longhand?

2. Go over questions about this.

Degenerate perturbation theory

1. Divergence of denominator!

2. Comes about because of “wrong basis”

3. Equation for 1st order correction involves not 1 but many (degen) states => mix up the degen states.

4. **Diagonalize the perturbation Hamiltonian in the degenerate subspace**