

- (a) Who first predicted, and who first observed the 21 cm line from galactic hydrogen? Cite the papers in which the discoveries were reported. (Be impressed that the theorist was a student!)

(b) In a class exercise, we related the 21-cm photon wavelength to an energy (in electron volts), a temperature (in K) and a frequency (in Hz). This energy scale is characteristic of the so-called "hyperfine" structure of the hydrogen atom. Read ahead to find the energy scale of the "fine structure" and repeat the exercise. (P.S. repeat every time you come across a new physical phenomenon!)
- Suppose the quantum state of a hydrogen atom is be represented as  $|n, \ell, m_\ell, s, m_s, I, m_I\rangle$ .

  - Explain carefully what each quantum number means.
  - In this notation, what is a representation of the state which the chemists call the 3d state that has the electron and proton with opposite projections of their spin angular momenta on the  $z$ -axis and which has zero projection of the orbital angular momentum on the  $z$ -axis?

We started Problem 3 in class on Wednesday, we'll continue on Friday, and you can finish the rest. Problem 4 is really a continuation of problem 3.

- Construct the matrices** for  $S_z, I_z, S_+, I_+, S_-, I_-, I^2, S^2$  in the basis of the 4 kets  $|s = \frac{1}{2}, m_s = \pm \frac{1}{2}; I = \frac{1}{2}, m_I = \pm \frac{1}{2}\rangle$ , representing a system of spin-1/2 electron ( $S$ ) **and** a spin-1/2 proton ( $I$ ). Remember that we are interested only in the spin part of the quantum system. For notational simplicity, you might prefer to label the states by the  $m$ -quantum numbers only because  $s = \frac{1}{2}, I = \frac{1}{2}$  always. That is  $|m_s = \pm \frac{1}{2}; m_I = \pm \frac{1}{2}\rangle$ . You could use even simpler notation of "+" and "-" if you like, but you must not forget the values that the symbols represent.
- McIntyre 11.8** Formally construct  $H'$  due to the hyperfine interaction and **ALSO** formally solve for the eigenvalues and eigenfunctions, just so you've actually done it. Put in all the constants and make sure you agree that the splitting is indeed 21 cm! For all your fundamental constants needs go here <http://physics.nist.gov/cuu/Constants/> and **ONLY** here. This is THE official place.

You do not have to have solved the previous problems to do #5, but you do have to know that the hyperfine perturbation Hamiltonian is  $H'_{hf} = \text{const } \vec{S} \cdot \vec{I}$ , remember how to use vectors, and remember the basics from the spins paradigm.

- McIntyre 11.9** (practice with commutators – which operators commute with  $H'_{\text{hyperfine}}$ ?)