PH451/551 Homework #2

Your homework grade includes an explicit 10% that reflects your ability to communicate the work well, including an interpretative statement about your result/mathematics

- 1. McIntyre 9.4 (probability calculation with HO eigenfunctions)
- 2. McIntyre 9.9 (expectation values by integration and by operator method). For part (a), you may use Mathematica/Maple *etc*. for the integrals. BUT IF THE INTEGRALS ARE ZERO, then you should have been able to do them without using Mathematica, so explain WHY they are zero. If you do use a computer, please include enough information to demonstrate that you know the forms of the wave functions. No answers of the type "When you substitute the functions and put it all into Mathematica, the answer is 4"! Remember your audience (refer to the writing guide if you've forgotten who your audience is).
- 3. McIntyre 9.13 (Spectrum of a modified HO well)
- 4. McIntyre 9.14a (expectation values) not 9.14b
- 5. Commutator algebra
 - (a) Show that the following are true. You don't need to reflect on the physics,

$$\begin{bmatrix} \hat{A}, (\hat{B} + \hat{C}) \end{bmatrix} = \begin{bmatrix} \hat{A}, \hat{B} \end{bmatrix} + \begin{bmatrix} \hat{A}, \hat{C} \end{bmatrix}$$
$$\begin{bmatrix} \hat{A}, \hat{B}\hat{C} \end{bmatrix} = \begin{bmatrix} \hat{A}, \hat{B} \end{bmatrix}\hat{C} + \hat{B}\begin{bmatrix} \hat{A}, \hat{C} \end{bmatrix}$$

- (b) Show that $[\hat{x}, \hat{p}] = i\hbar$. Hint: operate on a general wave function and use the position representation of the operators.
- 6. Create a table for the HO similar to the two tables in hwk 1 in which you represent the important quantities for the HO in ket (abstract), wave function, graphical and matrix notation. Decide what the important operators and states are.

Not to be turned in, but here are problems you should be able to do:

- McIntyre 9.1, 9.2, 9.5 (raising operator on eigenstate)
- More practice with commutators

 $\begin{bmatrix} \hat{A}, \hat{A}^n \end{bmatrix} = 0 \qquad \begin{bmatrix} \hat{A}, \hat{B} \end{bmatrix} = -\begin{bmatrix} \hat{B}, \hat{A} \end{bmatrix}$ $\begin{bmatrix} \hat{A}, c\hat{B} \end{bmatrix} = c\begin{bmatrix} \hat{A}, \hat{B} \end{bmatrix} \qquad \begin{bmatrix} \hat{A}, \begin{bmatrix} \hat{B}, \hat{C} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \hat{B}, \begin{bmatrix} \hat{C}, \hat{A} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \hat{C}, \begin{bmatrix} \hat{A}, \hat{B} \end{bmatrix} \end{bmatrix} = 0$

Challenge problem: the 2-D Quantum HO This will become important soon, so it's good to start thinking about it, especially if the above seem routine.

The Hamiltonian for a 2-dimensional isotropic harmonic oscillator is

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 (x^2 + y^2).$$

- (a) Find the energy eigenvalues for the system.
- (b) What is the degeneracy?
- (c) What is the ground state eigenfunction?