

(As of 1/18/18; not final yet)

Equations and integrals that may be useful:

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\int_0^\infty e^{-a^2 x^2} dx = \frac{1}{2a} \sqrt{\pi}$$

$$\int_0^\infty x e^{-x^2} dx = \frac{1}{2}$$

$$\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int \sin mx \sin nx dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)}, \quad (m^2 \neq n^2)$$

$$\int \cos mx \cos nx dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)}, \quad (m^2 \neq n^2)$$

$$\int \sin mx \cos nx dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)}, \quad (m^2 \neq n^2)$$

$$\langle \psi | \hat{A}^\dagger | \phi \rangle = \langle \phi | \hat{A} | \psi \rangle^* \quad \sum_n |n\rangle \langle n| = 1 \quad \Delta Q = \sqrt{\langle (Q - \langle Q \rangle)^2 \rangle}$$

$$E_n = n^2 \frac{\hbar^2 \pi^2}{2mL^2}$$

$$E_n = -\frac{1}{n^2} \frac{1}{2} \alpha^2 mc^2$$

$$E_n = (n + \frac{1}{2}) \hbar \omega$$

$$\varphi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

$$V(x) = \frac{1}{2} m\omega^2 x^2$$

$$a = \sqrt{\frac{m\omega}{2\hbar}} \left(x + i \frac{p}{m\omega} \right)$$

$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left(x - i \frac{p}{m\omega} \right)$$

$$a|n\rangle = \sqrt{n}|n-1\rangle$$

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\varphi_0(x) = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} e^{-\frac{m\omega}{2\hbar} x^2}$$

$$\varphi_1(x) = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \sqrt{\frac{m\omega}{\hbar}} \frac{1}{\sqrt{2}} 2xe^{-\frac{m\omega}{2\hbar} x^2}$$

$$\varphi_n(x) = \frac{1}{2^{n/2}} \frac{1}{\sqrt{n!}} \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} H_n \left(\sqrt{\frac{m\omega}{\hbar}} x \right) e^{-\frac{m\omega}{2\hbar} x^2}$$

$$E_n^{(1)} = \langle n^{(0)} | H' | n^{(0)} \rangle$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|\langle n^{(0)} | H' | m^{(0)} \rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$|n^{(1)}\rangle = \sum_{m \neq n} c_{nm}^{(1)} |m^{(0)}\rangle \quad c_{nm}^{(1)} = \frac{\langle m^{(0)} | H' | n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}}$$

$$S_x \doteq \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad S_y \doteq \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$S_z \doteq \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbf{S}^2 \doteq \frac{3\hbar^2}{4} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S_x \doteq \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad S_y \doteq \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$S_z \doteq \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad S^2 \doteq 2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$