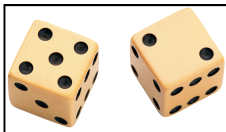


# PH424 Summary



## Non dispersive wave equation

$$\frac{\partial^2}{\partial x^2}\psi(x,t) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\psi(x,t)$$

Basic definitions, *k*, wavelength, frequency, angular freq, *period, etc*
Direction of travel
Traveling, standing or mixed?
Phase
Initial conditions, boundary conditions

Separation of variables

$$\psi(x,t) = \sum_{\omega} \begin{matrix} A_\omega \cos\frac{\omega}{v}x\cos\omega t + B_\omega \cos\frac{\omega}{v}x\sin\omega t \\ +C_\omega \sin\frac{\omega}{v}x\cos\omega t + D_\omega \sin\frac{\omega}{v}x\sin\omega t \end{matrix}$$

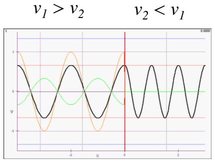
Standing waves, traveling waves, or mixtures, depending on coefficients

$$\psi_{\text{standing}}(x,t) = Af(kx)g(\omega t)$$
$$\psi_{\text{traveling}}(x,t) = Af(kx \pm \omega t)$$

Phase velocity $v_{phase} = \omega/k$ group velocity $v_{group} = d\omega/dk$
For this equation, they are the same and equal to *v*.

## Reflection and transmission

$v_1 > v_2$ $v_2 < v_1$



$$Z_{rope} = \frac{\tau}{v} = \sqrt{\tau\mu}$$
$$Z_{cable} = \frac{V}{I} = \sqrt{\frac{L_0}{C_0}}$$

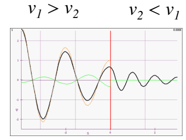
Boundary conditions

$$\psi_{Left}(x,t) = e^{i(-\omega t + k_1 x)} + \frac{Z_1 - Z_2}{Z_1 + Z_2} e^{i(-\omega t - k_1 x)}$$
$$\psi_{Right}(x,t) = \frac{2Z_1}{Z_1 + Z_2} e^{i(-\omega t + k_2 x)}$$
$$F_{Left}(x,t) = e^{i(-\omega t + k_1 x)} + \frac{Z_2 - Z_1}{Z_1 + Z_2} e^{i(-\omega t - k_1 x)}$$
$$F_{Right}(x,t) = \frac{2Z_2}{Z_1 + Z_2} e^{i(-\omega t + k_2 x)}$$

## Damping

$v_1 > v_2$ $v_2 < v_1$

$$\frac{\partial^2\psi(x,t)}{\partial x^2} - \frac{\Gamma}{v^2}\frac{\partial\psi(x,t)}{\partial t} = \frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial t^2}$$



$$Z_{rope} = \frac{\tau}{v} = \sqrt{\tau\mu}\left(1 + i\frac{\Gamma}{2v}\right)$$
$$Z_{cable} = \sqrt{\frac{L_0}{C_0}}\left(1 + i\frac{\Gamma}{2v}\right)$$

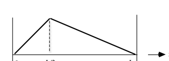
$$\psi_{Left}(x,t) = e^{-\frac{\Gamma}{2v_1}} e^{i(-\omega t + k_1 x)} + \frac{Z_1 - Z_2}{Z_1 + Z_2} e^{+\frac{\Gamma}{2v_1}} e^{i(-\omega t - k_1 x)}$$
$$\psi_{Right}(x,t) = \frac{2Z_1}{Z_1 + Z_2} e^{-\frac{\Gamma}{2v_2}} e^{i(-\omega t + k_2 x)}$$
$$F_{Left}(x,t) = e^{-\frac{\Gamma}{2v_1}} e^{i(-\omega t + k_1 x)} + \frac{Z_2 - Z_1}{Z_1 + Z_2} e^{+\frac{\Gamma}{2v_1}} e^{i(-\omega t - k_1 x)}$$
$$F_{Right}(x,t) = e^{-\frac{\Gamma}{2v_2}} \frac{2Z_2}{Z_1 + Z_2} e^{i(-\omega t + k_2 x)}$$

## Energy, energy density

$$\underbrace{W(x,t)}_{\text{Total energy density}} = \underbrace{\frac{1}{2}\mu\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2}_{\text{KE density}} + \underbrace{\frac{1}{2}\tau\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2}_{\text{PE density}}$$

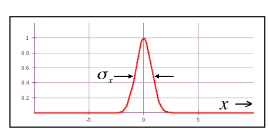
Energy in standing and traveling waves in ropes and coax cables as a function of position and time

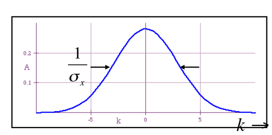
## Superposition




Quantitative analysis of standing wave superpositions (Fourier series)

Concept of Fourier Transform



## Schrödinger (dispersive) wave equation

$$\hat{H}\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

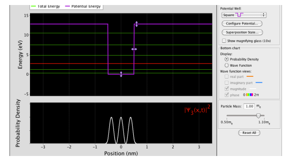
$$\psi(x,t) = \sum_n c_n\varphi_n(x)e^{-iE_nt/\hbar}$$

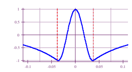
$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}(x) \qquad \hat{H}\varphi_n(x) = E_n\varphi_n(x)$$

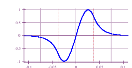
$$\hat{p} \doteq -i\hbar\frac{d}{dx}$$

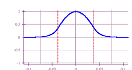
$$\hat{H} \doteq -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

## Finite well eigenstates









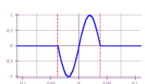
Apply boundary conditions
Understand reasons for these
Discuss types of solutions
Symmetry
Time dependence
Orthogonality etc

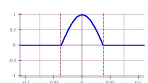
## Infinite well eigenstates

$$\varphi_n(x) = \sqrt{\frac{2}{2a}}\sin\frac{n\pi x}{2a} = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2m(2a)^2} = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$\text{Prob.density}(x) = \varphi_n^*(x)\varphi_n(x) = \frac{2}{L}\sin^2\frac{n\pi x}{L}$$





Know everything about these solutions and work with them in gory detail
Symmetry
Time dependence
Orthogonality etc

## Abstract bra-ket & position state (wavefunction) representations

| | | |
|---|---|---|
| State superposition | $\lvert\Phi\rangle = \sum_n c_n\lvert\varphi_n\rangle$ | $\Phi(x) = \sum_n c_n\varphi_n(x)$ |
| Eigenstates & eigenvalues | $\hat{Q}\lvert\varphi_n\rangle = q_n\lvert\varphi_n\rangle$ | $\hat{Q}\varphi_n(x) = q_n\varphi_n(x)$ |
| Eigenstate orthogonality | $\langle\varphi_n\lvert\varphi_m\rangle = \delta_{n,m}$ | $\int_{-\infty}^{\infty}\varphi_n^*(x)\varphi_m(x)dx = \delta_{n,m}$ |
| Normalization | $\langle\Phi\lvert\Phi\rangle = 1$ → $\sum_n \lvert c_n\rvert^2 = 1$ | $\int_{-\infty}^{\infty}\underbrace{\Phi^*(x)\Phi(x)}_{\text{Probability density}}dx = 1$ (Probability) |
| Expectation value | $\langle Q\rangle = \langle\Phi\lvert\hat{Q}\lvert\Phi\rangle$ → $\langle Q\rangle = \sum_n \lvert c_n\rvert^2 q_n$ | $\langle Q\rangle = \int_{-\infty}^{\infty}\Phi^*(x)\hat{Q}\Phi(x)dx$ |

10

## Odds and ends

$$\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2} \qquad \Delta p\Delta x \geq \frac{\hbar}{2}$$

- Lots of qualitative things – is this probability larger or smaller than this one? (Integrals are often hard to do)
- Which energy is more likely to be measured in this superposition state?
- Why does more certainty in location mean less certainty in position?