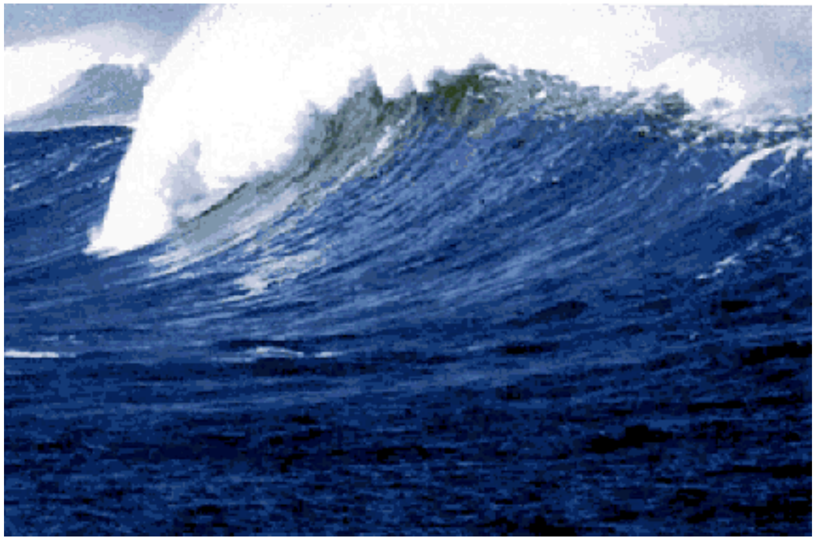
ENERGY

Reading:
Main 9.4



www.jamstec.go.jp/ jamstec/MTD/Whale/

ENERGY

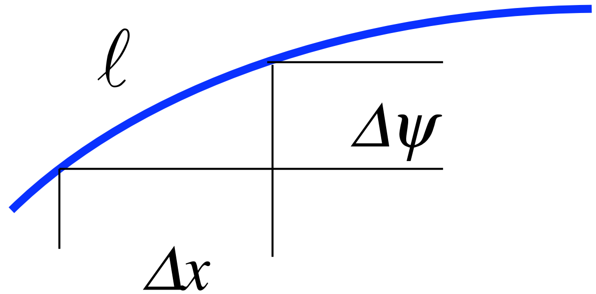
$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi(x,t)}{\partial t^2}$$ No attenuation

$$\psi(x,t) = Ae^{i(-\omega t + kx)} + Be^{i(-\omega t - kx)}$$

What about energy and energy transfer?

Define energy density (energy per unit length) as sum of kinetic and potential energy density. These are stored in the motion and stretchiness of the rope and are different at different points on the rope.

$$\underbrace{W(x,t)}_{\text{Total energy density}} = \underbrace{\frac{1}{2}\mu\left(\frac{\partial \psi(x,t)}{\partial t}\right)^2}_{\text{KE density}} + \underbrace{\frac{1}{2}\overset{\boxed{\textit{tension}}}{\tau}\left(\frac{\partial \psi(x,t)}{\partial x}\right)^2}_{\text{PE density}}$$



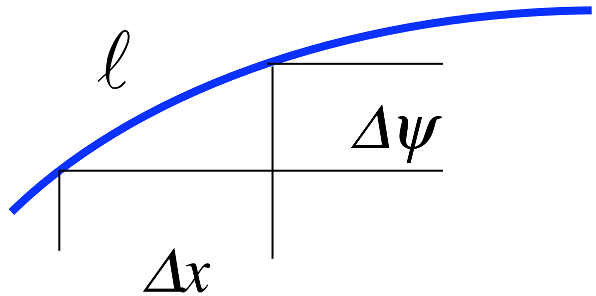
POTENTIAL ENERGY
Force × extension

$$\ell^2 = \Delta x^2 + \Delta\psi^2 = \Delta x^2\left(1+\left[\frac{\Delta\psi}{\Delta x}\right]^2\right)$$

$$\ell = \Delta x\left(1+\left[\frac{\Delta\psi}{\Delta x}\right]^2\right)^{\frac{1}{2}} \approx \Delta x\left(1+\frac{1}{2}\left[\frac{\Delta\psi}{\Delta x}\right]^2\right)$$

$$\ell - \Delta x \approx \frac{1}{2}\left[\frac{\Delta\psi}{\Delta x}\right]^2 \Delta x$$

Extension



$$PE = \tau\left(\ell - \Delta x\right) = \tau \Delta x \frac{1}{2}\left(\frac{\partial \psi}{\partial x}\right)^2$$

$$KE = \frac{1}{2}\mu \Delta x\left(\frac{\partial \psi}{\partial t}\right)^2$$

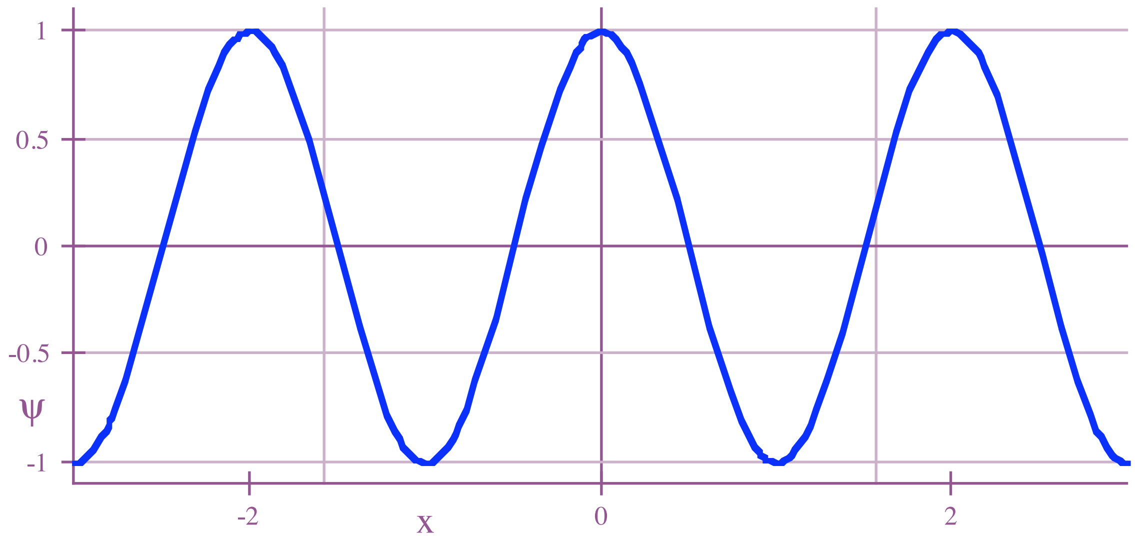
$$\boxed{W(x,t) = \frac{1}{2}\mu\left(\frac{\partial \psi(x,t)}{\partial t}\right)^2 + \frac{1}{2}\tau\left(\frac{\partial \psi(x,t)}{\partial x}\right)^2}$$

rope

$$\boxed{W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial \psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial \psi(x,t)}{\partial x}\right)^2\right]}$$

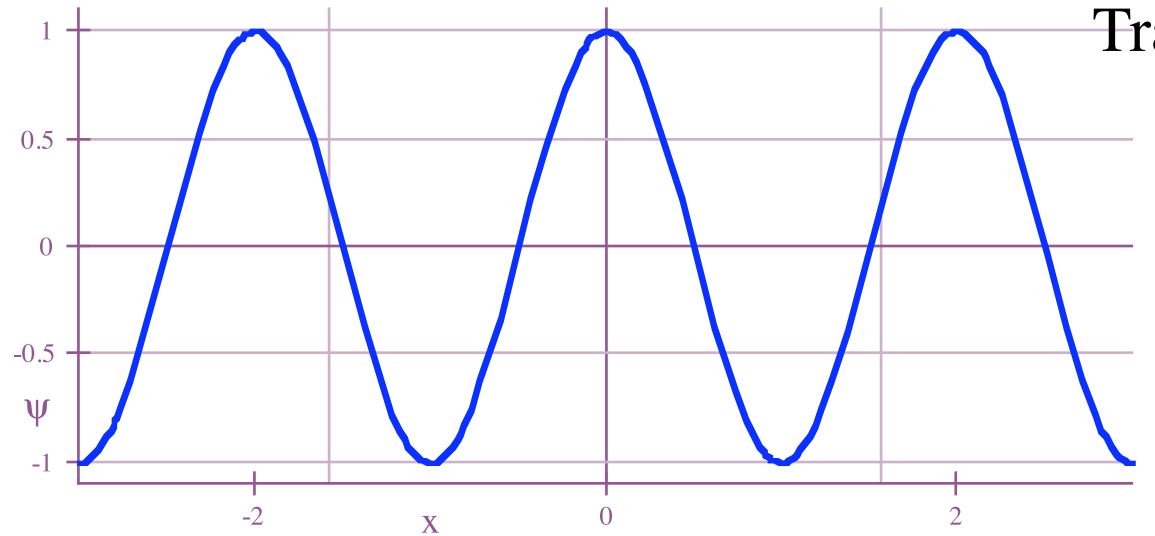
generic

$$\boxed{W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$



Group exercises (8): This is a traveling/standing wave. Find KE, PE at $t = 0T, 0.25T, 0.5T, 0.75T$.
Traveling wave is moving right; standing wave is momentarily at rest.

$$\boxed{W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$



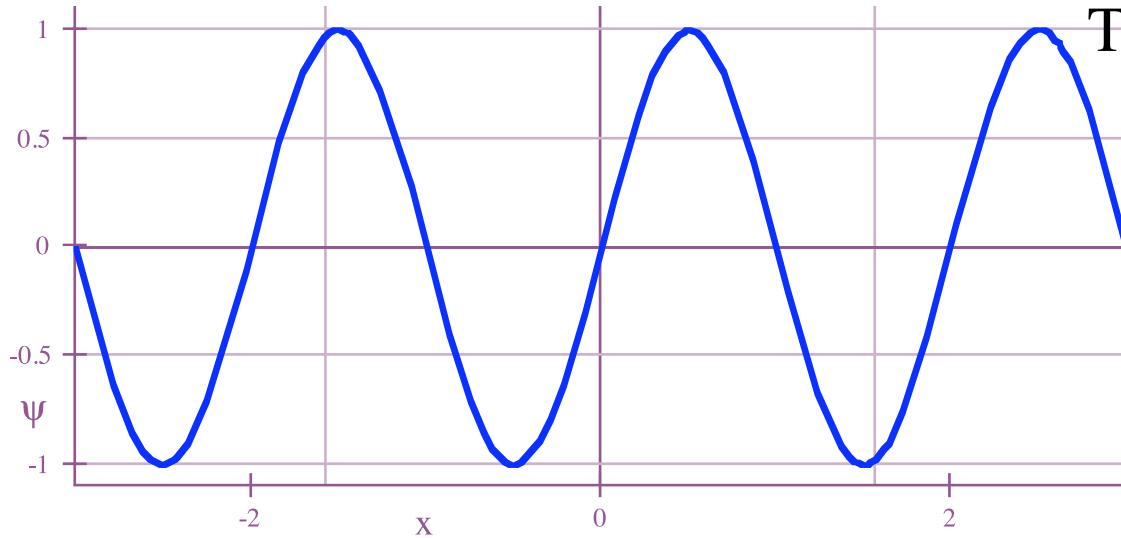
Traveling wave

t = 0 T

KE max

PE max

$$\boxed{W(x,t)=\frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2+v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$



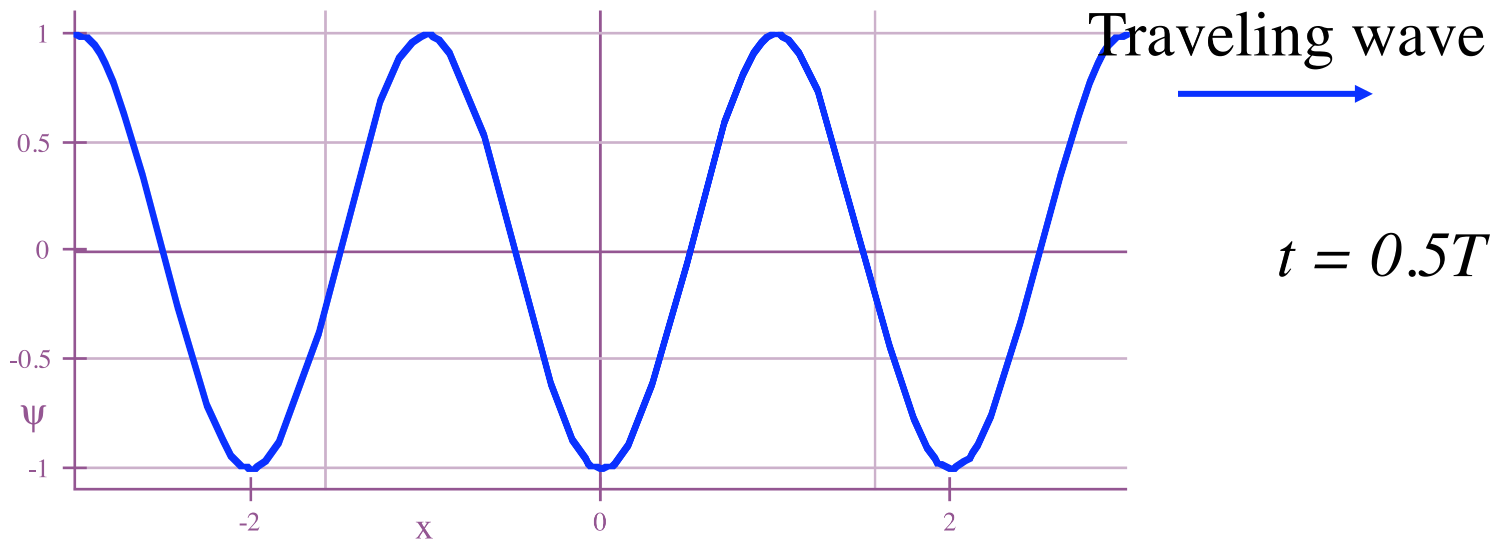
Traveling wave

t = 0.25T

KE max

PE max

$$W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]$$

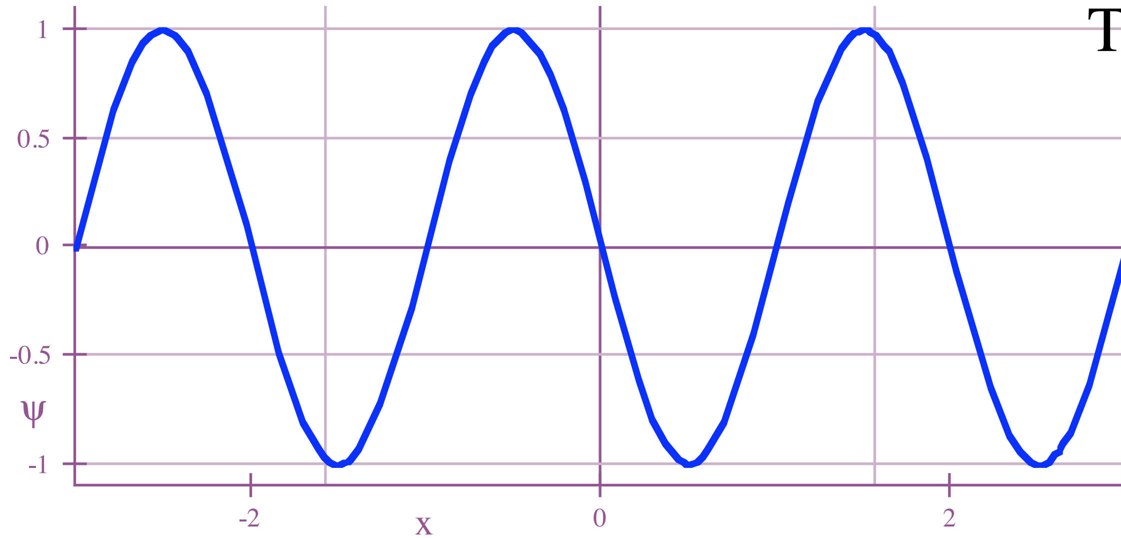


$t = 0.5T$

KE max

PE max

$$W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]$$



Traveling wave

t = 0.75T

KE max

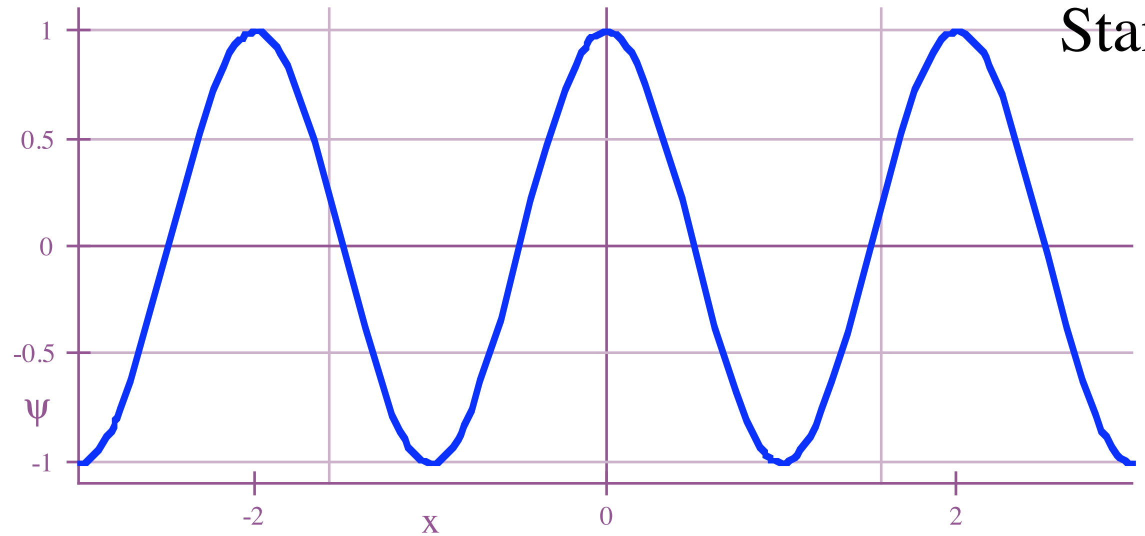
PE max

$$\boxed{W(x,t)=\frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2+v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$

$$\frac{\partial\psi(x\pm vt)}{\partial t}=\pm v\frac{\partial\psi(x\pm vt)}{\partial x}\qquad\boxed{W(x,t)=Zv\left[\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$

Traveling wave: KE & PE density are equal to each other at any given place and time, and vary from place to place at a given time, and from time to time at a given place. Energy propagates in direction of phase velocity.

$$W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]$$



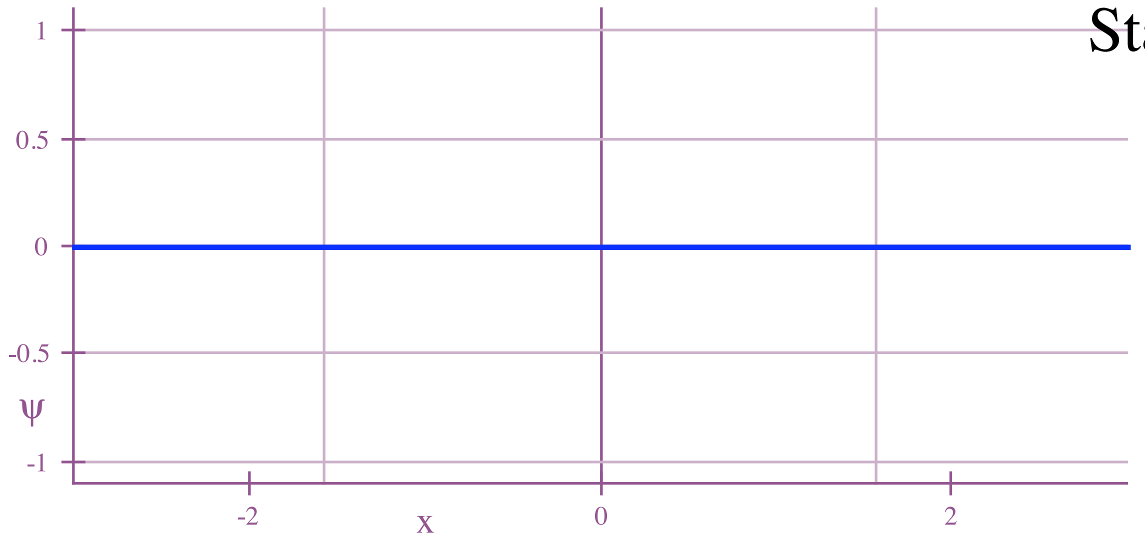
Standing wave

t = 0

KE	0	0	0	0	0	0	0	0	0	0
PE	0	max	0	max	0	max	0	max	0	max

$$W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]$$

Standing wave

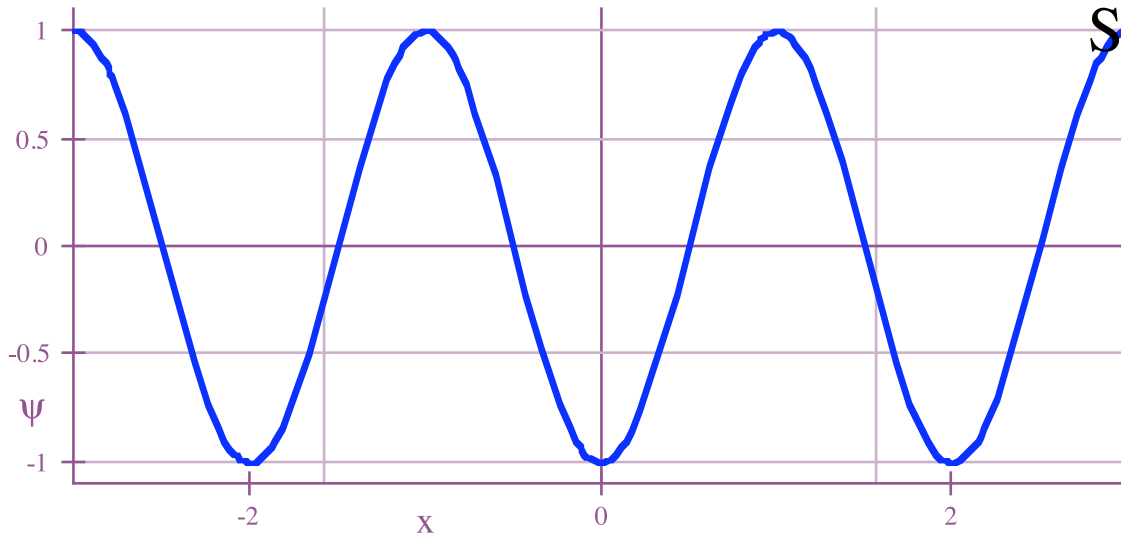


$t = 0.25T$

KE max 0 max 0 max 0 max 0 max 0

PE 0 0 0 0 0 0 0 0 0 0

$$\boxed{W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$



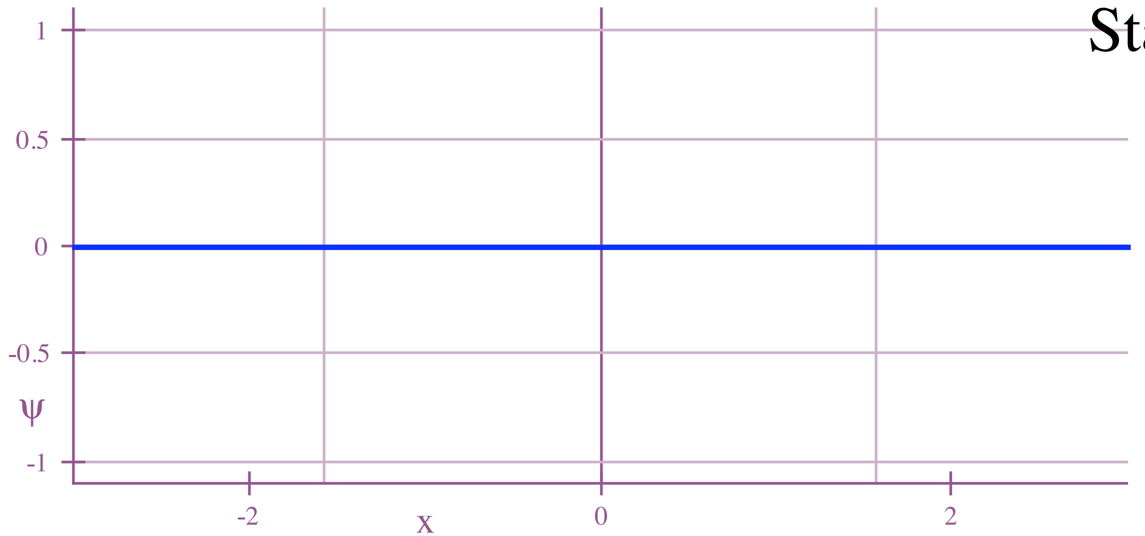
Standing wave

t = 0.5T

KE 0 0 0 0 0 0 0 0 0 0

PE 0 max 0 max 0 max 0 max 0 max

$$W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]$$



Standing wave

t = 0.75T

KE max 0 max 0 max 0 max 0 max 0

PE 0 0 0 0 0 0 0 0 0 0

$$\boxed{W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$

Standing wave : KE & PE density are different from each other at any given place and time, and vary from place to place at a given time, and from time to time at a given place. No energy propagation, but oscillation to and fro (see Maple sheet).

Further consideration:
Homework: Calculate average power transferred to traveling wave
Homework: (extra credit) consider reflection and transmission
Independent study: Incorporate damping

$$\boxed{W(x,t) = \frac{Z}{2v}\left[\left(\frac{\partial\psi(x,t)}{\partial t}\right)^2 + v^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2\right]}$$

ENERGY - REVIEW

- *How is energy stored in a rope? In other systems?*
- *Kinetic energy density, potential energy density*
- *Total energy density*
- *Energy transfer*
- *Mathematical representations of the above as space & time functions*
- *Not much focus on mathematical representation of damped systems, but interested students should study this independently*