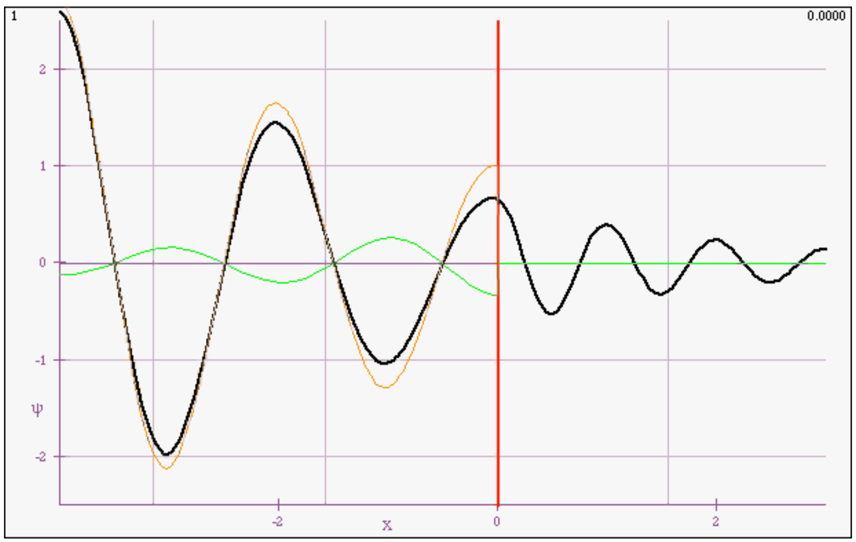
DAMPING & ATTENUATION

Reading:
Main 9.5
GEM 9.2.1
(TM 13.8)



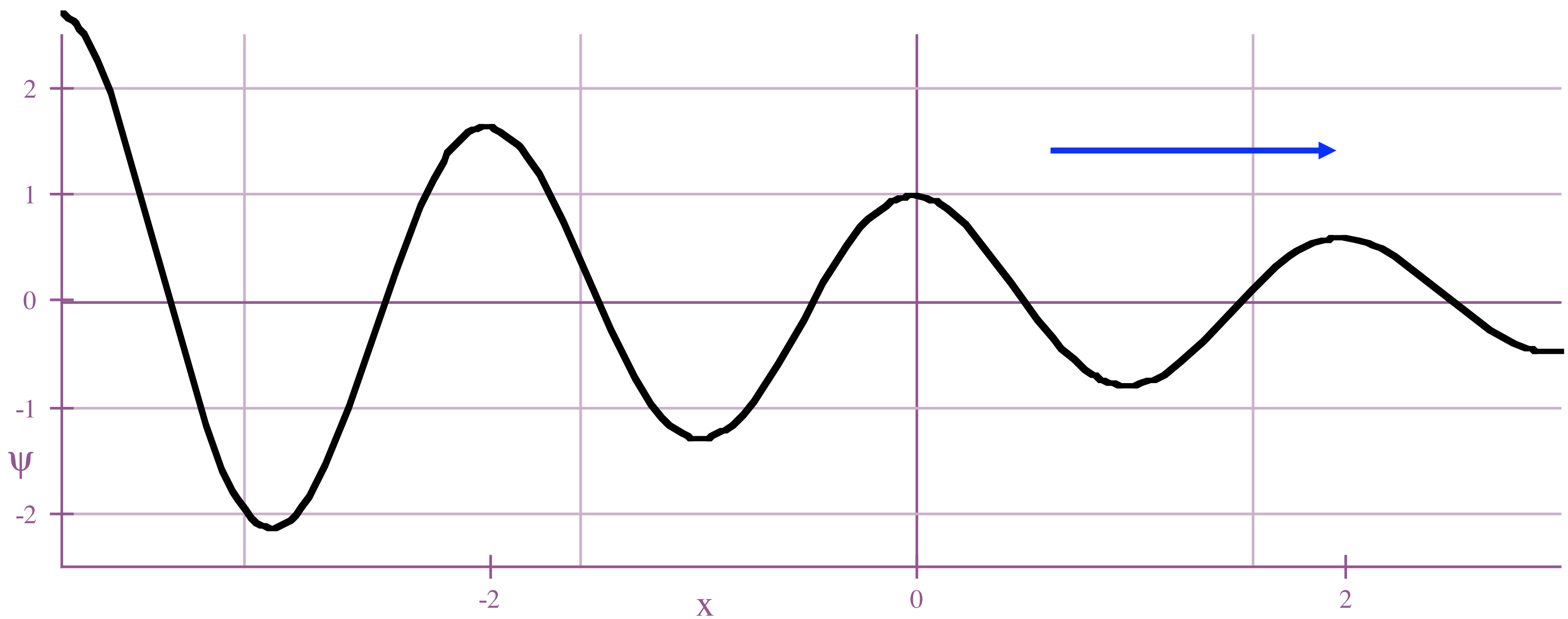
How do we describe, or model, energy dissipation and damping?

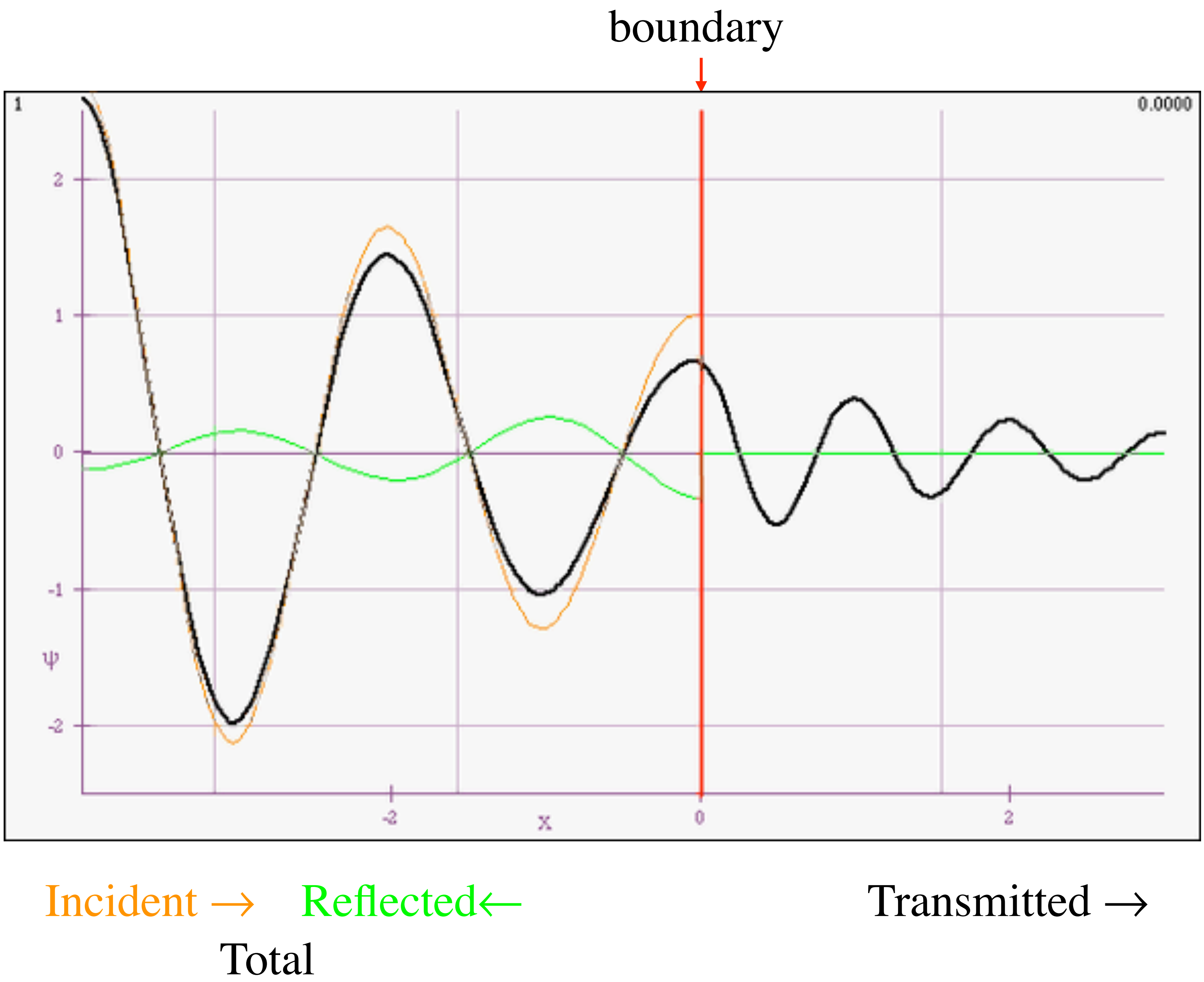
What would a traveling waveform look like if k were complex?

$$\psi(x,t) = Ae^{i(-\omega t + k_c x)}$$

$$k_c = k \pm i\kappa \quad (k,\ \kappa \text{ real, positive})$$

Exponentially damped or growing traveling waveform - former corresponds to energy dissipation or damping.




boundary
1
0.0000
2
1
0
-1
-2
ψ
-2
x
0
2
Incident →
Reflected←
Transmitted →
Total

Reflection and transmission coefficients exactly as before, but now that k values are complex, there is (i) a decrease in amplitude in the direction of travel, and (ii) a phase shift introduced into transmitted and reflected waves, both displacement and force.

$$\boxed{R_\psi = \frac{k_1 - k_2}{k_1 + k_2}} \quad \boxed{T_\psi = \frac{2k_1}{k_1 + k_2}} \quad \boxed{R_F = \frac{k_2 - k_1}{k_1 + k_2}} \quad \boxed{T_F = \frac{2k_2}{k_1 + k_2}}$$

$$\psi_{Left}(x,t) = e^{i(-\omega t + k_1 x)} + \frac{k_1 - k_2}{k_1 + k_2} e^{i(-\omega t - k_1 x)}$$

$$\psi_{Right}(x,t) = \frac{2k_1}{k_1 + k_2} e^{i(-\omega t + k_2 x)}$$

For *light* damping, the most important effect is the amplitude decay over long distances. The change in magnitude of R and T relative to no damping is quite small.

$$\frac{\partial^2\psi(x,t)}{\partial x^2} - \frac{\Gamma}{v^2}\frac{\partial\psi(x,t)}{\partial t} = \frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial t^2}$$

New, friction term proportional to material/field velocity in our model.
Γrepresents magnitude of frictional term - what are dimensions?
v is still velocity of propagation.

$$\psi(x,t) = Ae^{i(-\omega t+kx)}$$

will solve the equation but ONLY if k is complex! And complex k means attenuation, which is what happens if friction is present!

Let's see …….

$$\frac{\partial^2\psi(x,t)}{\partial x^2}-\frac{\Gamma}{v^2}\frac{\partial\psi(x,t)}{\partial t}=\frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial t^2}$$

$$\psi(x,t)=Ae^{i(-\omega t+kx)}$$

$$(ik)^2\,\psi(x,t)+i\omega\frac{\Gamma}{v^2}\psi(x,t)=\frac{(-i\omega)^2}{v^2}\psi(x,t)$$ substitute

$$\frac{\omega^2}{v^2}+i\omega\frac{\Gamma}{v^2}=k^2$$

$$k=[\mathrm{Re}(k)]+i[\mathrm{Im}(k)]$$

$$\omega^2 + i\omega\Gamma = v^2 k^2$$

$$\omega^2\left(1 + i\frac{\Gamma}{\omega}\right) = v^2 k^2$$

$$\omega\left(1 + i\frac{\Gamma}{\omega}\right)^{1/2} = v\left[\operatorname{Re}(k) + i\operatorname{Im}(k)\right]$$

$$\omega\left(1 + i\frac{\Gamma}{2\omega}\right) = v\left[\operatorname{Re}(k) + i\operatorname{Im}(k)\right]$$ Light damping - Γ/ω is **<<** 1

real and imaginary parts are **separately** zero

$$\operatorname{Re}(k) = \pm\frac{\omega}{v} \qquad \operatorname{Im}(k) = \pm\frac{\Gamma}{2v}$$

Choose signs so that wave propagates in desired direction and loses amplitude in the direction of travel.

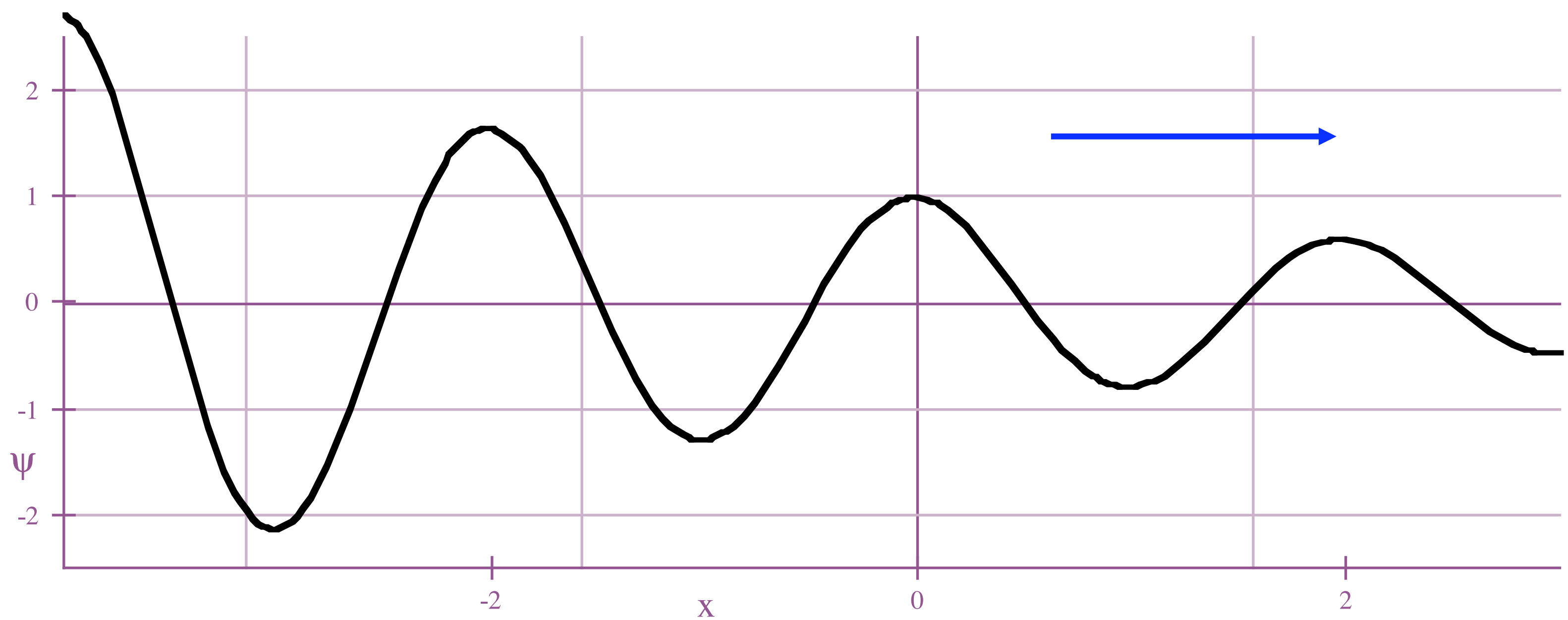
$$\psi(x,t) = Ae^{i(-\omega t + kx)}$$

$$\mathrm{Re}(k) = +\frac{\omega}{v} \qquad \mathrm{Im}(k) = +\frac{\Gamma}{2v}$$

$$\psi(x,t) = Ae^{i\left(-\omega t + \frac{\omega}{v}x + i\frac{\Gamma}{2v}x\right)}$$

$$\psi(x,t) = Ae^{-\frac{\Gamma}{2v}x}e^{i\left(-\omega t + \frac{\omega}{v}x\right)}$$

Important aspect is the loss of energy (represented by the amplitude) in the direction of travel



Impedance, Z

$$F_{appl} \equiv Z\frac{\partial\psi}{\partial t}$$

For the rope example, recall

The subscript "c" is to remind you that k is complex (and therefore so is Z)

$$Z = \frac{\tau k_c}{\omega} \Rightarrow Z = \frac{\tau}{\omega}\left(\frac{\omega}{v} + i\frac{\Gamma}{2v}\right)$$

$$Z = \frac{\tau}{v}\left(1 + i\frac{\Gamma}{2\omega}\right) = \sqrt{\tau\mu}\left(1 + i\frac{\Gamma}{2\omega}\right)$$

"τ" is tension here. It's the same in both ropes, as is ω. Do you remember why?

$$\psi_{Left}(x,t) = e^{i(-\omega t + k_1 x)} + \frac{Z_1 - Z_2}{Z_1 + Z_2} e^{i(-\omega t - k_1 x)}$$

$$\psi_{Right}(x,t) = \frac{2Z_1}{Z_1 + Z_2} e^{i(-\omega t + k_2 x)}$$

$$F_{Left}(x,t) = e^{i(-\omega t + k_1 x)} + \frac{Z_2 - Z_1}{Z_1 + Z_2} e^{i(-\omega t - k_1 x)}$$

$$F_{Right}(x,t) = \frac{2Z_2}{Z_1 + Z_2} e^{i(-\omega t + k_2 x)}$$

Your job is to work out how all this applies to the electrical co-ax cable system.

In the generic wave equation, how are the generic parameters v and G related to the properties of the cable?

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} - \frac{\Gamma}{v^2}\frac{\partial \psi(x,t)}{\partial t} = \frac{1}{v^2}\frac{\partial^2 \psi(x,t)}{\partial t^2}$$

How do we express the complex k and complex Z in terms of the properties of the cable?

How much resistance would you cable have to have to account for the attenuation you measured? What would the impedance have to be to account for the velocity you measured (and the condition of zero reflectance)? Do you have any other information to corroborate your findings?

A closer look at the effect of damping on the transmission coefficient:

$$\psi_{Right}(x,t) = \frac{2Z_1}{Z_1 + Z_2} e^{i(-\omega t + k_2 x)}$$

$$\frac{2Z_1}{Z_1 + Z_2} = \frac{2\sqrt{\mu_1}\left(1 + i\frac{\Gamma_1}{2\omega}\right)}{\sqrt{\mu_1}\left(1 + i\frac{\Gamma_1}{2\omega}\right) + \sqrt{\mu_2}\left(1 + i\frac{\Gamma_2}{2\omega}\right)} =$$

$$\frac{2\left(\mu_1^{1/2} + i\frac{\mu_1^{1/2}\Gamma_1}{2\omega}\right)}{\left(\mu_1^{1/2} + \mu_2^{1/2}\right) + i\left(\frac{\mu_1^{1/2}\Gamma_1}{2\omega} + \frac{\mu_2^{1/2}\Gamma_2}{2\omega}\right)} = \frac{2\left|Z_1\right| e^{i\phi_1}}{\left|Z_1 + Z_2\right| e^{i\phi_{12}}}$$

A closer look at the effect of damping on the transmission coefficient:

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$$\frac{2Z_1}{Z_1 + Z_2} = \frac{2\sqrt{\mu_1}\left(1 + i\frac{\Gamma_1}{2\omega}\right)}{\sqrt{\mu_1}\left(1 + i\frac{\Gamma_1}{2\omega}\right) + \sqrt{\mu_2}\left(1 + i\frac{\Gamma_2}{2\omega}\right)} =$$

$$\frac{2\left(\mu_1^{1/2} + i\frac{\mu_1^{1/2}\Gamma_1}{2\omega}\right)}{\left(\mu_1^{1/2} + \mu_2^{1/2}\right) + i\left(\frac{\mu_1^{1/2}\Gamma_1}{2\omega} + \frac{\mu_2^{1/2}\Gamma_2}{2\omega}\right)} = \frac{2\left|Z_1\right| e^{i\phi_1}}{\left|Z_1 + Z_2\right| e^{i\phi_{12}}}$$

DAMPING & ATTENUATION - REVIEW

- *Damping removes energy from the wave and is evident over large distances*
- *Amplitude decay in direction of travel is represented by a complex k vector*
- *Reflection and transmission coefficients for ψ and $d\psi/dx$ are worked out in a similar fashion to the case for no damping. The magnitudes are not affected very much for light damping, but there can be a phase shift that is different from the undamped case*
- *Light damping is our focus*
- *Mathematical representations of the above*