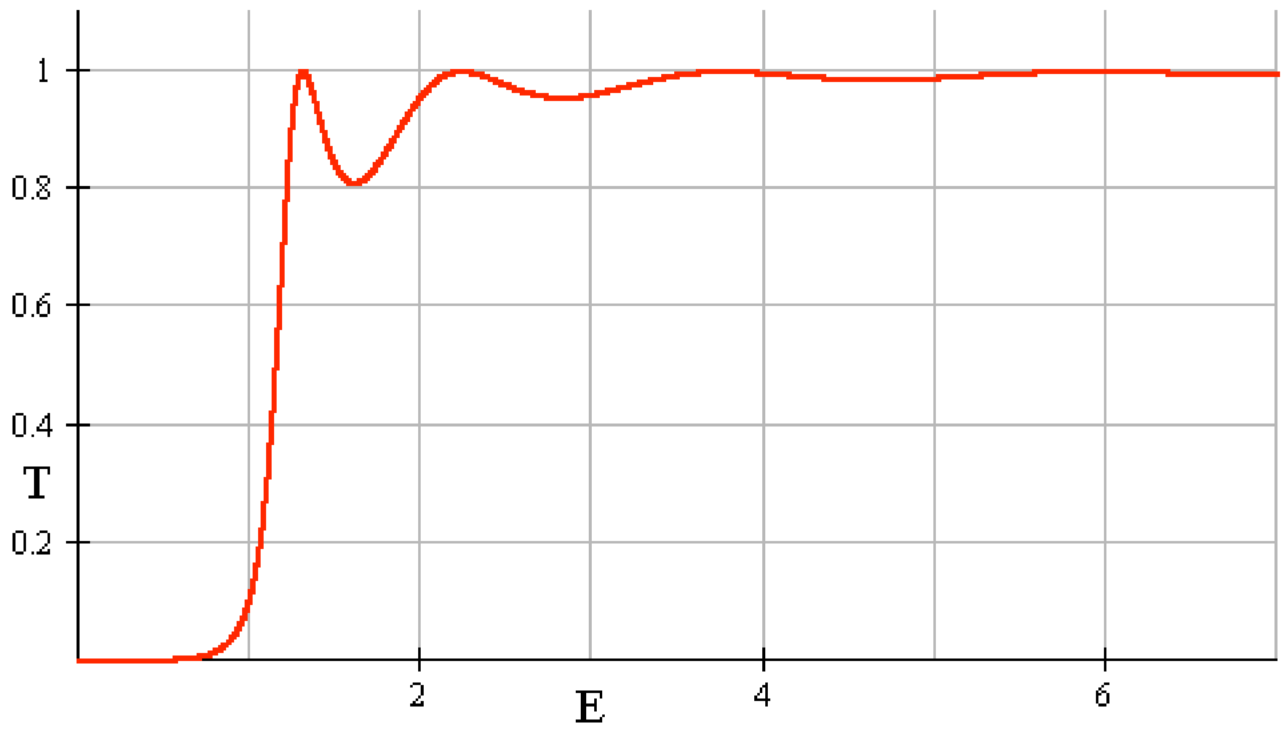
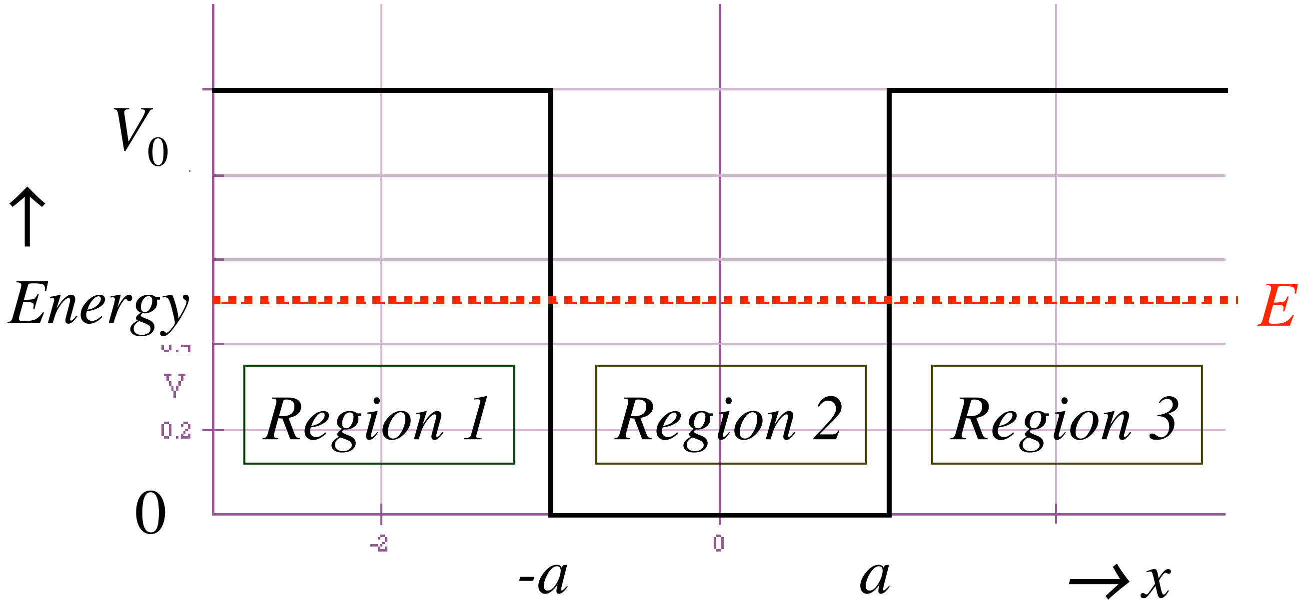
BARRIERS & TUNNELING

Reading:
QM Course Packet- ch 6.4, 6.5



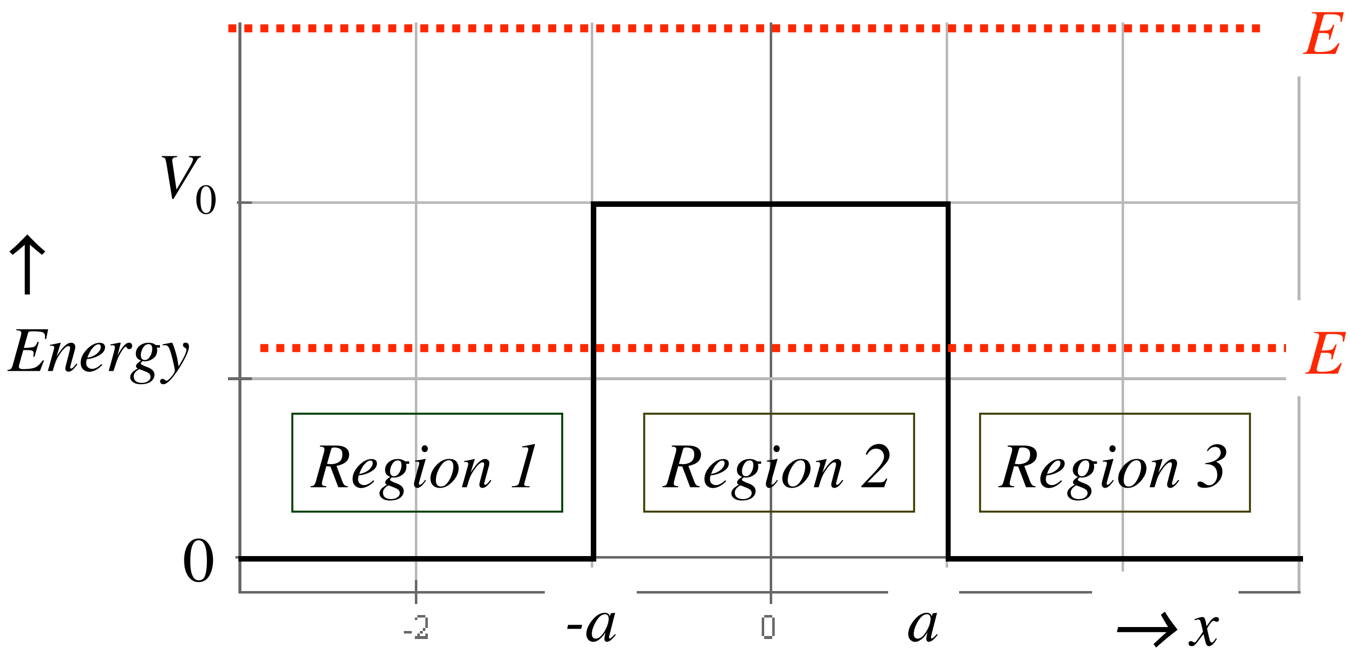
Remember we solved the eigenvalue equation to find the wave functions and energies for this finite well potential? We looked only at bound states, $E < V_0$.



$$V(x) = \begin{cases} V_0 & |x| > a \\ 0 & |x| < a \end{cases}$$

$$\varphi(x) = \begin{cases} \varphi_1 & x < -a \\ \varphi_2 & -a < x < a \\ \varphi_3 & x > a \end{cases}$$

Now let's invert the well, and consider all energies.



$$V(x) = \begin{cases} V_0 & |x| < a \\ 0 & |x| > a \end{cases}$$

$$\varphi(x) = \begin{cases} \varphi_1 & x < -a \\ \varphi_2 & -a < x < a \\ \varphi_3 & x > a \end{cases}$$

$$\hat{H}\varphi(x) = E\varphi(x)$$

$$\frac{d^2\varphi}{dx^2} = -\frac{2m}{\hbar^2}(E - V)\varphi$$

defines k^2

$$k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$$

V_0 if region 2

0 if region 1,3

$$\frac{d^2\varphi}{dx^2} = -k^2\varphi$$

$$k_1 = k_3 = \sqrt{\frac{2m}{\hbar^2}E}$$

$$k_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_0)}$$

real if $E > V_0$;
imag if $E < V_0$

$$\varphi(x) = Ce^{-ikx} + C'e^{+ikx}$$

$$\varphi_1(x) = Ae^{+ik_1x} + Be^{-ik_1x}$$

Incident *Reflected*

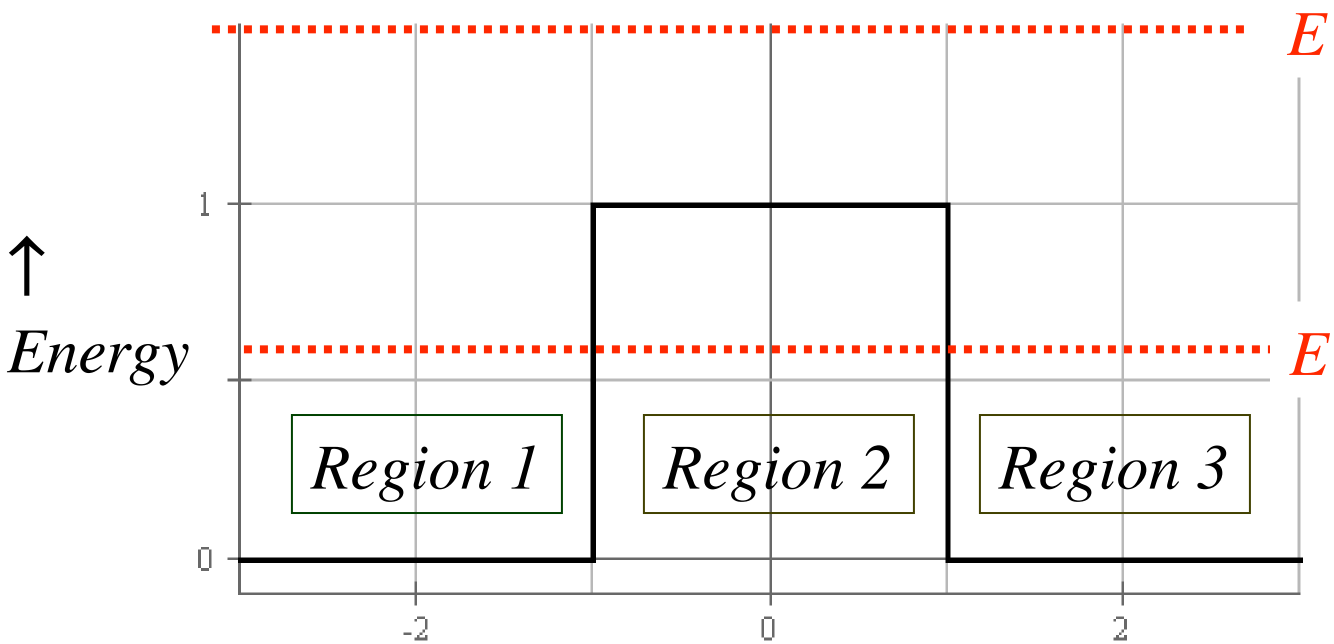
$$\varphi_3(x) = 0e^{-ik_3x} + Fe^{+ik_3x}$$

transmitted

$$\varphi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$$

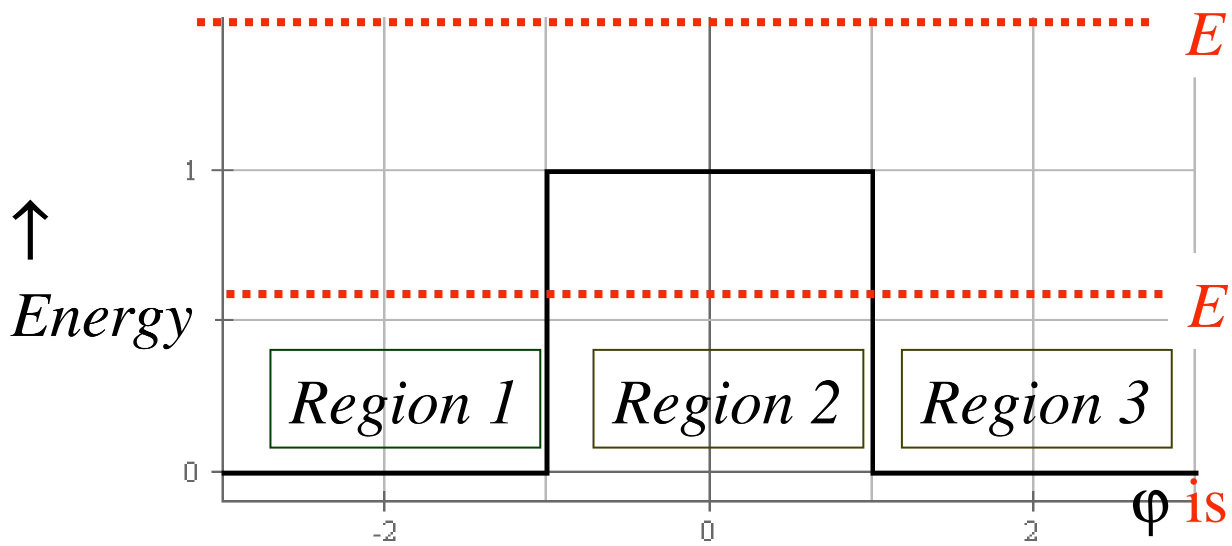
Imaginary k means exponential growth or exponential decay! (classically forbidden region) But now we allow both terms …. Why?

Real k means oscillatory behavior (classically allowed region)



φ is continuous everywhere
φ has a continuous derivative

$$\varphi(x) = \begin{cases} \varphi_1 & x < -a \\ \varphi_2 & -a < x < a \\ \varphi_3 & x > a \end{cases}$$



φ is continuous everywhere

φ has a continuous derivative

$$\varphi_1(-a) = \varphi_2(-a) \qquad Ae^{-ik_1 a} + Be^{ik_1 a} = Ce^{-ik_2 a} + De^{ik_2 a}$$

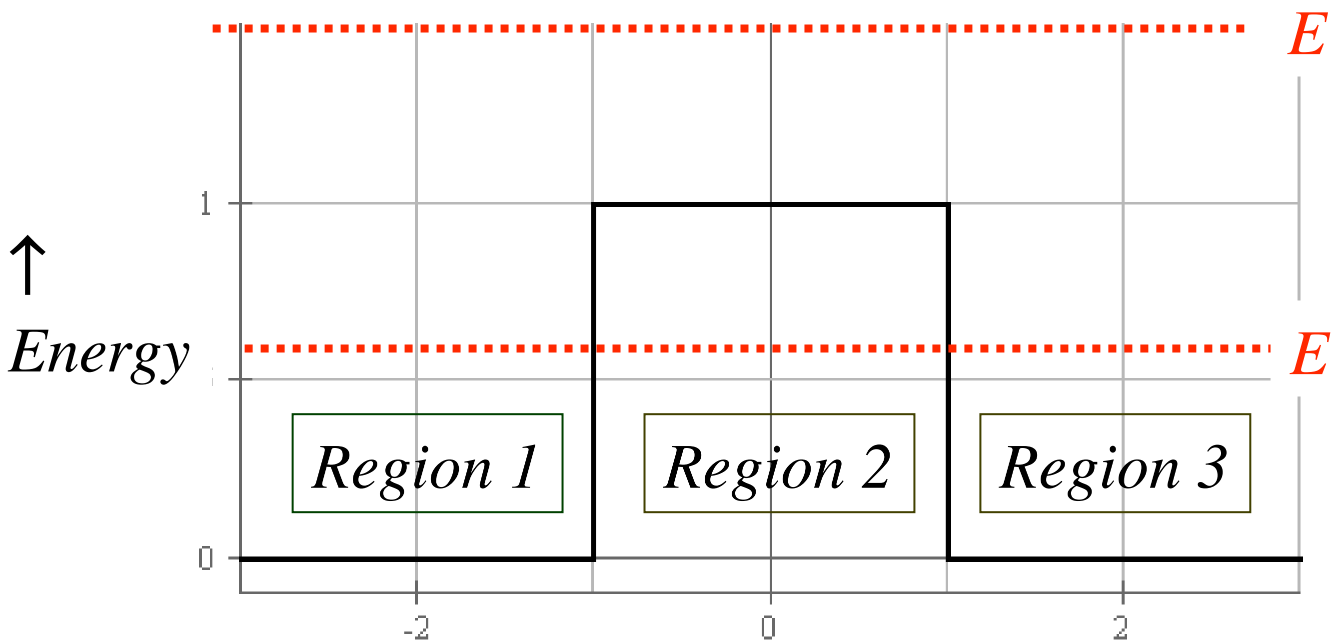
$$\varphi_3(a) = \varphi_2(a) \qquad Ce^{ik_2 a} + De^{-ik_2 a} = Fe^{ik_3 a}$$

$$\varphi_1'(-a) = \varphi_2'(-a) \qquad ik_1 Ae^{-ik_1 a} - ik_1 Be^{ik_1 a} = ik_2 Ce^{-ik_2 a} - ik_2 De^{ik_2 a}$$

$$\varphi_3'(a) = \varphi_2'(a) \qquad ik_2 Ce^{ik_2 a} - ik_2 De^{-ik_2 a} = ik_1 Fe^{ik_1 a}$$

4 equations, 5 unknowns (A, B, C, D, F). **No condition on E this time, because no boundary condition at infinity**.

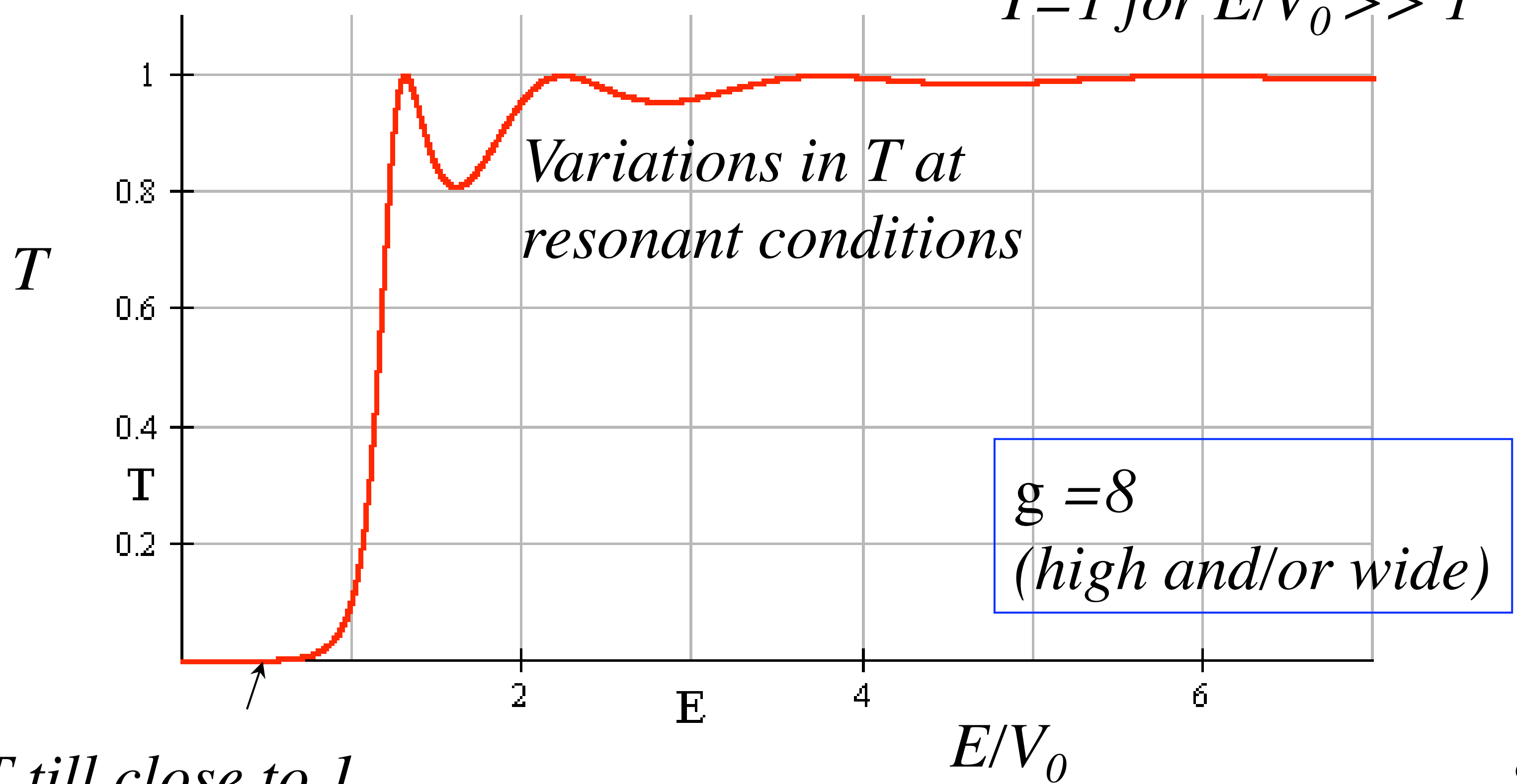
We can't solve for all 5 unknowns, but we can solve for B/A, C/A, D/A, F/A, and we are most interested in $|F/A|^2$, the transmitted flux.



$$T = \begin{cases} \dfrac{1}{1 + \dfrac{1}{4E(1-E)}\left(\sinh\left[K\dfrac{g}{\sqrt{2}}\right]\right)^2} & (E < 1) \\[2ex] \dfrac{1}{1 + \dfrac{1}{4E(E-1)}\left(\sin\left[k\dfrac{g}{\sqrt{2}}\right]\right)^2} & (E \geq 1) \end{cases}$$

$$g = \sqrt{\frac{4\,mV_0(2a)^2}{hbar^2}}$$

$T=1$ for $E/V_0 >> 1$



low T till close to 1

$$T = \begin{cases} \dfrac{1}{1+\dfrac{1}{4E(1-E)}\left(\sinh\left[K\dfrac{g}{\sqrt{2}}\right]\right)^2} & (E < 1) \\ \dfrac{1}{1+\dfrac{1}{4E(E-1)}\left(\sin\left[k\dfrac{g}{\sqrt{2}}\right]\right)^2} & (E \geq 1) \end{cases}$$

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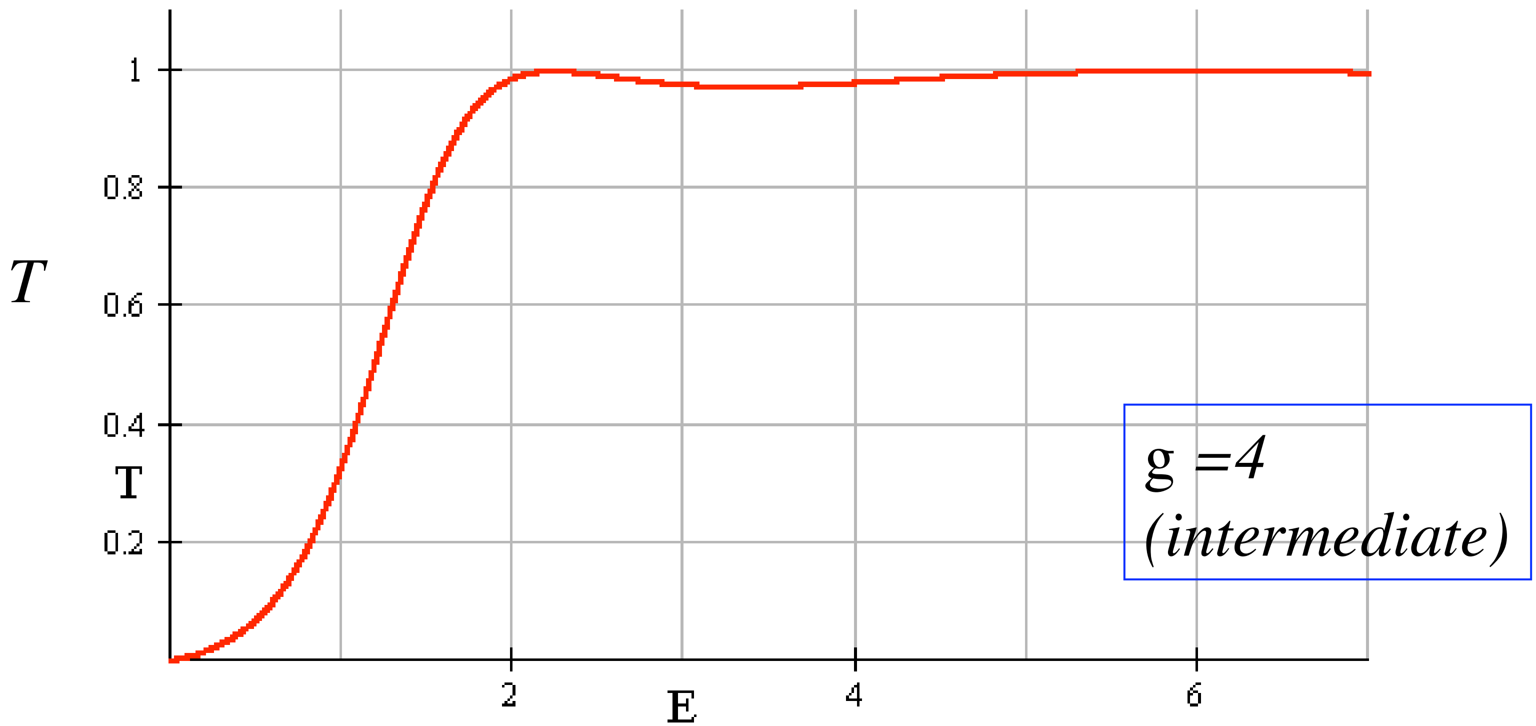
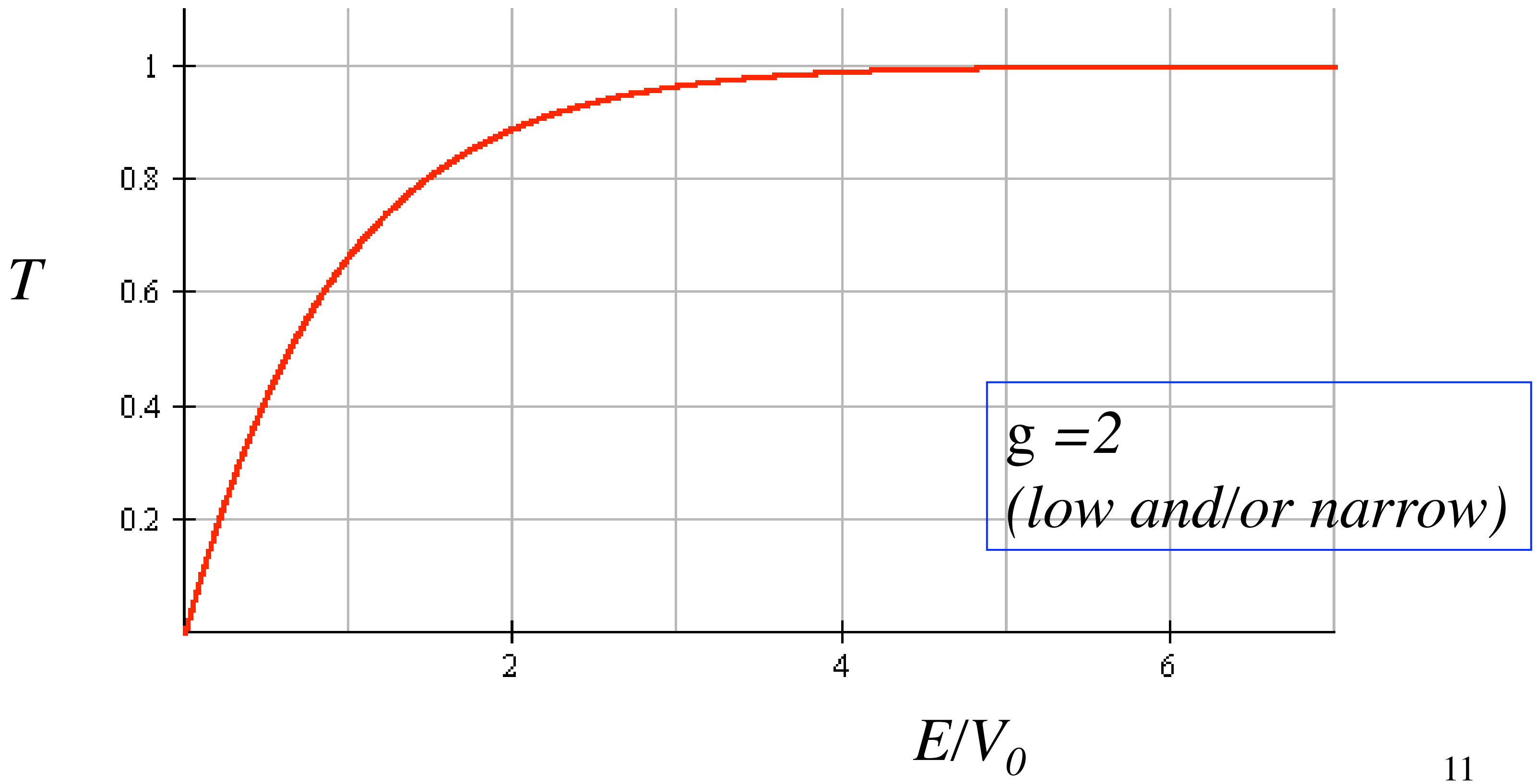


E/V_0

T significant even for $E/V_0 < 1$

$$T = \begin{cases} \dfrac{1}{1 + \dfrac{1}{4E(1-E)}\left(\sinh\left[K\dfrac{g}{\sqrt{2}}\right]\right)^2} & (E < 1) \\ \dfrac{1}{1 + \dfrac{1}{4E(E-1)}\left(\sin\left[k\dfrac{g}{\sqrt{2}}\right]\right)^2} & (E \geq 1) \end{cases}$$

$$g = \sqrt{\frac{4\,mV_0(2a)^2}{hbar^2}}$$



BARRIERS & TUNNELING - REVIEW

- *Energy eigenvalue (time independent Schrödinger) equation for barrier*
- *No boundary at infinity => no condition on E*
- *Eigenstates*
- *Reflection at barrier even for* $E>V_0$
- *Tunneling through/ reflection at barrier* $E < V_0$
- *Mathemathical representations of the above*