PH424/524: 1-DIMENSIONAL WAVES

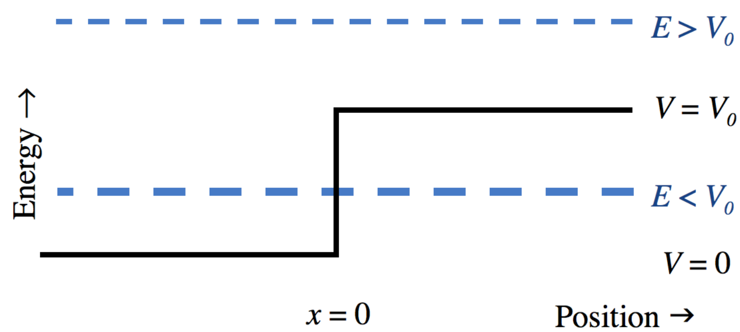
PH424/525: 1-Dimensional Waves — Winter 2012

Homework #3 — Assigned Saturday 02/18/12

Q 1-5 – due Wednesday 02/22/11; Q 6,7 - due Friday 02/24/12

1. Reflection of a quantum mechanical particle from a potential step

The potential energy part of the Hamiltonian operator (a potential step of height V_0) for a system is depicted as a solid line in the energy/position diagram. This is an example of an *unbounded* system, so there is no condition on the energy eigenvalue. A particle of mass m and energy E is incident from the left. There are 2 cases: $E > V_0$, and $E < V_0$.
(As you work through this problem, think carefully about the problem we discussed where a wave in a rope or cable is incident on a different rope or cable. There are important similarities and equally important differences.)



(i) Set up a wave incident from the left, a wave reflected to the left, and one transmitted to the right, so that the total wave function is: $\psi(x,t) = \begin{cases} Ae^{ik_1x} + Be^{-ik_1x} & x \le 0 \\ Ce^{ik_2x} & x \ge 0 \end{cases}$

Verify that these are indeed solutions to the energy eignevalue equation for this problem, provided that $k_1 = \sqrt{\dfrac{2mE}{\hbar^2}}; k_2 = \sqrt{\dfrac{2m(E - V_0)}{\hbar^2}}$.

(ii) What are the boundary conditions that establish the relationships among the coefficients?

(iii) The probability to observe the particle reflected is $r \equiv \left|\dfrac{B}{A}\right|^2$ (remember we measure probabilities and not amplitudes). Find r for both cases: $E > V_0$, and $E < V_0$. Also find the probability of transmission $t = 1 - r$. (See note below*)

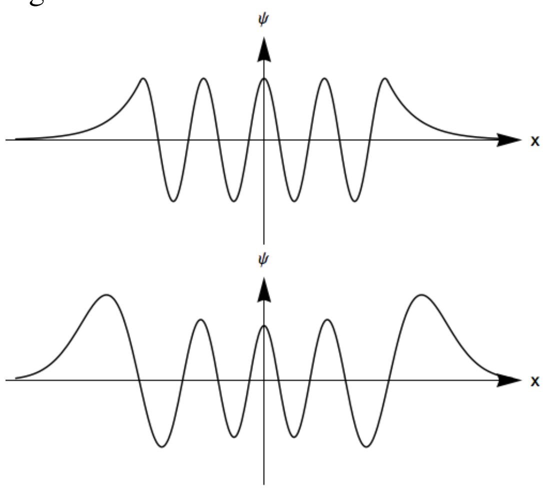
(iv) Interpret your results, and also discuss the limiting cases $V_0 = 0$, and $E >> V_0$.

* Do not try to define the transmission coefficient as $|C/A|^2$. You'll get the wrong answer. The reason is related to the discussion we had about displacement and force reflection and transmission coefficients, but in quantum mechanics, there's a slightly different rationale. Ask me about this if you want to know the details.

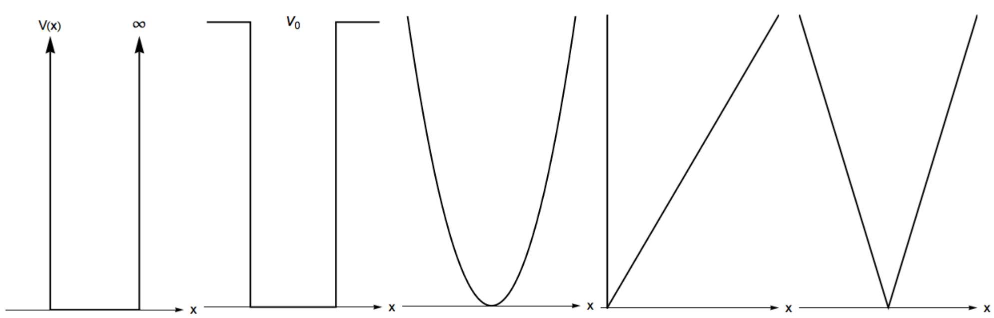
2. **Qualitative discussion of energy eigenfunctions**

   Below are two eigenfunctions of *different* Hamiltonian operators. Consider the Hamiltonian operators corresponding to the 5 potential wells drawn below the functions. For each function, decide whether or not it could be the eigenfunction of each the 5 Hamiltonian operators and say why.

   Eigenfunctions:



   Potential energy functions:



   The *x*-axes of the potential functions do *not* correspond to those of the wave functions. The potentials correspond to, from left to right, the infinite square well (ISW), the finite square well (FSW), the quadratic or harmonic oscillator potential (HO), and a 1-sided linear potential well (1LW), and a 2-sided linear well (2LW).

3. **McIntyre 5.6 (only for the first 2 states): Probability of finding a particle in a certain range - infinite well**

4. **McIntyre 5.5: Expectation values, uncertainties for infinite well**

This is a perfect example of where you should use Mathematica to do integrals. The point here is NOT to demonstrate prowess at doing integrals, but to use a tool to evaluate integrals to show some interesting physics. Note that if an integral is often zero, or evaluates to something quite simple, then there is some physics or symmetry that you should be seeing, and you should seek it out.

Extra practice: Having found Δx and Δp, also calculate the uncertainty product $\Delta x \Delta p$ for the first two states and demonstrate that $\Delta x \Delta p > \frac{\hbar}{2}$.

5. **McIntyre 5.18: Energies for a particular finite well**
   Hints: Use Mathematica to plot either Eqs. 5.88 as a function of z or the equations we used in class – you'll get the same result. Zoom in to find intersection points and hence find energies. Show your working.

DUE FRIDAY

6. **McIntyre 5.2, (d) and (e): Time evolution of superposition state**

7. **McIntyre 5.8: Time evolution of superposition state; expectation value and time evolution**

This problem (5.8) is very long. Let's change it to something simpler.

Instead, (using the waveform given in 5.8 at t = 0):

What is the probability of measuring the energy $E_1 = \pi^2/2mL^2$ if you a measurement of the energy at $t = 0$?

What is the probability of measuring the energy $E_2 = 4\pi^2/2mL^2$ if you a measurement of the energy at $t = 0$?