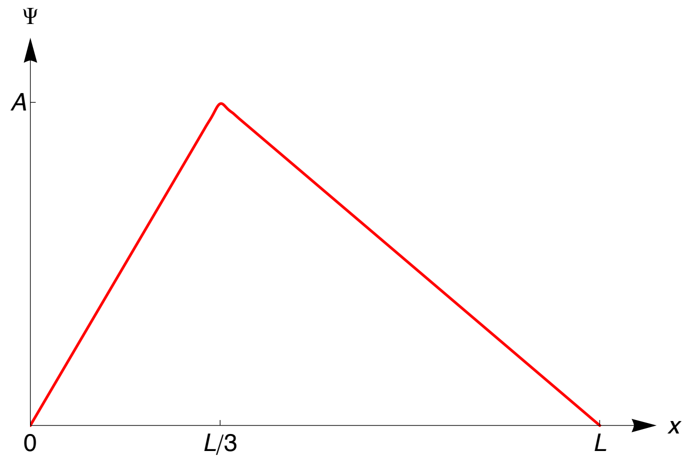
PH424/525: 1-Dimensional Waves — Winter 2012
Homework #2 (Friday) — Assigned Friday 02/10/12
Due Friday 02/17/12

1. **Analysis of normal modes of a string (start in class)**:

   A string (mass per unit length μ, under tension T) is anchored at $x = 0$ and $x = L$. It is then displaced so that it has the following profile at $t = 0$, and the transverse velocity at all points is zero at $t = 0$. Give a complete mathematical description of the motion of the string, including a Mathematica animation.



   Here is a guide to help you:
   (a) Write this wave form very generally as a superposition of the allowed standing wave modes of vibration of this string (Remember the group activity in class? Use that and generalize to many wavelengths).
   (b) Make sure that the *time* dependence satisfies what you know about the velocity at $t = 0$, and that the *space* dependence satisfies what you know about where the string is anchored.
   (c) Turning to the space dependence only, find out what contribution each "normal mode" makes at $t = 0$. (Fourier series will be useful here even though this is not a periodic function – why?)
   (d) Now return to the full wave function, including the time dependence of *each* normal mode. Plot the function (you need to include enough terms to form a reasonable approximation to the function), and animate it. Describe what you see.

2. **Energy**
   Main 9.11

3. **Quantum waves**
   McIntyre 5.1 and 5.2 (a), (b), (c).

Extra practice for exam: All the problems in Main.
Challenge problem on next page, for those who want to go to the next level.

PARADIGMS IN PHYSICS

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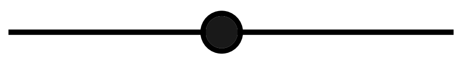
Challenge problem:

Reflection and transmission of waves from a point mass on a rope:

Consider a rope has a uniform mass density μ and has a mass m attached to it at position $x = 0$. A harmonic, single frequency, traveling wave is incident from the left. You are to discuss, qualitatively and quantitatively by following (a) – (f), the reflection and transmission of the wave. Understand that this mass is a point mass – that is, its mass is concentrated at a true mathematical point. Experimentally, this might be approximated by a small, dense ball bearing crimped onto a very much lighter string.



$x = 0$

(a) First, think about the limits your answer should yield. What would happen if the mass were not there ($i.e.$ $m = 0$)? What if the mass were very heavy?

(b) Second, set up the general problem ...

On the left, there is an incident wave (with unit amplitude) and a reflected one $\psi_{left}(x,t) = \psi_{inc}(x,t) + \psi_{ref}(x,t)$. On the right there is a transmitted wave $\psi_{right}(x,t)$. Write down an expression for $\psi_{right}(x,t)$ and $\psi_{right}(x,t)$. It is understood that when we actually plot the displacement, the *real* part of these expressions will be plotted.

(c) What can you say about the frequencies and k vectors of the incident, transmitted and reflected waves, and why is this so?

(d) Consider what conditions must be placed on the displacement $\psi(0,t)$ at $x = 0$, and on the slope $\left.\frac{\partial\psi}{dx}\right|_{x=0}$. (This second part that is different from what we did in class ... you have to apply Newton's law to the *mass*).

(e) Solve your equations to show that $R = \frac{-i\varepsilon}{2+i\varepsilon}$; $T = \frac{2}{2+i\varepsilon}$, where $\varepsilon = \frac{m\omega^2}{k\tau}$ and τ is the tension in the rope. Show that the phase angle of the reflected and transmitted waves changes in general. Was your intuition in (a) justified?

(f) Plot and animate the string profile for various values of ε (small intermediate, large) and describe what you see. In particular, for large values of epsilon, can you see when the discontinuity in the slope at $x = 0$ is greatest?

Animation notes: I found it most useful to plot a little more than one wavelength on either side of zero and to animate for a time corresponding to at least 2 periods of the oscillation. I also increased the number of points and plotted points rather than a line, because the point style seemed better at highlighting discontinuities in the slope, which is what you want to see. Also move the axes away from the interesting region, so you can see the effect better.