Structure of this homework: Problems 1-3, due Wednesday, are designed for routine practice. Problems 4-6, due Friday, require a bit more thought, and are practice for material we discuss later in the week. You should work on them all through the week. You may have to read ahead to get an idea of how to solve these problems.

Reminder: The paradigms courses are 2-unit courses, but they meet for only three weeks in the term. Thus they are the time equivalent of two 3-unit courses running for a full term. *The time equivalent is thus 6 units or 2 courses.*

Animating functions of two variables: You will find it very useful to be able to view functions of 2 variables as an animated sequence. Mathematica and other programs are very good at this. You **must** become familiar with this animation capability.

Mathematica has an excellent interface command for animations, with slider bars. Look up the `Animate and Manipulate` commands in the help files. In the `Manipulate` mode, you can vary many parameters at once. For example if the function is `A*sin(k*x-ω*t)`, you could plot the function as a function of x, with t varying, and set a particular value of k with a slider bar. Then you can move the slider to change omega, and run the t animation again. See the demo worksheet on the class page.

REQUIRED

For #1 and #2, print out the *Mathematica* code and output and turn it in as part of your writeup.

1. (a) $\psi(x,t) = A\cos(-kx + \omega t + \pi/4)$ for A = 1 unit; k = 2π m^{-1}; ω = π rad s^{-1}. What are the wavelength, period and amplitude of the disturbance? Discuss the dimensions of A.

(b) Plot in *Mathematica* two spatial cycles of the waveform and animate for two time periods.

(c) Which direction does the wave travel and with what speed? Which direction does it travel if you change the sign of the position term? Of the time term? Of both? Why?

(d) Focus on the position $x = 0$ m. At what rate is the quantity represented by ψ changing at t = 0, ¼,1/2, and 1s? Describe the variation over one cycle.

2. Write down & plot in *Mathematica* a sinusoidal waveform $\psi(x,t)$ that has the following properties:

(a) Amplitude 2 m, wavelength 10 m, travels to the right at 1 m/s, ψ = 2 m at x = 5m and t = 0 s.

(b) Standing wave, amplitude 5 m, period 1 s, wavelength 1 m that is momentarily flat at t = 0 s.

3. Describe the following waveforms in words (waveform, period, phase angle, direction & speed of travel … *etc*.). Demonstrate whether they are, or are not, solutions to the non-dispersive wave equation $\frac{\partial^2}{\partial t^2}\psi(x,t) = v^2\frac{\partial^2}{\partial x^2}\psi(x,t)$.

(a) $\psi(x,t) = 4\cos(4\pi x + 3\pi t) - 4\sin(4\pi x + 3\pi t)$

(b) $\psi(x,t) = 3\cos(2\pi x)\sin(\pi t)$

(c) $\psi(x,t) = 3e^{-\alpha x}\cos\left(\frac{2\pi}{3}x - \pi t\right)$, α a constant

4. **Standing Waves in a rope:**
Download from the class website the Excel sheet that has the results of the standing wave experiment we did in class on Tuesday.
(a) Tabulate the raw data (# nodes, frequency, wavelength, for several standing waves) and also do the necessary conversions (explain, please!) to plot the "dispersion relation" (ω *vs*. k) for the rope and obtain the phase velocity of the waves in the rope from the graph.
(b) Explain how the application of Newton's 2nd law to the system predicts a value for the velocity in terms of the physical parameters of the vibrating string system. Measure those parameters to predict the phase velocity and compare with the result from (a).
(c) Why are the waves you observed not traveling?
(d) What is the significance of a linear dispersion relation? Give an example of a system where the dispersion is not linear.

5. **Traveling and Standing Waves:**
(a) "A standing wave is the sum of two traveling waves propagating at the same speed in opposite directions".
Prove analytically that this statement is true or untrue by explicitly adding the waves $\psi_1 = A_1\sin(kx - \omega t)$ and $\psi_2 = A_2\sin(-kx - \omega t)$. You could look at an animated function to try some possibilities, but you also need an analytical proof.
(b) "A traveling wave is the sum of standing waves".
Demonstrate analytically the truth or otherwise of this statement.

6. **Main 9.9**

7. **Propagation of electromagnetic waves in vacuum (the wave equation):**
(This is relevant for the lab exercise. This is really a 3-dimensional example because the wave has components in two spatial dimensions and propagates in the third. However, the components are described by the 1-d wave equation. You will meet this particular example again in the E&M and optics courses.)
Show that in a medium in which there is *no* free charge and *no* free current, electromagnetic waves propagate with a velocity of propagation $v = \frac{1}{\sqrt{\mu\varepsilon}}$, where ε is the permittivity and μ the permeability of the vacuum. (In vacuum, $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$, in which case the velocity has the special symbol *c*.)

The following guides you through the problem. The basic approach is to show that the electric and magnetic field vectors separately obey the 1-d non-dispersive wave equation, and identify the velocity.

To begin this problem, think back to previous courses -- *all* electric and magnetic fields obey the Maxwell equations, and you will find them in Griffiths E&M. PH320 and PH422 dealt with electro*statics* and magneto*statics*, but you will recall from PH213 that there are *dynamic* (time varying) terms in two of the equations, describing motional EMF (Faraday's Law) and a "displacement current", respectively. These are crucial for discussing propagation of electromagnetic waves.

Write down the 4 Maxwell equations with the source (current and charge) terms set to zero. This problem is most easily done with the equations in *differential* form, not *integral* form (you must relate derivatives to each other). The equations are coupled -- E and B appear together in some. You must manipulate the equations to obtain equations in E alone and B alone that relate the second time derivative to the second space derivative. Find the constant that relates the two and hence identify the velocity. [Hint: you will have to look up a vector identity for the "curl of the curl" of a vector: $\nabla \times \nabla \times \vec{A} = ?$]

EXAMPLES FOR EXTRA PRACTICE

Main 9.1 through 9.10; 9.13; 9.14