PH421: Oscillations – lecture 1

Reading: *Taylor 4.6*
(Thornton and Marion 2.6)
Knight 10.7

Goals for the pendulum module:

(1) CALCULATE the period of oscillation if we know the potential energy; specific example is the pendulum

(2) MEASURE the period of oscillation as a function of oscillation amplitude

(3) COMPARE the measured period to models that make different assumptions about the potential

(4) PRESENT the data and a discussion of the models in a coherent form consistent with the norms in physics writing

(5) CALCULATE the (approximate) motion of a pendulum by solving Newton's F=ma equation

How do you calculate how long it takes to get from one point to another?

$$\Delta t = \frac{\Delta x}{v}$$

But what if v is not constant?

$$\frac{dx}{dt} = v(x)$$

$$\boxed{dt = \frac{dx}{v(x)}}$$

Separation of x and t variables!

$$\int_{t_{start}}^{t_{finish}} dt = \int_{x(t=t_{start})}^{x(t=t_{finish})} \frac{dx}{v(x)}$$

The case of a conservative force

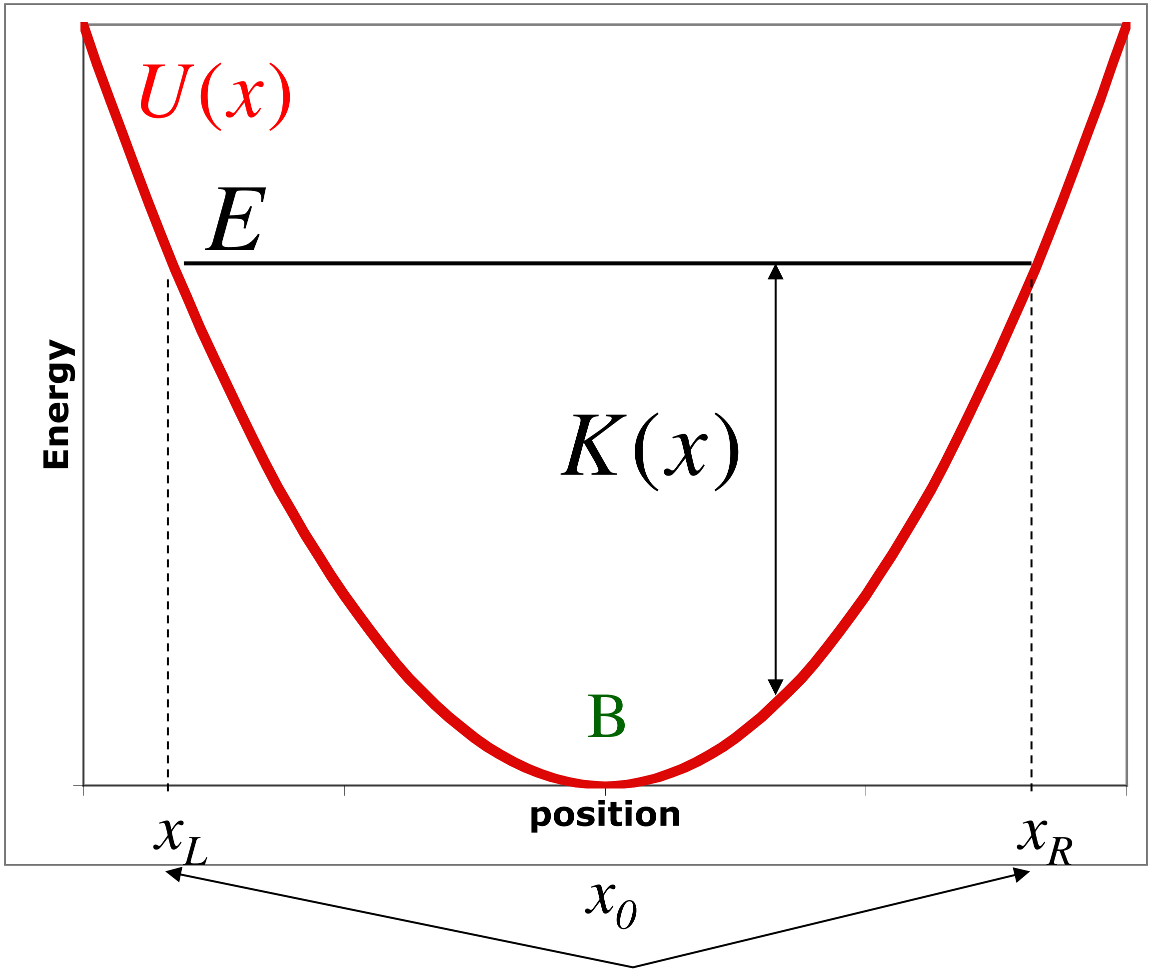
$$E = K(x) + U(x)$$

Suppose total energy is CONSTANT (we have to know it, or be able to find out what it is)

$$K(x) = \tfrac{1}{2} m \left[\frac{dx}{dt}\right]^2 \Rightarrow \frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

$$\boxed{dt = \frac{dx}{\pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}}}$$

Example: $U(x) = ½\ kx^2$, the harmonic oscillator

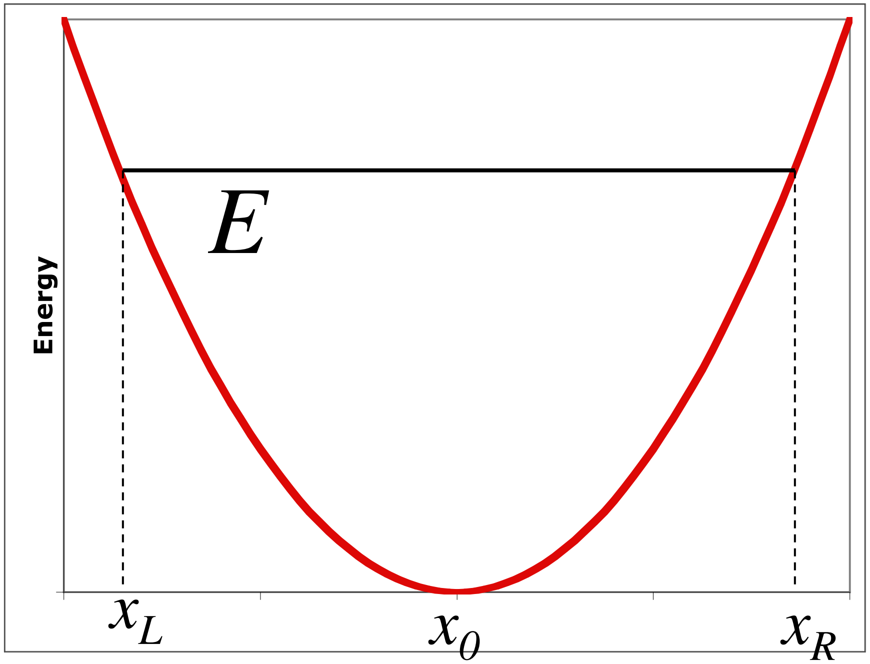


$$\boxed{U(x) = \tfrac{1}{2}kx^2 \quad \text{PE}}$$

$$\boxed{F(x) = -\frac{dU}{dx} = -kx}$$

$$\boxed{K(x) = \tfrac{1}{2}m\dot{x}^2 \ \text{KE}}$$

$$\boxed{\dot{x} \equiv \frac{dx}{dt}}$$



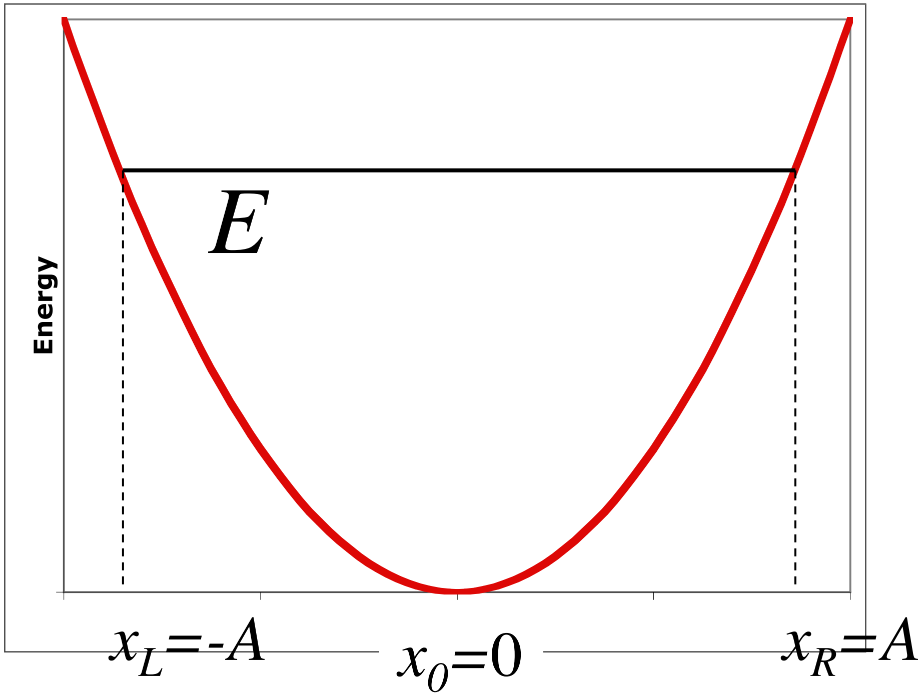
Symmetry - time to go there is the same as time to go back (no damping)

$$T = 2\int_{x_L}^{x_R} \frac{dx}{\sqrt{\frac{2}{m}\left[E - U(x)\right]}}$$

SHO - symmetry about x_0

x_L -> x_0 same time as for x_0 -> x_R

$$T = 4\int_{x_0}^{x_R} \frac{dx}{\sqrt{\frac{2}{m}\left[E - \frac{1}{2}kx^2\right]}}$$



SHO - do we get what we expect?

$$T = 4\int_{x=0}^{x=A} \frac{dx}{\sqrt{\frac{2}{m}\left[E - \frac{1}{2}kx^2\right]}}$$

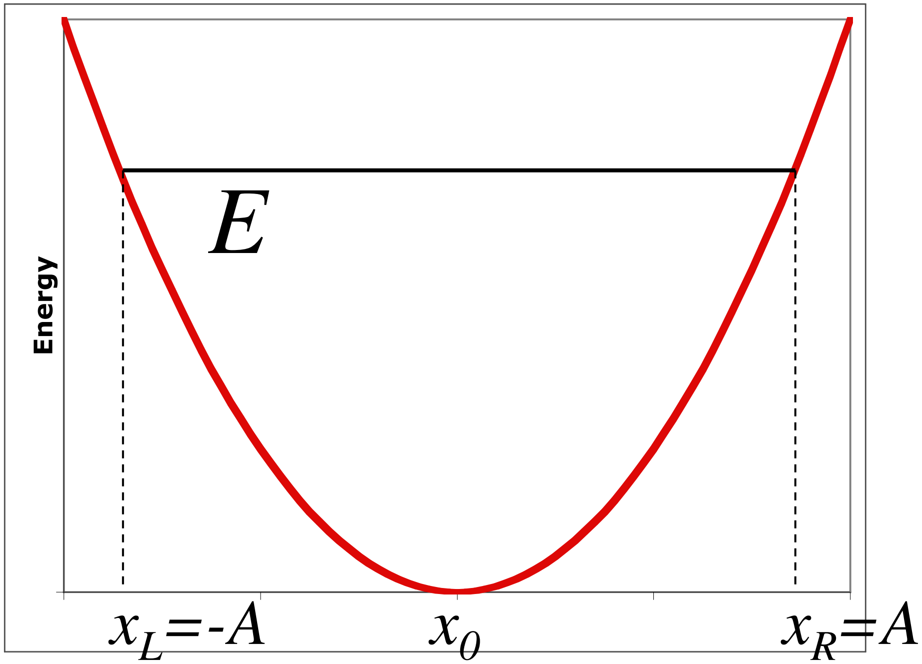
$$u = \frac{x}{A} \qquad \boxed{E = \tfrac{1}{2}kA^2}$$

Another way to specify E is via the amplitude A

$$\boxed{T = 4\int_{u=0}^{u=1} \frac{du}{\sqrt{\frac{k}{m}\left[1 - u^2\right]}}}$$

Independent of A!

http://www.wellesley.edu/Physics/Yhu/Animations/sho.html



$$T = 4\int_{u=0}^{u=1} \frac{du}{\sqrt{\frac{k}{m}\left[1-u^2\right]}}$$

$$u = \sin\phi$$

$$du = \cos\phi d\phi$$

$$\sqrt{1-u^2} = \cos\phi$$

$$\downarrow$$

$$T = 4\int_{\phi=0}^{\varphi=\pi/2} \frac{\cos\phi d\phi}{\sqrt{\frac{k}{m}}\cos\phi}$$

$$\boxed{T_{SHO} = 2\pi\sqrt{\frac{m}{k}} \equiv \frac{2\pi}{\omega_0}}$$

You have seen this before in intro PH, but you didn't derive it this way.

$$\boxed{T_{SHO} = \frac{2\pi}{\omega_0}}$$

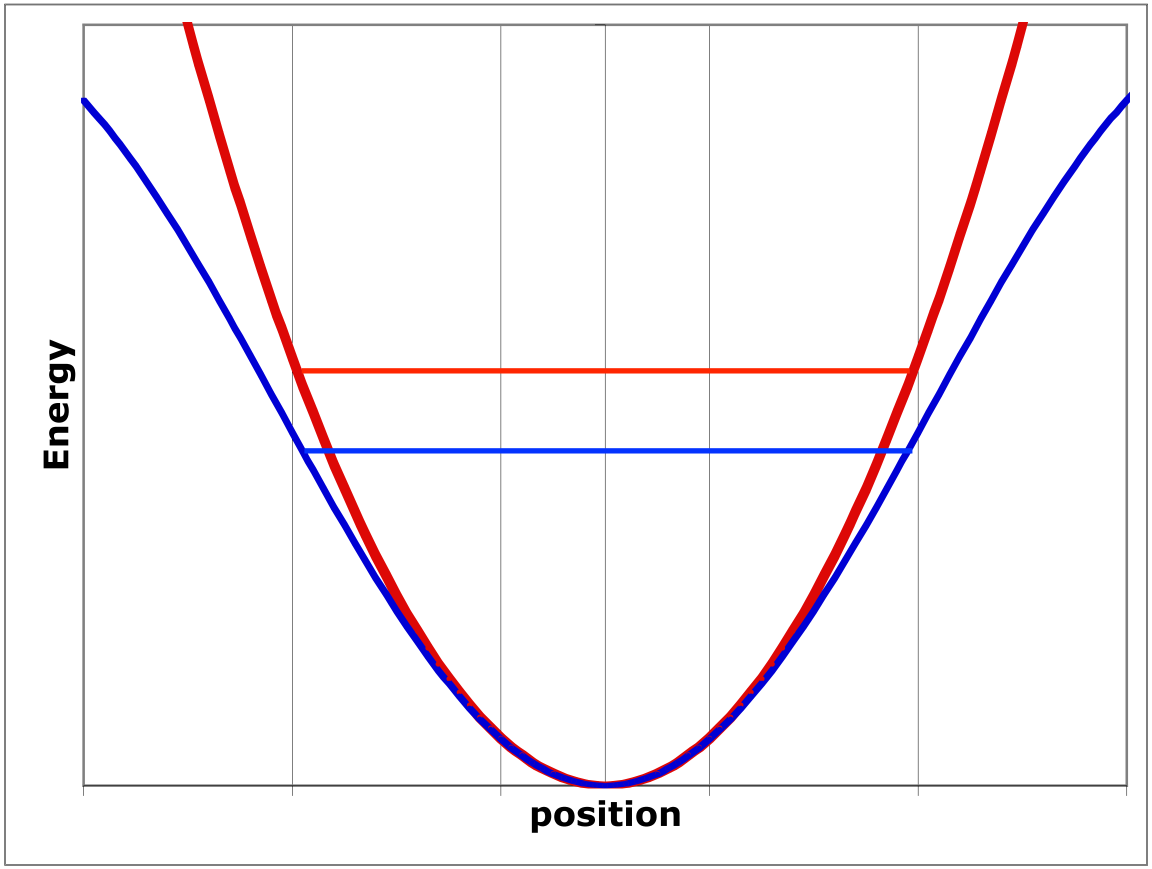
Period of SHO is INDEPENDENT OF AMPLITUDE
Why is this surprising or interesting?

As *A* increases, the distance and velocity change. How does this affect the period for ANY potential?

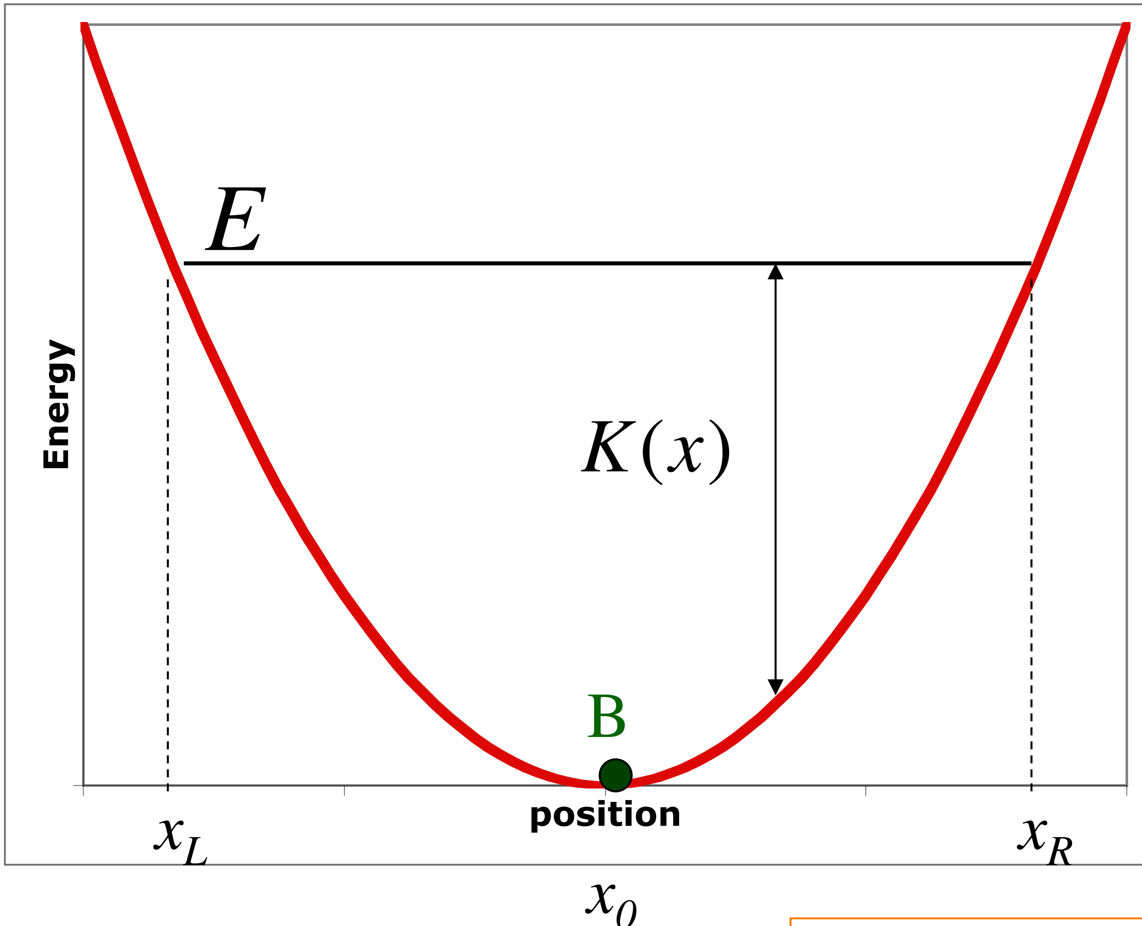
A increases -> further to travel -> distance increases
-> period increases
A increases -> more energy -> velocity increases
-> period decreases

Which one wins, or is it a tie?

Compare $1 - \cos\theta$ with $\frac{1}{2!}\theta^2$



Period increases because *v(x)* is smaller at every *x* (*why?*) in the trajectory. Effect is magnified for larger amplitudes.



Eqm: $\left.\dfrac{dU}{dx}\right|_{x_0} = 0$

If $\left.\dfrac{d^2U}{dx^2}\right|_{x_0} = k$

"Everything is a SHO!"

$$\boxed{U(x) = U(x_0) + \tfrac{1}{2}k(x - x_0)^2}$$

$$\boxed{U(x) = U(x_0) + (x - x_0)\left.\frac{dU}{dx}\right|_{x_0} + \frac{(x-x_0)^2}{2!}\left.\frac{d^2U}{dx^2}\right|_{x_0} + \frac{(x-x_0)^2}{3!}\left.\frac{d^3U}{dx^3}\right|_{x_0}}$$