The aim of this exercise is to calculate **quantitatively** how the period T of a pendulum's oscillation depends on the amplitude of its motion. We write the exact integral, and then use a power series to approximate the integrand. This is to be included in your first homework. [You might wonder why, in this computer age, we bother with approximation methods. The answer is partly that your brain needs exercise … just because you can lift 100 lb with a winch is no reason to stop your weight training class! It is partly that approximation methods lend physical insight that computational techniques cannot.]

Step I: Short recap (<3 min).
Consider a pendulum like the one used in the lab, which has mass M and moment of inertia I, and whose center of mass is a distance D from the axle. The angle from the vertical is and the amplitude of the oscillation is θ_{max}.

THIS WAS HOMEWORK: Start with the time differential $dt = d\theta / \dot{\theta}$. The velocity $\dot{\theta}$ is found from energy conservation. Reassure yourselves that you can derive the expression for the period: $\frac{T}{2} = \sqrt{\frac{I}{2MgD}} \int_{-\theta_{max}}^{\theta_{max}} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_{max}}}$. (what are the dimensions of the prefactor?)

Step II:
We now want to make this expression look similar to the one we had in class for the simple harmonic oscillator. Here's where you'll apply your knowledge of series expansions in a slightly more complex situation than in the previous paradigms courses.
THIS IS PART OF YOUR REPORT: Approximate the integral using the small angle approximation, keeping just the term that corresponds to harmonic motion.

THIS ISN"T, BUT IT'S A GOOD THING TO DO AT HOME: keep the next term <u>past</u> what would give you the answer we found in class for the pure harmonic oscillator. Try to make your expression for T look like this:

$$T = (\text{constants}) \times \int_0^1 \frac{dy}{\sqrt{1-y^2}} \left(1 + (const)y + (consts)y^2 + ...\right)$$

Step III:
Solve the integral. You can use Mathematica, but the substitutions to solve analystically are not hard.

Step IV:
THIS IS PART OF YOUR REPORT: Evaluate the full integral (known as an elliptic integral) numerically. You should graph your results for the exact and approximate values of the period as a function of the maximum amplitude. On the same graph you can plot your experimental data. This same graph will be included in your lab report.