• In this assignment, it a good idea to use Mathematica or other software to plot functions to gain insight into their shape and behavior, and to evaluate how well series approximate functions. It is also OK to use it to evaluate integrals, but you should not use the built-in Fourier coefficients functions (except to check), since this won't help you understand the process.

PRACTICE

1. Kronecker delta worksheet.
2. All of the integrals in the worksheet we did in class, both graphically and analytically.

REQUIRED

Q1 and 2 due by Wednesday 11/21

1. Prove analytically that $\frac{2}{T}\int_0^T \sin(n\omega t)\sin(m\omega t)\,dt = \delta_{m,n}$. Here $\omega = 2\pi/T$, n and m are integers greater than zero and $\delta_{m,n}$ (the "Kronecker delta") is the function that is 1 if $m = n$ and 0 if $m \neq n$. You have to treat the two cases separately. Do not choose specific values of m and n to show examples, prove in general for ANY m and n.

   Hint: $\sin(n\omega t) = \frac{e^{in\omega t} - e^{-in\omega t}}{2i}$ will make the integrals easier. Use software packages to check your results, but do integrals analytically to help you understand how Fourier series work.

2. Integrate over one period ($2\pi/\omega$), the product of the function $f(t) = 2\sin\omega t\cos\omega t$ and
   (i) $\sin(\omega t)$,
   (ii) $\cos(\omega t)$,
   (iii) $\sin(2\omega t)$.
   Argue conceptually why these results are expected or not. Try to think in terms of projecting one function onto another.

Q3 due by Tuesday 11/27

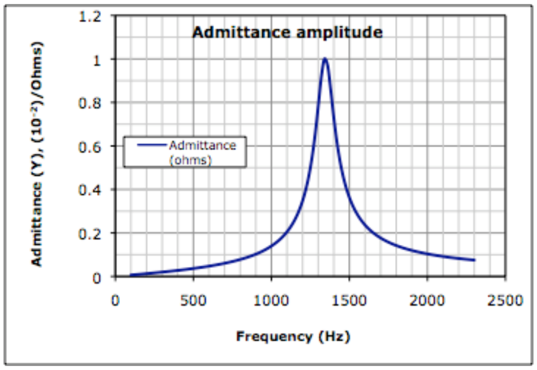
3. Suppose a driving voltage in the form of a triangle wave of amplitude A (voltage values range from A to $-A$) and period T is applied to a series LCR circuit.
   i) Calculate the Fourier coefficients for this voltage triangle wave by hand. You should choose the time origin to make the function either even or odd. Say which and choose the appropriate form of the Fourier series.
   ii) Make a plot of the voltage waveform using just a few terms (2 or 3) and compare to the plot using a larger number (10 or more). This will give you a visual evaluation of how well the series converges.

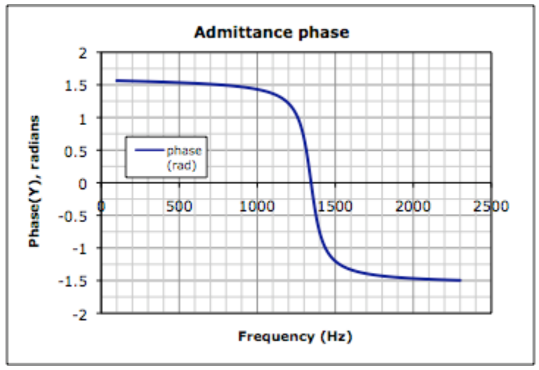
   For the next part, we use values of ω_0, $|Y(\omega)|$, $\phi(\omega)$ similar to the ones of the circuit we studied in the lab. The graphs below were generated with $L = 0.14$ H, $C = 0.1$ μF, $R = 100$ Ω. You can read off values of phase and admittance or you can calculate them.

iii) Write an expression for the current in the circuit $I(t)$ including enough terms so that adding more terms will not be significant. Explain why you stopped where you did, by estimating the size of the next term relative to the last one you used.

(A) The applied voltage is a triangle form with a period $2\pi/\omega_0$ where ω_0 is the resonant frequency of the LCR circuit, and

(B) the applied voltage is a triangle form with a period $\pi/2\omega_0$.





(Note – there are 2 Mathematica worksheets posted on the class web page - fourier_triangle.nb and fourier_square.nb. These expand upon the "guess" worksheet idea that we used in class, allowing you to compare your proposed series with the actual function. You may find it helpful to work through either or both.)

Extra to practice for exam, but you don't have to turn in.

1. Find the Fourier coefficients for a square wave. Compare the coefficients found in with the coefficients for a triangle wave. There is an important difference in how the *nth* order term in each series depends on n. What is this difference and what is the physical significance?
2. (a) The periodic function known as a truncated sine function is given over one period by

$$f(t) = \begin{cases} \sin\omega t & 0 < t < \pi/\omega \\ 0 & \pi/\omega < t < 2\pi/\omega \end{cases}$$ (and then repeated infinitely). Sketch the function. Is it an even or odd function of t or neither? Choose the appropriate form for the Fourier series:

$f(t) = \sum_{n=0}^{\infty} a_n \cos n\omega t$, $f(t) = \sum_{n=0}^{\infty} b_n \sin n\omega t$ or $f(t) = \sum_{n=0}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$. Find the first 3 coefficients.
3. Taylor 5.42

PARADIGMS IN PHYSICS