REQUIRED:
By Tuesday in class: Question 0

0. Visit the class webpage http://www.physics.oregonstate.edu/ph421.
   • Read the syllabus, noting the reading assignments and due dates for homework & lab assignments. I post my lecture notes under the class date link. **Please complete the reading assignments BEFORE class each day.**
   • Read through general information handout, the Pendulum Lab instructions, write-up instructions and rubric.
   • Visit all the links on visible on the side bar under "PH421" and make sure you know what info is on each page. Make sure you read the writing guide.
   • The syllabus, general information, this homework have been provided as hardcopy. It is your responsibility to download other assignments.

Due Wednesday 4 pm: Questions 1, 2

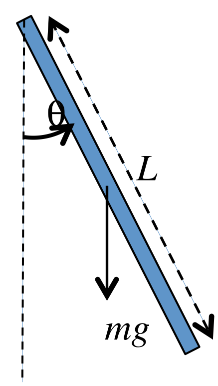
1. (a) A particle of mass m and total energy E oscillates under the influence a potential energy

$$U(x) = \begin{cases} \infty & x < 0 \\ mgx & x \geq 0 \end{cases}$$ where mg is a known constant.

Draw a diagram of the potential energy as function of position. Label any quantities you define in your solution. Calculate the period of motion and discuss whether the period is amplitude-independent.

(b) Suppose the particle oscillates in a harmonic potential with the same amplitude as in the above potential energy, and at the particular amplitude in question, the total energy in each case happens to be the same. Draw an energy/position graph that represents this information, and discuss whether the period is longer, shorter, or the same.

2. A pendulum has a mass m, length L, moment of inertia I about the axis shown. (Define any other parameters you might need.) Its position from equilibrium is specified by a single variable - the angle θ. This is therefore a 1-dimensional problem. The pendulum swings freely; any friction is negligible. Start from the statement that the infinitesimal time for the pendulum to swing through an infinitesimal angle is $dt = \frac{d\theta}{\dot{\theta}}$, where $\dot{\theta}$ is the angular velocity (sometimes called ω). Express the conservation of energy in terms of $\dot{\theta}, \theta, I, L, m$ and other variables you might need. Show that the period is



$$T = 4\sqrt{\frac{I}{MgL}} \int_0^{\theta_{max}} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_{max}}}$$

where θ_{max} is the amplitude of oscillation.
You'll work further with this expression in class.

Due Friday 4 pm: Questions 3, 4, 5, 6

3. (a) A pendulum as in #2 is observed at a particular time to be at an angle $\theta = \frac{\pi}{4\sqrt{2}} rad$ and to have an angular velocity $\dot{\theta} = \frac{\pi^2}{2\sqrt{2}} rad/s$. Assume that the motion is simple harmonic, and that the period is T = 1 s, and write the motion in the "B" form. Now convert to the "A" and "C" forms, but explain each step – do not simply quote a formula.

   (b) Based on the amplitude you found in (a), discuss whether your simple harmonic assumption was justified.

4. Represent the following four complex numbers in rectangular form $a + ib$, polar form $|z|e^{i\phi}$, and on an Argand diagram:
   (a) $e^{i\pi}$ (b) i
   (c) $\sin \pi/2$ (d) $\cos(\pi/4) - i\sin(-\pi/4)$.

5. Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ is very important and you must be able to write it down without even thinking.

   (a) Starting from Euler's formula, find expressions for
   (i) $\cos\theta$ and (ii) $\sin\theta$
   in terms of exponential functions.

   (b) The hyperbolic trigonometric functions are:
   (i) $\cosh(\theta) = \cos(i\theta)$ and (ii) $\sinh(\theta) = -i\sin(i\theta)$.
   Using (a), find expressions for them in terms of the exponential function.

6. Solve the following complex equation for the unknown quantities $|q_0|$ and ϕ in terms of the known quantities a, b, c.

$$|q_0|e^{i\phi} = \frac{a}{b + ic}$$

EXTRA EXAMPLES TO HELP STUDY FOR THE EXAM.

- Main, Problem 1.1, 1.2, 1.3, 1.6
- An oscillator's motion is described as the superposition of two simple harmonic displacements of the same frequency, but different amplitudes and different phases. Is the resultant motion simple harmonic? If not, what type of motion is it? If so, what is the amplitude and phase of the resultant motion? [Important: you must rigorously justify your result mathematically, even though the question is not posed with equations and symbols.]
- Repeat problem 1(a) for other potentials that support oscillatory motion.
  Repeat problem 1(b) to compare a square well and a harmonic potential.

- Given any initial conditions (*i.e.* the velocity and position at a one given time t_0 that is not necessarily zero), you should be able to find the velocity and position of a harmonic oscillator at any other time.
- All the problems in the *Harmonic Functions* Drill on the class website. This is the level of algebra that must be second nature to you, and will be needed to solve problems. Practice until you are very good at the problems, and can do the drills quite quickly.