NAME:____

Final Exam Fall 2012/12/04
PH421/521: Oscillations

• Answer all 5 questions; each starts on a new page. Questions are 20 points each, so budget ON AVERAGE 20 minutes per question, but they are NOT of equal length or difficulty. It is imperative that you make a reasonable attempt at all questions.

• Some level of explanation is always required - judge from the point assignment how detailed you should be (about 1 point/minute on this exam).

• "Sketch" means that you do not need to plot accurately, but you must pay attention to overall shape, maxima, minima, & zero crossings. It also means large, clear graphs, with appropriately labeled axes.

• NO CALCULATORS allowed. Where numbers are requested, the values are chosen for easy evaluation, or you are supposed to use approximations.

• Use this exam handout for your answers and keep all questions separate. Insert extra pages if needed (2 extra provided, more available from the instructor). Put your name on each page. Turn in everything at the end of the test. The test and solutions will be posted on the class web page after the exam.

• Use the formulae on the next page without derivation unless you are specifically requested to derive them. Their presence does not imply a particular way to do a problem, nor is there any guarantee that they are relevant to the exam questions. These are provided ahead of the exam; it is your responsibility to know where they apply and what the symbols mean. You are allowed one side of an 8.5 by 11 sheet of handwritten notes, which you must turn in with your exam. You may not assume that it is permissible to quote anything from your sheet without proof.

NAME	
Q1	
Q2	
Q3	
Q4	
Q5	
TOTAL	

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$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$f(t) = A\cos(\omega t + \phi) \quad f(t) = B_p \cos\omega t + B_q \sin\omega t$$

$$f(t) = Ce^{i\omega t} + C^* e^{-i\omega t} \quad f(t) = \mathrm{Re}\left[De^{i\omega t}\right]$$

$$\ddot{\psi} + 2\beta\dot{\psi} + \omega_0^2\psi = 0 \qquad \tau = \frac{1}{\beta} \qquad Q = \frac{\omega_0}{2\beta} = \frac{\omega_0}{\Delta\omega}$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right] = \sum_{n=-\infty}^{\infty} c_n e^{in\omega t}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,dt \qquad b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt \qquad c_n = \frac{1}{T}\int_0^T f(t)e^{-in\omega t}\,dt$$

$$Y(\omega) = \frac{I_{out}}{V_{in}} = |Y(\omega)|e^{i\phi_I(\omega)} \qquad |Y(\omega)| = \frac{\omega / L}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]^{1/2}} \qquad \phi_I(\omega) = \frac{\pi}{2} + \arctan\frac{-2\beta\omega}{\omega_0^2 - \omega^2}$$

$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta \quad \sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

$$\int \sin(ax)\cos(bx)\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}; \quad a \neq \pm b$$

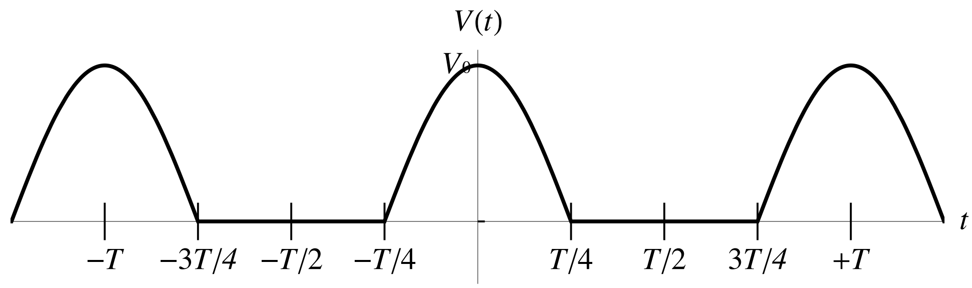
$$\int \cos(ax)\cos(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}; \quad a \neq \pm b$$

$$\int \sin(ax)\sin(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}; \quad a \neq \pm b$$

$$\int_0^1 \frac{du}{\sqrt{1-u^2}} = \frac{\pi}{2} \qquad \int_0^a \frac{dx}{\sqrt{1 - x^2/a^2}} = \frac{\pi a}{2}$$

1. [20 points]

A time-periodic signal $V(t)$ with period T is the truncated sinusoid shown below.



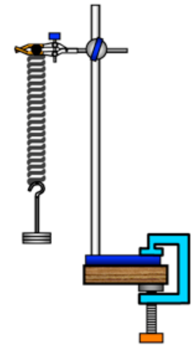
(a) [7 points] Choose a convenient 1-period interval and write $V(t)$ as a piecewise function of t over this time interval.

(b) [13 points] Use (a) to find the Fourier coefficients a_n and b_n for $V(t)$ for n =1, and n=2.
(You are not expected to simplify terms like 14/π etc., but you must simply terms like sin(nπ) or cos((n+1)π) that simplify to 0,1,-1 etc.)

Q1 cont'd

2 [20 points]

A mass m attached to a spring with spring constant k is set into motion such that its displacement is described by $\ddot{x}(t)+2\beta\dot{x}(t)+\omega_0^2 x(t)=0$.



(a) [3 points] How is this equation a statement of Newton's law?
(b) [12 points] The undamped system (β=0) oscillates with period T_0 = 1.000 s. Now you introduce a little damping and the period changes to T_1 = 1.0002 s. What is the damping constant?
(c) [5 points] How many cycles does it take for the oscillation to decay to e^{-2} of its initial amplitude?
(b&c: evaluate to 1 significant figure. The numbers have been chosen to make the evaluations reasonable without a calculator. Don't forget the lessons from PH320 about expansions in small quantities.)

Q2 cont'd

3. [20 points]

A mass undergoes simple harmonic oscillation at frequency ω (no damping).

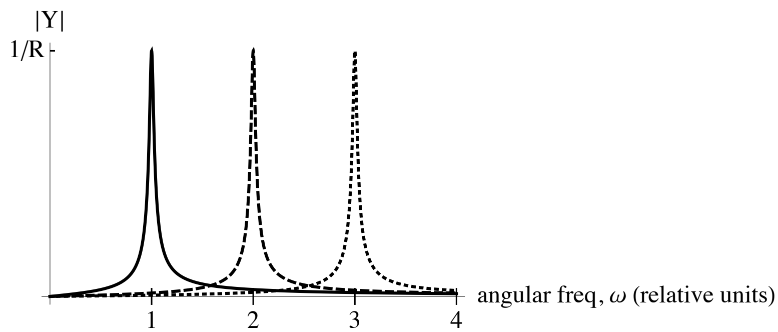
(a) [6 points] Write the general equation for the motion $x(t)$ - in both the "A form" (amplitude/phase) and the "B form" (sine/cosine) – and relate the arbitrary constants in the A form to those in the B form.

(b) [14 pts] Now evaluate the arbitrary coefficients so that the following specific information is reflected. At $t = 0$, the mass is released from a position $x_0 > 0$. It is not released from rest, but rather with a velocity directed towards the equilibrium position that results in an oscillation of amplitude $2x_0$.

Q3 cont'd

4. [20 points]

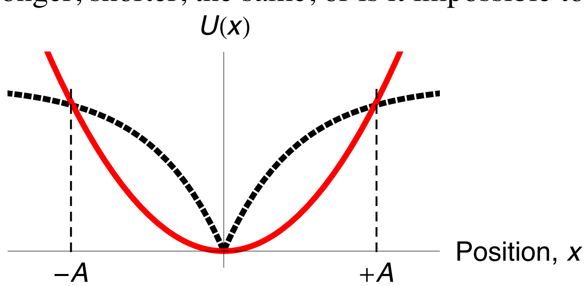
(a) [5 points] Below are the admittance magnitude curves for three different series LRC circuits driven by sinusoidal driving forces at frequencies around their respective resonance frequencies, which are 1,2,3 in some units. What is the ordering of their (i) damping constants? (ii) Q-values?



(b) [5 points] What was the approximate damping time (in seconds) for the series LRC circuit you studied in class?

(c) [5 points] If a mass-spring system is driven by a sinusoidal driving force well above its resonance frequency, which quantity is in phase with the driving force – displacement, velocity or acceleration?

(d) [5 points] A particle has potential energy $U(x)$ given by either the dashed curve or the solid curve (below). In both cases, it oscillates between $\pm A$. Is the period in the dashed potential longer, shorter, the same, or is it impossible to be definitive without a calculation? Why?



Q4 cont’d

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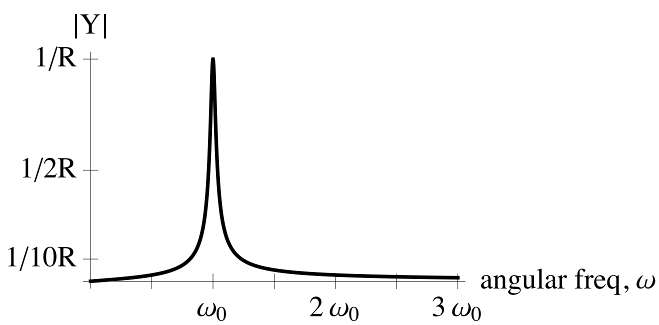
5. [20 points]

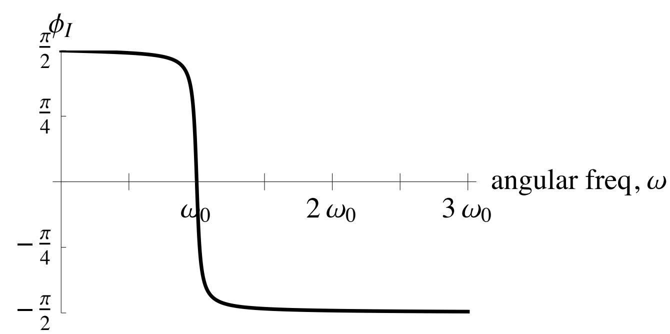
(a) [10pts] Use Euler's formula to show that the periodic waveform $V(t) = V_0 \cos(\Omega t)\cos(3\Omega t)$ can be expressed as the sum of two sinusoidal functions with equal amplitude.

(b) [10pts] The driving voltage in (a) is applied across all three elements of a series LRC circuit with resonance frequency ω_0. Describe, with the help of formulae and/or sketches where appropriate, the resulting waveform across the resistor if :

(i) $\Omega = \frac{\omega_0}{4}$ (ii) $\Omega = \frac{\omega_0}{3}$

Below are sketches of the magnitude and phase of the admittance function for this particular circuit. Use them to estimate admittances.





Q5 cont'd

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