KRONECKER DELTA solutions

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Evaluate the following expressions:

a. $\sum_{n=0}^{\infty} \delta_{n,2} = 1$ **b.** $\sum_{n=0}^{\infty} n\,\delta_{n,2} = 2$ **c.** $\sum_{n=-\infty}^{\infty} n^2\,\delta_{n,2} = 4$

d. $\sum_{n=4}^{10} \delta_{n,2} = 0$ **e.** $\sum_{n=-\infty}^{\infty} \delta_{n^2,2} = 0$ **f.** $\sum_{n=-\infty}^{\infty} n^2\,\delta_{n^2,2} = 0$

g. $\sum_{n=-\infty}^{\infty} \delta_{n^2,4} = 2$ **h.** $\sum_{n=-\infty}^{\infty} n\,\delta_{n^2,4} = 0$ **i.** $\sum_{n=-\infty}^{\infty} n^2\,\delta_{n^2,4} = 8$

j. $\sum_{n=1}^{\infty} n \sin\left(\frac{n\pi}{2}\right)\delta_{n,1} = 1$ **k.** $\sum_{n=1}^{\infty} n \sin\left(\frac{n\pi}{2}\right)\delta_{n,2} = 0$ **l.** $\sum_{n=1}^{\infty} n \sin\left(\frac{n\pi}{2}\right)\delta_{n,3} = -3$

Write the following series in the sigma notation. You need not evaluate the sums.

m. $1 - 1/3 + 1/9 - 1/27 + \ldots = \sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n$

n. $1 - 1/k + 1/k^2 - 1/k^3 + \ldots = \sum_{n=0}^{\infty} \left(-\frac{1}{k}\right)^n$

o. $1/k - 1/k^2 + 1/k^3 - 1/k^4 + \ldots = -\sum_{n=1}^{\infty} \left(-\frac{1}{k}\right)^n$

p. $1 - \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos 4\theta - \frac{1}{6}\cos 6\theta + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{\cos(2nq)}{2n + d_{n0}}$

q. $\sin\theta - \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta - \frac{1}{7}\sin 7\theta + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{\sin(2n+1)q}{2n+1}$

r. $1 - \frac{1}{2}\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{8}\cos 3\theta + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{\cos(nq)}{2^n}$