

All quantities appearing in parentheses are real numbers.

definitions: principal value  $-\pi/2 \leq \arctan x \leq \pi/2$

$$\operatorname{sgn}(x) \equiv \frac{x}{|x|} = \begin{cases} +1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

**1.**  $A \sin(\theta+B) = a \cos(\theta + b)$  .  $a = A$  ,  $b = B - \pi/2$

**2.**  $A \sin(\theta+B) = a \sin(\theta) + b \cos(\theta)$  .  $a = A \cos B$  ,  $b = A \sin B$

**3.**  $A \cos(\theta+B) = a \sin(\theta) + b \cos(\theta)$  .  $a = -A \sin B$  ,  $b = A \cos B$

**4.**  $A \sin(\theta+B) + C \sin(\theta+D) = a \sin(\theta+b)$  .

$$a = \sqrt{A^2+C^2+2AC \cos(B-D)} \operatorname{sgn}(A \cos B + C \cos D) ,$$

$$b = \operatorname{Atan}\left(\frac{A \sin B + C \sin D}{A \cos B + C \cos D}\right) \quad \text{Hint: picture as Im parts of complex numbers}$$

**5.**  $A \cos(\theta+B) + C \cos(\theta+D) = a \cos(\theta+b)$  . Same as **4**, from Re parts

**6.**  $A \sin(\theta) + B \cos(\theta) = a \sin(\theta+b) = c \cos(\theta + d)$  .

$$a = \sqrt{A^2+B^2} \operatorname{sgn} A , b = \arctan(B/A), c = \sqrt{A^2+B^2} \operatorname{sgn} B, d = -\arctan(A/B)$$

*Hint:* these are special cases of **4** and **5**

**7.**  $A \exp i(\theta+B) = a \exp i(\theta)$  .  $a = A \exp iB$

**8.**  $A \exp i(\theta) + B \exp -i(\theta) = a \sin(\theta) + b \cos(\theta)$  .  $a = i(A-B)$ ,  $b = A + B$

**9.**  $A \sin(\theta) + B \cos(\theta) = a \exp i(\theta) + b \exp -i(\theta)$  .  $a = \frac{B-iA}{2}$  ,  $b = \frac{B+iA}{2}$

**10.**  $A \sin(\theta) + B \cos(\theta) = a \exp i(\theta) + b \exp -i(\theta)$  .  $a = \frac{B-iA}{2}$  ,  $b = \frac{B+iA}{2}$

**11.**  $A \cos(\theta+B) = a \exp i(\theta) + b \exp -i(\theta)$  .  $a = \frac{A}{2} \exp iB$  ,  $b = \frac{A}{2} \exp -iB$