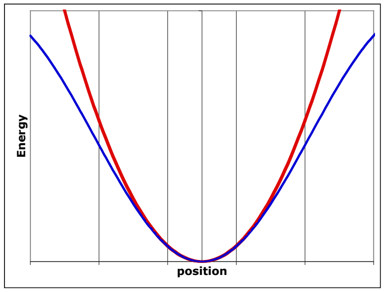
WEEK 1 SUMMARY

Energy approach to equation of motion:
i.e. find trajectory x(t) if U(x) is known

- $$dt = \frac{dx}{\pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}}$$
- Special case of $U(x)=(½)kx^2$
  found period T (indep of A),
  found $x(t) = A\cos(\omega t+\phi)$
- Found 3 other forms of $A\cos(\omega t+\phi)$
- Learned to apply initial conditions to determine A, ϕ, and also the arbitrary parameters in other 3 forms

Energy approach to equation of motion:

- Harder case of $U(\theta) = MgL_{cm}(1\text{-}\cos\theta)$
  found -> HO for small theta
  found T numerically
  measured T for pendulum
- Learned to argue qualitatively about time to move certain distances, comparing T for diff U
- Did NOT learn equation of motion $\theta(t)$



Complex numbers:

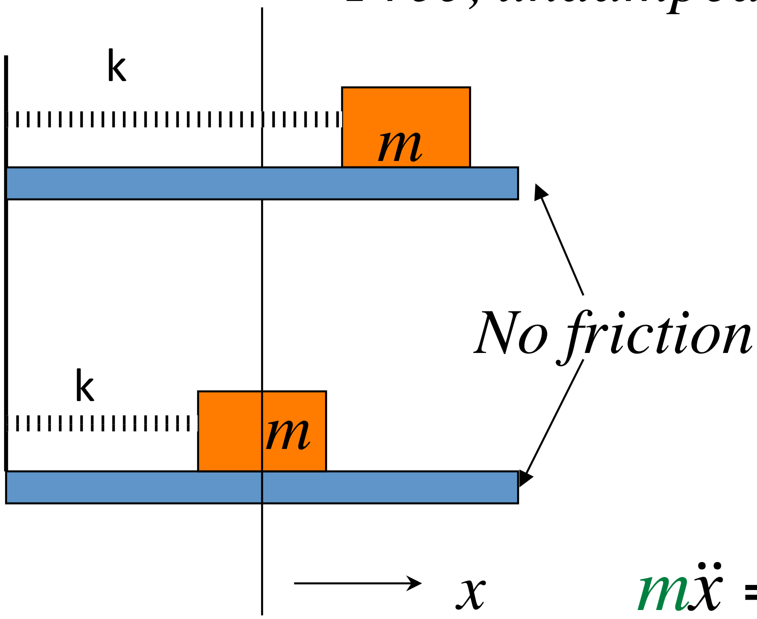
- rectangular and polar form and Argand diag.
- complex conjugate
- Euler relation

$$\exp(i\phi) = \cos\phi + i\sin\phi$$

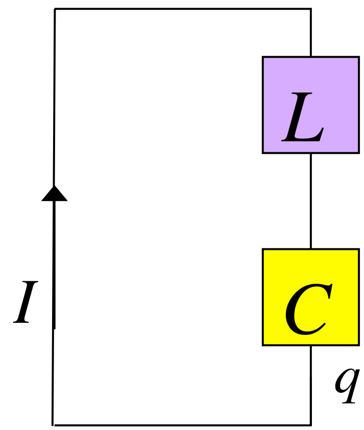
- solving one complex equation is actually solving 2 simultaneous equations

WEEK 2 SUMMARY

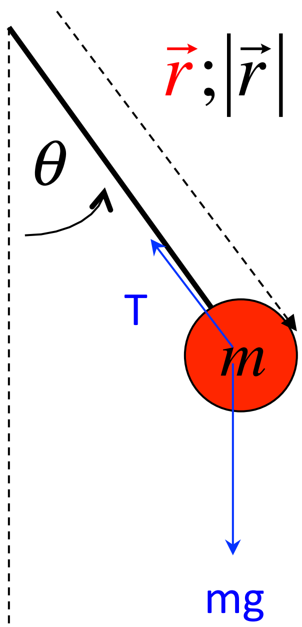
Free, undamped oscillators



$$m\ddot{x} = -kx$$



$$\ddot{q} = -\frac{1}{LC}q$$



$$\vec{r};|\vec{r}| = L$$

$$\ddot{\theta} \approx -\frac{g}{L}\theta$$

Common notation for all

$$\ddot{\psi} + \omega_0^2\psi = 0$$

Force approach to equation of motion of FREE, UNDAMPED HARMONIC OSCILLATOR:

$i.e.$ find trajectory θ(t) if F(θ) is known

- Special case of F(θ)=-sin(θ) -> small angle approx:

F(θ)=-θ   =>2nd order DE,

Found sinusoidal motion

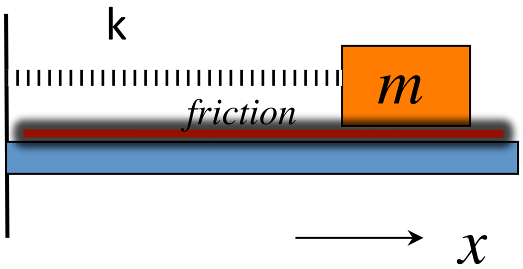
$$\theta(t) = Ce^{i\omega_0 t} + C^* e^{-i\omega_0 t}$$

$$\theta(t) = A\cos\left(\omega_0 t + \phi\right)$$

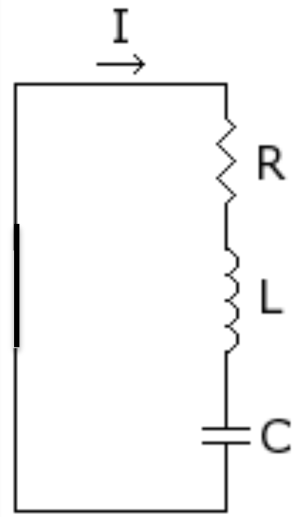
Applied initial conditions as before.

$$E = K + U = \tfrac{1}{2} I\dot{\theta}^2 + \tfrac{1}{2} mgL\theta^2$$

Free, damped oscillators

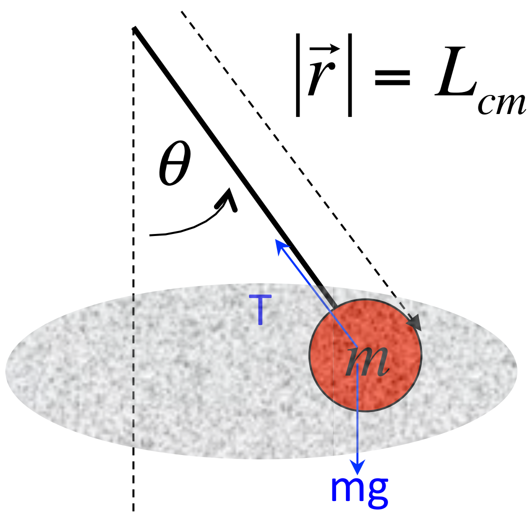


$$m\ddot{x} = -kx - b\dot{x}$$



$$L\dot{I} + \frac{1}{C}q + RI = 0$$

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = 0$$



$$|\vec{r}| = L_{cm}$$

$$\ddot{\theta} \approx -\frac{g}{L}\theta - b'\dot{\theta}$$

Common notation for all

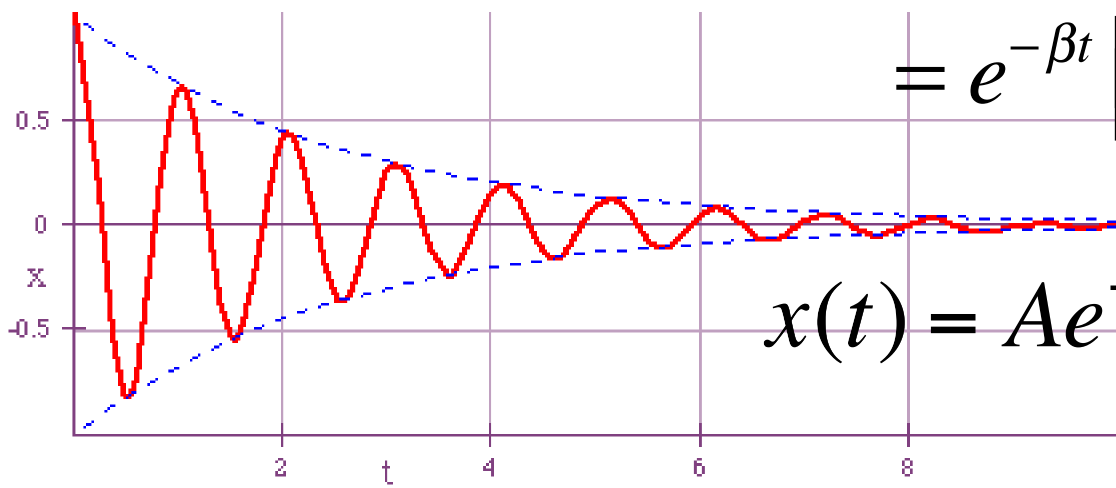
$$\ddot{\psi} + 2\beta\dot{\psi} + \omega_0^2\psi = 0$$

Force approach to equation of motion of FREE, DAMPED OSCILLATOR

- Add damping force to eqn of motion
- Found decaying sinusoid

$$x(t) = Ce^{-\beta t + i\omega_1 t} + C^* e^{-\beta t - i\omega_1 t}$$

$$= e^{-\beta t}\left[Ce^{+i\omega_1 t} + C^* e^{-i\omega_1 t} \right]$$

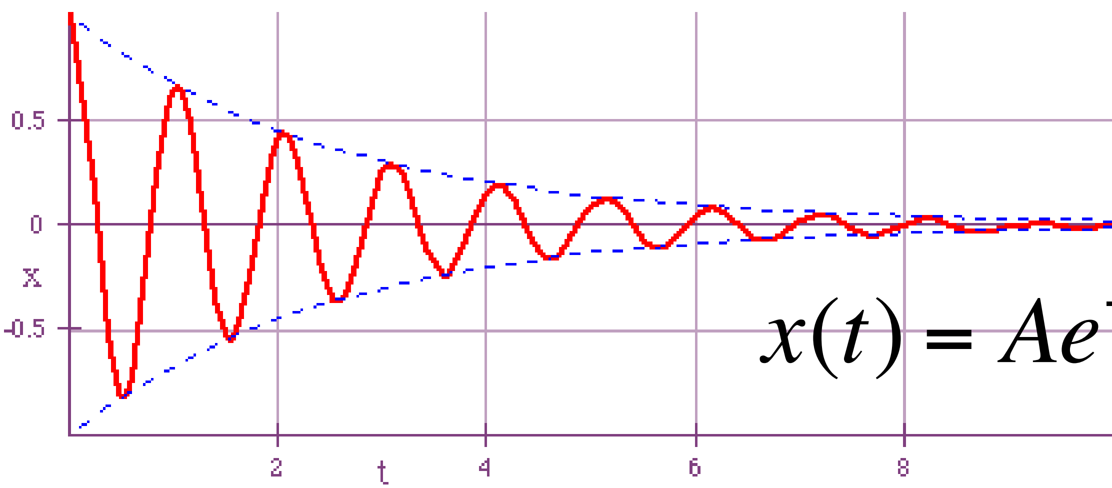


$$x(t) = Ae^{-\beta t}\left[\cos(\omega_1 t + \delta)\right]$$

FREE, DAMPED OSCILLATOR

- Damping time $\tau = 1/\beta$
- measures number of oscillations in decay time
- apply initial conditions, energy decay

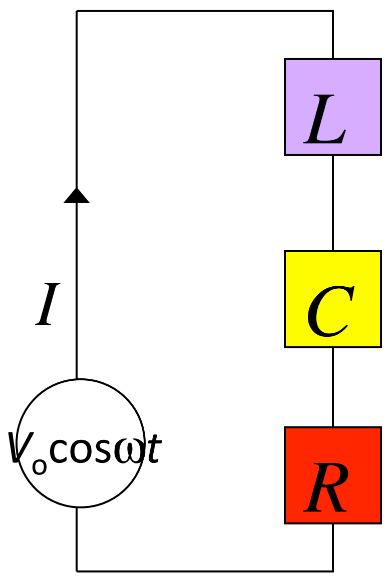
$$Q = \pi \frac{\tau}{T} = \frac{\omega_0}{2\beta}$$



$$x(t) = Ae^{-\beta t}\left[\cos(\omega_1 t + \delta)\right]$$

WEEK 3 SUMMARY

DRIVEN, DAMPED OSCILLATOR



$$V_0 e^{i\omega t} - L\ddot{q} - \frac{q}{C} - R\dot{q} = 0$$

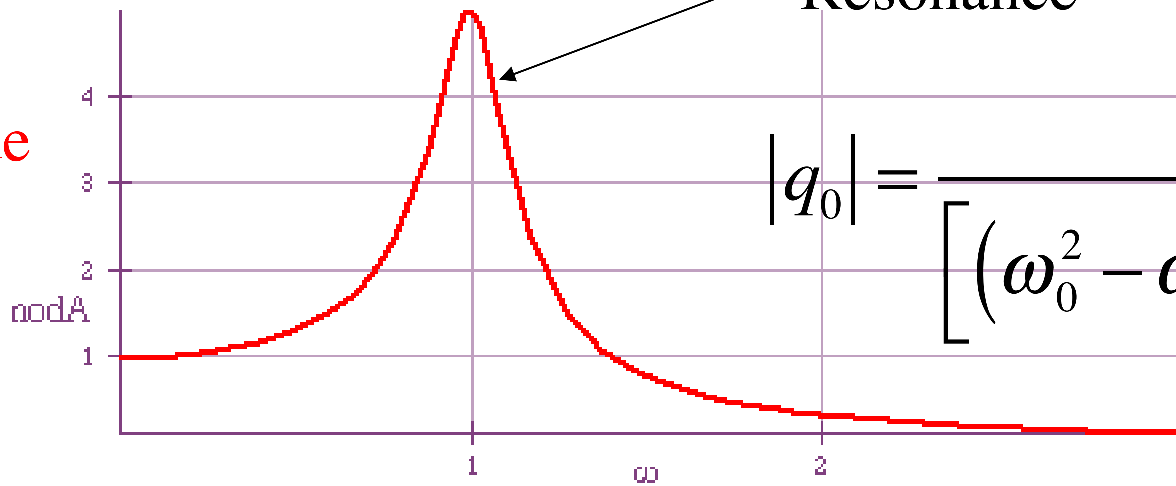
$$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = \frac{V_0}{L} e^{i\omega t}$$

$$q(t) = \mathrm{Re}\left[|q_0| e^{i\phi_q} e^{i\omega t} \right]$$

$$|q_0| = \frac{V_0 / L}{\left[\left(\omega_0^2 - \omega^2 \right)^2 + 4\beta^2\omega^2 \right]^{1/2}}; \quad \tan\phi_q = \frac{-2\beta\omega}{\omega_0^2 - \omega^2}$$

CHARGE

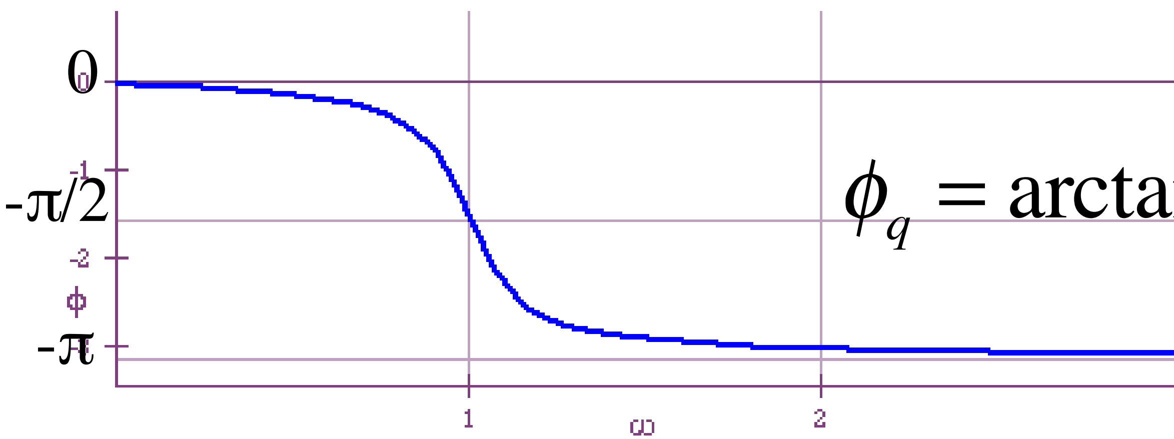
Charge Amplitude $|q_0|$



$$|q_0| = \frac{V_0 / L}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]^{1/2}}$$

Driving Frequency ω------>

Charge Phase ϕ_q

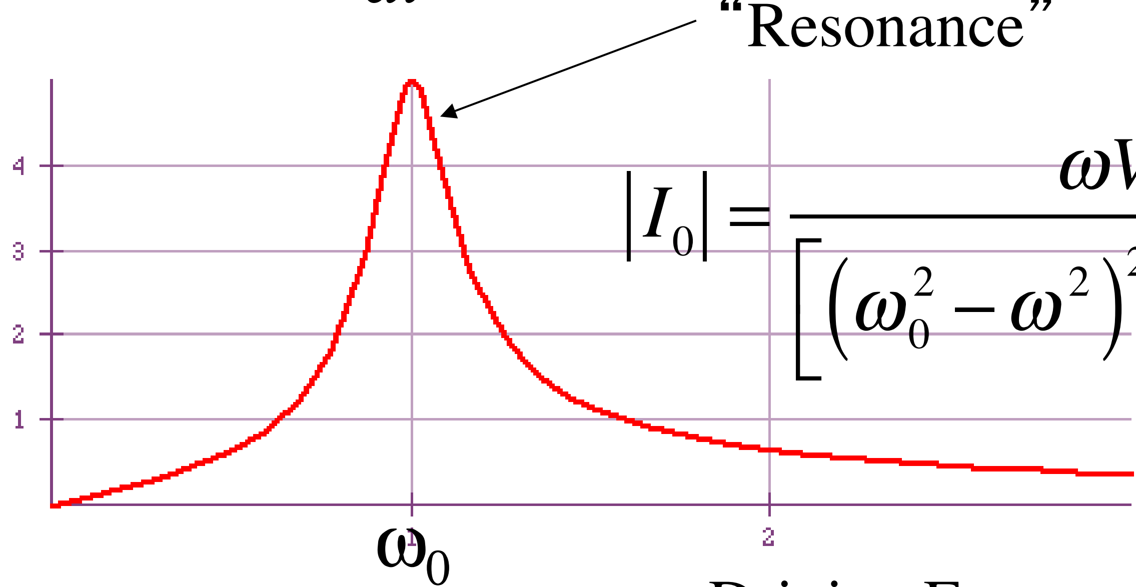


$$\phi_q = \arctan\left[\frac{-2\beta\omega}{\omega_0^2 - \omega^2}\right]$$

CURRENT

$$I(t)=\frac{dq(t)}{dt}=i\omega q(t)$$

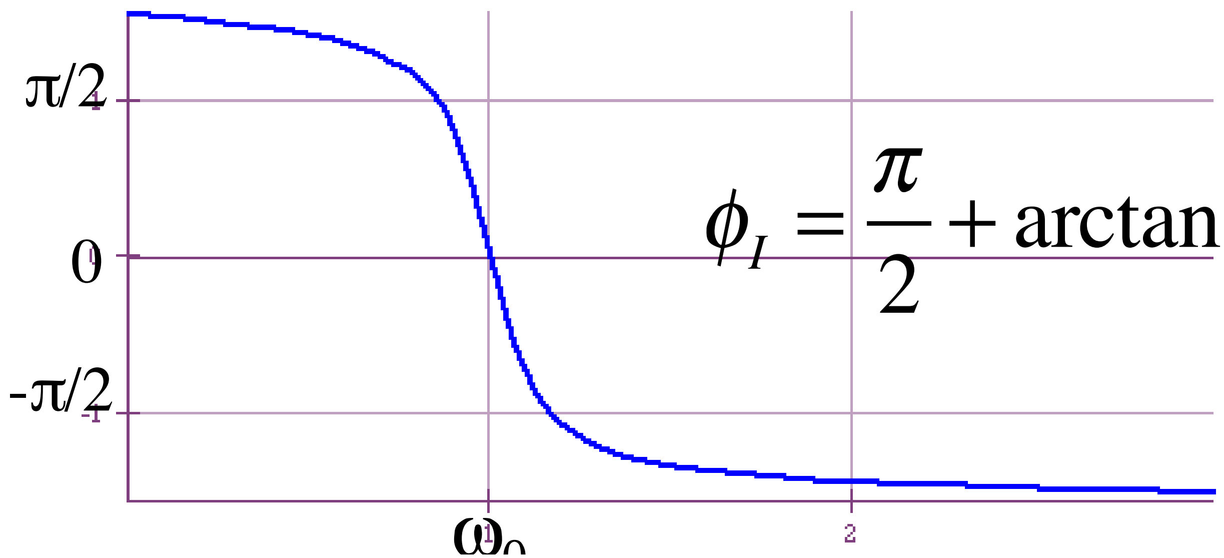
Current Amplitude $|I_0|$



$$|I_0|=\frac{\omega V_0/L}{\left[\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2\right]^{1/2}}$$

Driving Frequency ω------>

Current Phase ϕ_I

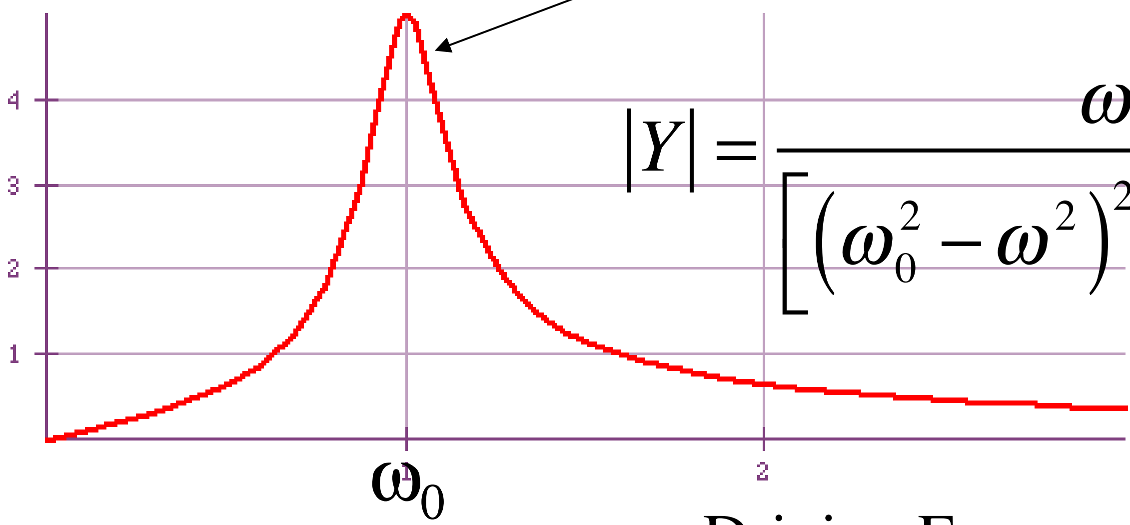


$$\phi_I=\frac{\pi}{2}+\arctan\left[\frac{-2\beta\omega}{\omega_0^2-\omega^2}\right]$$

ADMITTANCE $Y(\omega) = \dfrac{I}{V_{app}}$ NOT time dependent, but IS freq dependent.

"Resonance"

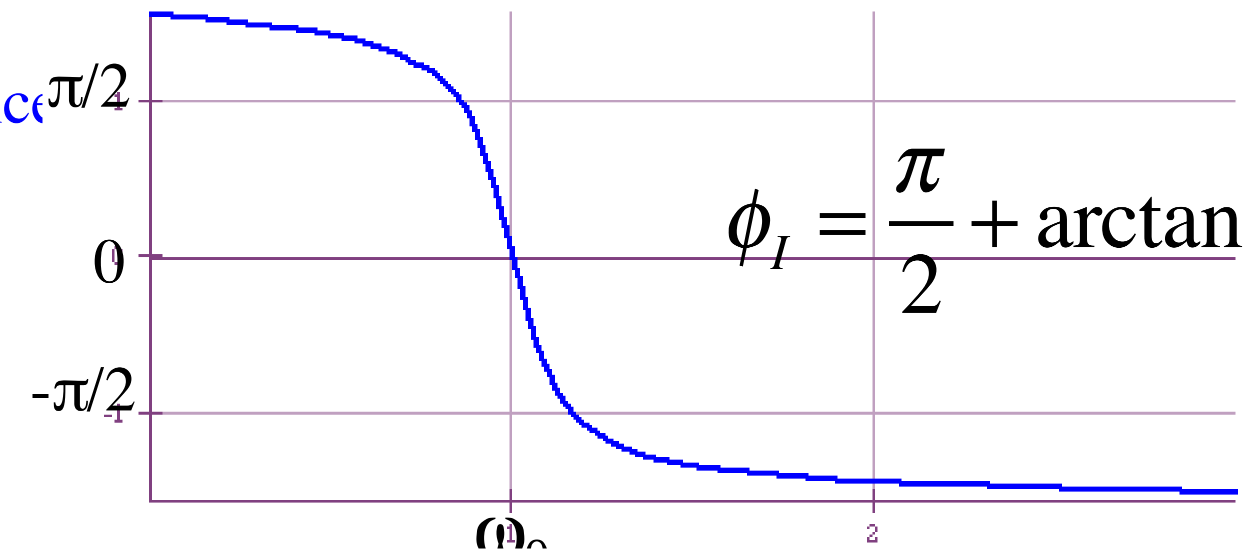
Admittance Amplitude $|Y_0|$



$$|Y| = \frac{\omega / L}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]^{1/2}}$$

Driving Frequency ω------>

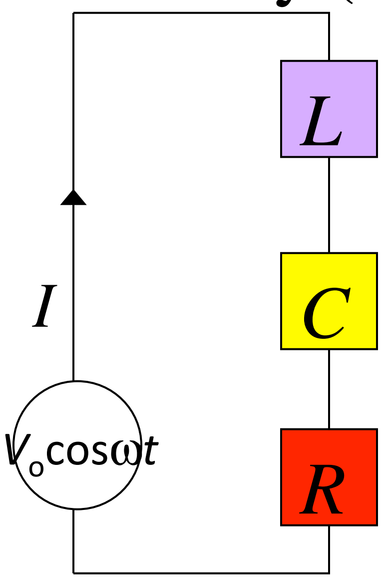
Addmittance Phase ϕ_I



$$\phi_I = \frac{\pi}{2} + \arctan\left[\frac{-2\beta\omega}{\omega_0^2 - \omega^2}\right]$$

DRIVEN, DAMPED OSCILLATOR

• can also rewrite diff eq in terms of I and solve directly (same result of course)



$$V_0 e^{i\omega t} - L\ddot{q} - \frac{q}{C} - R\dot{q} = 0$$

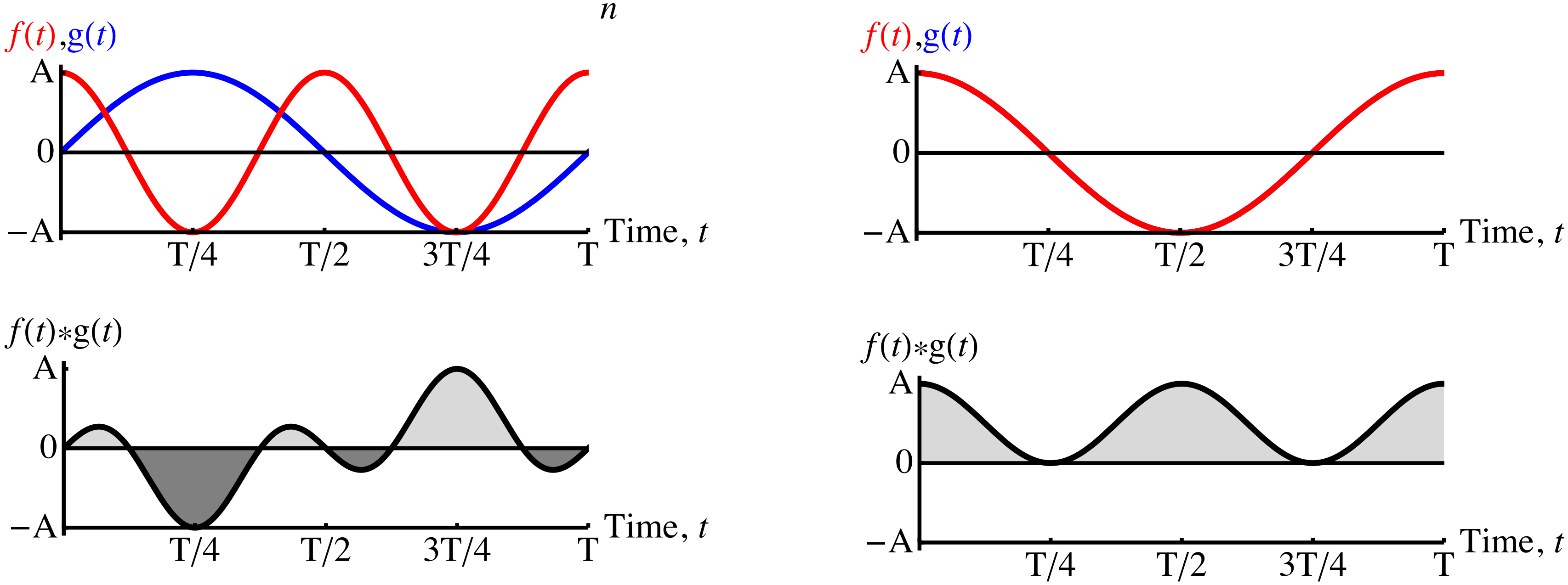
$$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = \frac{V_0}{L} e^{i\omega t}$$

$$\Rightarrow \dddot{q} + 2\beta\ddot{q} + \omega_0^2 \dot{q} = i\omega \frac{V_0}{L} e^{i\omega t}$$

$$\ddot{I} + 2\beta\dot{I} + \frac{I}{LC} = i\omega \frac{V_0}{L} e^{i\omega t}$$

FOURIER SERIES – periodic functions are sums of sines and cosines of integer multiples of a fundamental frequency. These "basis functions are orthonormal

$$f(t) = \sum_n a_n \cos n\omega t + b_n \sin n\omega t$$



$$\frac{2}{T}\int_0^T \sin(p\omega t)\sin(q\omega t)\,dt = \delta_{pq}$$

$$\frac{2}{T}\int_0^T \cos(p\omega t)\cos(q\omega t)\,dt = \delta_{pq}$$

ODD functions $f(t) = -f(-t)$. Their Fourier representation must also be in terms of odd functions, namely sines.

Suppose we have an odd periodic function $f(t)$ like our sawtooth wave and you have to find its Fourier series

$$\sum_{n=1,2\ldots} b_n \sin(n\omega t)$$

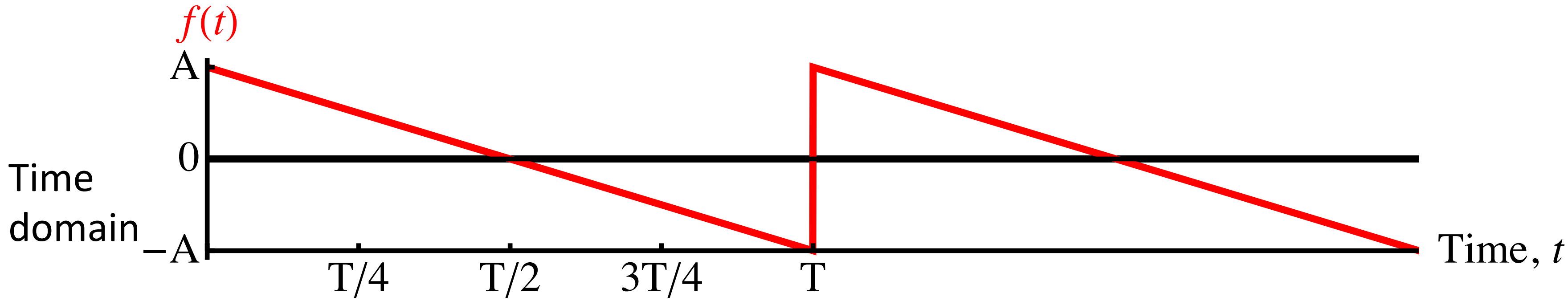
Then the unknown coefficients can be evaluated this way

Integrate over the period of the fundamental

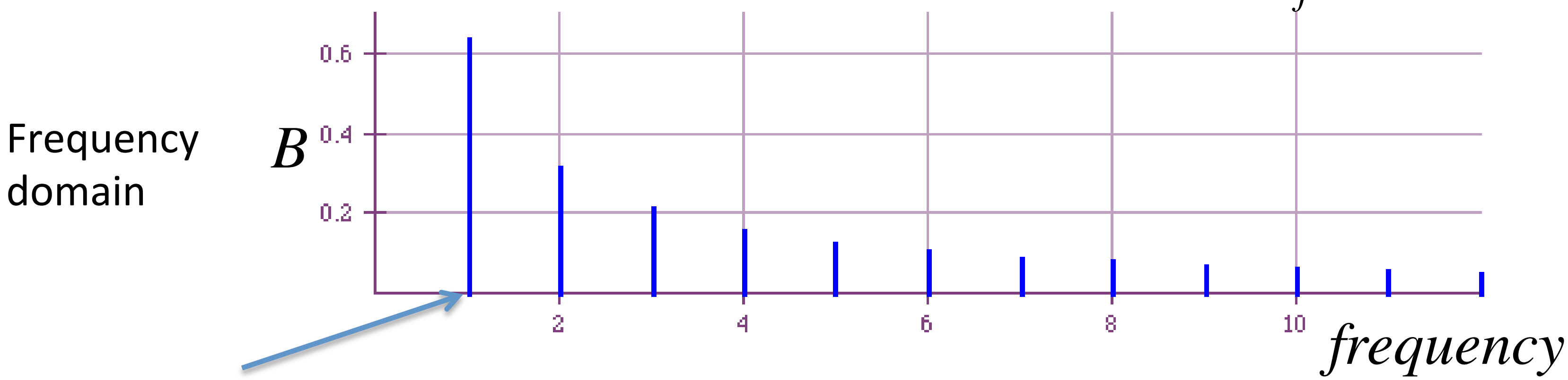
Here's the coefficient of the $\sin(\omega_n t)$ term! Plot it on your spectrum!

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt$$

normalize properly — the function — the harmonic



$$f(t) = \sum_{n} \frac{2}{n\pi} \sin\left(n\omega_f t\right) \qquad f(t) = A - \frac{2A}{T_f} t \quad 0 < t < T_f$$



Fundamantal freq = 2π/T

Integrate over the period of the fundamental

coefficient of the sin(ω_nt) term!

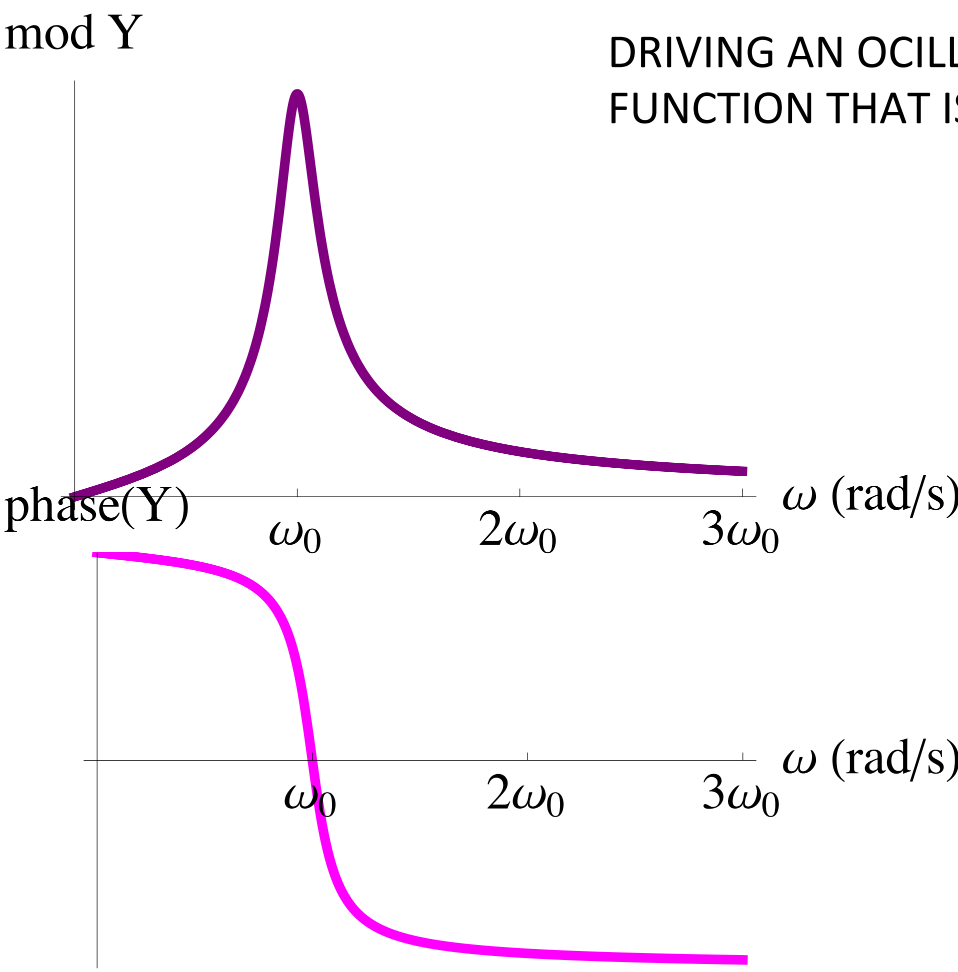
$$b_n = \frac{2}{T} \int_0^T f(t) \sin\left(n\omega t\right) dt$$

normalize properly

the function

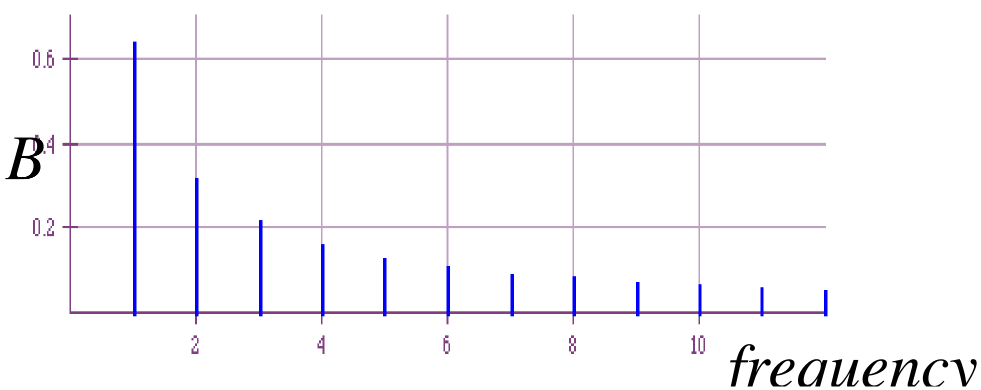
the harmonic

DRIVING AN OCILLATOR WITH A PERIODIC FORCING FUNCTION THAT IS NOT A PURE SINE



Oscillator response

Important to know where the fundamental freq of the forcing function lies in relation to the oscillator max response freq!



Forcing function

DRIVING AN OCILLATOR WITH A PERIODIC FORCING FUNCTION THAT IS NOT A PURE SINE

$$V_{app} = V_0 e^{i\omega_f t} + 2V_0 e^{i2\omega_f t} \quad \text{(given)}$$

$$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = V_0 e^{i\omega_f t} + 2V_0 e^{i2\omega_f t} \quad \text{(Kirchoff)}$$

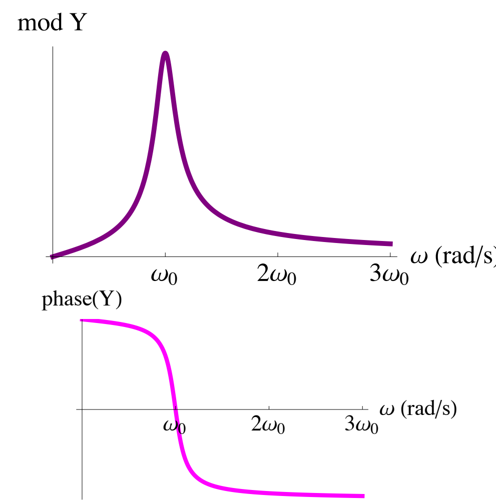
$$\Rightarrow q = q_\omega + q_{2\omega} \quad \text{(linear diff eq - superposition)}$$

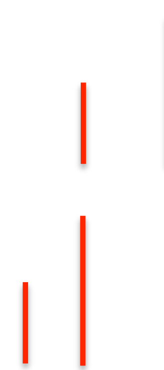
$$\Rightarrow I = I_{\omega_f} + I_{2\omega_f} \qquad I = \dot{q}$$

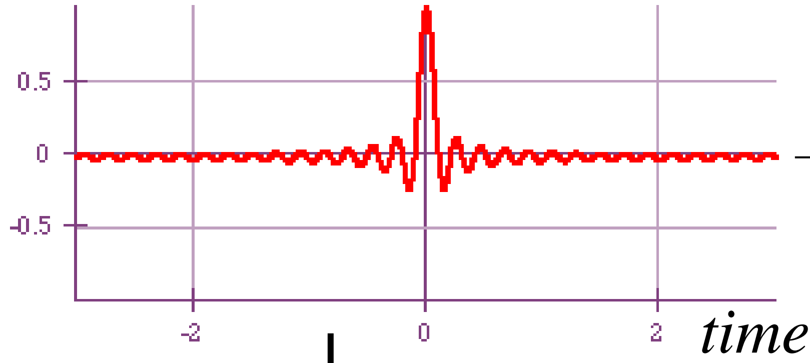
$$I = Y_{\omega_f} V_{app,\omega_f} + Y_{2\omega_f} V_{app,2\omega}$$

$$I = \frac{\omega_f / L}{\left[\left(\omega_0^2 - \omega_f^2\right)^2 + 4\beta^2\omega_f^2\right]^{1/2}} e^{i\left(\frac{\pi}{2} + \arctan\left[\frac{-2\beta\omega_f}{\omega_0^2 - \omega_f^2}\right]\right)} V_0 e^{i\omega_f t}$$

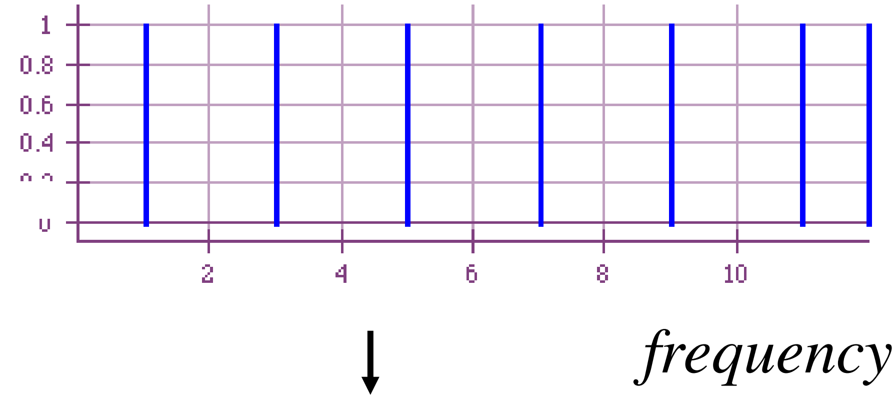
$$+ \frac{(2\omega_f)/L}{\left[\left(\omega_0^2 - (2\omega)_f^2\right)^2 + 4\beta^2(2\omega)_f^2\right]^{1/2}} e^{i\left(\frac{\pi}{2} + \arctan\left[\frac{-2\beta 2\omega_f}{\omega_0^2 - (2\omega)_f^2}\right]\right)} 2V_0 e^{i(2\omega_f)t}$$

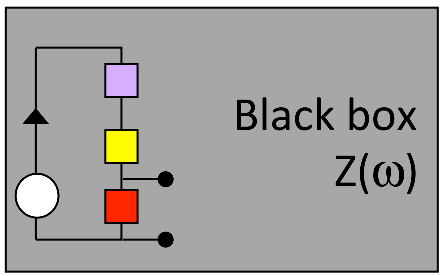




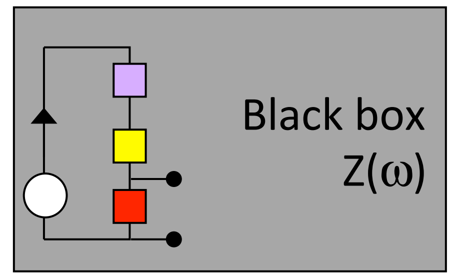


FT - you know this

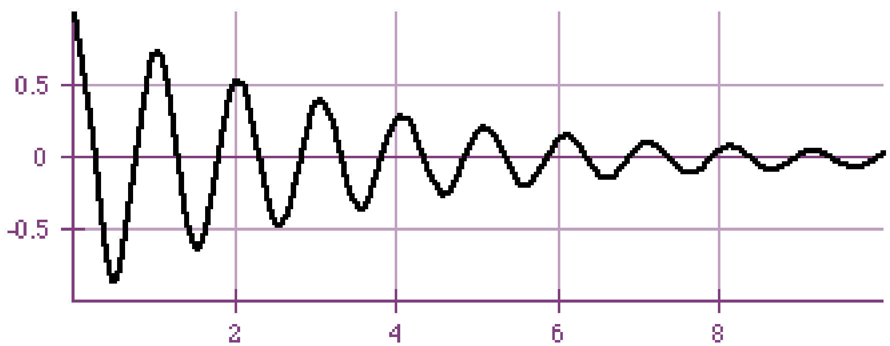




Observe what (LRC) black box does to an impulse function



This was harmonic response expt - you know what black box (LRC) does to a single freq



?

Are these connected by FT?? They' d better be - you find out!