

RELATIVISTIC MOMENTUM AND ENERGY: BETA DECAY

OBJECTIVE: In this experiment you will observe the emission of electrons in a radioactive decay process, and you will analyze their momentum and kinetic energy using the equations of special relativity.

REFERENCE: Krane, Section 2.7 (also 2.8); 12.8 (also 12.6 pp.386-387).
<http://phet.colorado.edu/en/simulation/beta-decay>

THEORY: One type of radioactive decay is called *beta decay*. In a beta decay process, a nucleus of one element is transformed into a nucleus of an element with one more proton or one fewer protons. An electron is emitted from the nucleus in this process. This electron is NOT one of the atomic electrons that surround the nucleus. Instead, it is a new electron “manufactured” inside the nucleus from the available decay energy. The rest energy of an electron is 0.511 MeV, so there must be at least this much energy available in the decay. In a typical beta decay process, the kinetic energy of the electron is on the order of 1 MeV, so the electron must be analyzed using the equations for relativistic energy and momentum.

Another particle, called a *neutrino*, is also emitted in the beta decay process. The neutrino has no electric charge and a very small mass. For the analysis in this experiment, we may assume that the neutrino mass is zero and that it travels with the speed of light. The neutrino energy and momentum are then related by $E_\nu = cp_\nu$.

The decay process can be represented as $X \rightarrow X' + e + \nu$, where X is the nucleus before the decay and X' is the nucleus after the decay. Figure 1 is a schematic picture of the products of the decay.

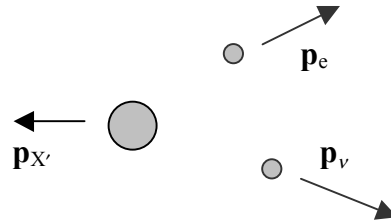


Figure 1

Conservation of momentum requires that, if the initial nucleus X is at rest, then the total momentum of the three final particles must be zero: $\mathbf{p}_{X'} + \mathbf{p}_e + \mathbf{p}_\nu = 0$. The total energy available to these three particles, after accounting for the energy necessary to create the electron, is called the *Q-value* of the decay: $Q = K_{X'} + K_e + E_\nu$. Because the product nucleus X' is so massive compared with the electron and the neutrino, its kinetic energy can be neglected, and so

$$Q \cong K_e + E_\nu \quad (1)$$

The electron and neutrino share the available energy. When the neutrino's energy is zero, the electron has its maximum kinetic energy, equal to Q . The object in this experiment is to determine this energy.

For the radioactive decay used in this experiment, the number of electrons with a particular value of the momentum is

$$N(p_e) = Cp_e^2(Q - K_e)^2(p_e^2 + p_v^2) \quad (2)$$

where C is a constant. Note that $N(p_e) = 0$ when $p_e = 0$, and also $N(p_e) = 0$ when p_e has its maximum value, corresponding to $K_e = Q$. Figure 2 shows a sketch of the behavior of $N(p_e)$. At $p_e = p_{e,\max}$, the neutrino carries no momentum or energy, so $p_v = E_v = 0$, and so according to Eq. 1, $K_{e,\max} = Q$. In this experiment you will measure $p_{e,\max}$, use that value to calculate $K_{e,\max}$, and thus determine Q .

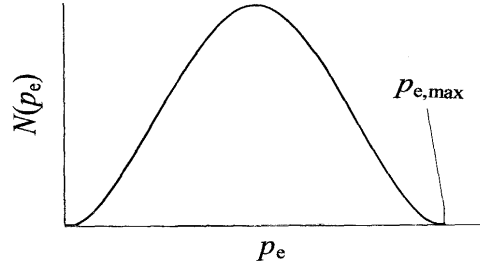


Figure 2

Equation 2 is really an equation in a single variable, because K_e is related to p_e by

$$K_e = E_e - m_e c^2 = \sqrt{(p_e c)^2 + (m_e c^2)^2} - m_e c^2 \quad (3)$$

and p_v is related to p_e by

$$p_v = \frac{E_v}{c} = \frac{Q - K_e}{c} = \frac{Q - \sqrt{(p_e c)^2 + (m_e c^2)^2} + m_e c^2}{c} \quad (4)$$

Determining the maximum energy or momentum from a plot like Figure 2 is difficult, because it is hard to find exactly where the curve hits the axis at the upper end. It would be easier to use a linear plot, but Equation 2 is not easy to put in linear form. We can approximate linear behavior by writing Equation 2 in the following form:

$$\sqrt{C}(Q - K_e) = \left[\frac{N(p_e)}{p_e^2(p_e^2 + p_v^2)} \right]^{1/2} \quad (5)$$

If we plot the quantity on the right side of Equation 5 on the y axis as a function of K_e on the x axis, we should get a straight line. Extending the straight line to where it meets the x axis gives the value of Q .

This process is because in order to determine the value of the quantity in brackets in Equation 5, we must first know the value of Q (needed to find p_v from Equation 4). So to determine Q , we first must know Q ! Fortunately, this process is not very sensitive to the estimated value of Q , so first make a plot like Figure 2 and estimate the maximum value of p_e . Then use that value to find the maximum value of K_e , which serves as an estimate for Q . Then use this estimated value of Q in Equation 4 and make a plot of Equation 5 to determine a better value for Q . You can repeat this process until Q stops changing.

Determining the momentum of the electron: When a charged particle moves in a uniform magnetic field, its path is a circular arc. Figure 3 is a side view schematic diagram of the apparatus for this experiment.

Electrons enter through the collimator at the bottom and pass through a circular aperture into the field region between the poles of the magnet. Varying the current through a coil changes the strength of the field. The electrons move in a circular arc of radius r and pass through the second aperture and into a counter.

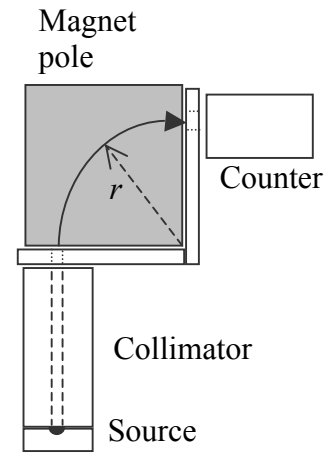


Figure 3

The relation between the magnetic field strength B and path radius r is

$$r = \frac{p_e}{eB} \quad \text{or} \quad p_e = erB \quad (6)$$

where all quantities are expressed in SI units (p_e in kg·m/s and B in tesla, with 1 tesla = 10^4 gauss) and e is the magnitude of the charge of the electron (1.602×10^{-19} C).

Equation 6 is correct even when the momentum is relativistic. In using Equations 3, 4, and 5, it is often easier to express the momentum in units of MeV/ c :

$$1 \text{ MeV}/c = 5.33 \times 10^{-22} \text{ kg}\cdot\text{m/s} \quad (7)$$

PROCEDURE:

1. Check the alignment of the system – make sure the collimator is directly below the bottom aperture and the counter is directly behind the back aperture. **WHEN YOU SIGHT DOWN THROUGH THE APERTURE TO LINE UP THE COLLIMATOR, PLACE THE SMALL PLASTIC SQUARE ON TOP OF THE MAGNET TO SHIELD YOUR EYES FROM THE RADIOACTIVE SOURCE.**
2. Measure the distance from the back lower corner of the magnet pole to the center of each aperture to determine the radius r of the path of the electron. Estimate the uncertainty in this value due to the holes in the apertures.
3. Take data every 0.25 A from 0-4.5 A. Count the number of electrons for 1 minute at each current.
4. Remove the source to a distant location and measure the background counts. Take four 1-minute background runs. This background in the detector comes from cosmic rays and from natural radioactivity in the concrete walls and floor.

ANALYSIS:

1. Add the four background counts together and determine the uncertainty (the uncertainty in N counts is \sqrt{N}). Divide this number by 4 to get the average background for a 1-minute count and its uncertainty (divide the total uncertainty by 4 also).
2. Subtract the background from each data point. If you get a value less than zero, set the value to zero. Determine the uncertainty of each background-corrected data point. Use the rule for finding the uncertainty in addition or subtraction: when $C = A \pm B$, then $\delta C = \sqrt{(\delta A)^2 + (\delta B)^2}$.
3. Plot the background-corrected $N(p_e)$ against p_e . Show the uncertainties of each point on your graph. From your graph, estimate the maximum momentum of the electrons, at the end point where the number becomes zero. From this value, find the estimate of Q .
4. For each data point, determine the electron kinetic energy K_e and the neutrino momentum p_ν . Plot Equation 5 and determine which points give the best linear region with negative slope. Draw a straight line through the points you have selected. From the x-intercept of your line, determine the value of Q .
5. Put uncertainty bars on each data point. We'll make a simplifying assumption that the uncertainties in the right side of Eq. 5 are due only to the counting uncertainties in N . Draw vertical error bars through your data points based on that uncertainty. There are also uncertainties in p_e due to the uncertainty in r from the size of the aperture openings. Use the uncertainty in r to find the uncertainties in p_e , and then determine the corresponding uncertainties in your values of K_e . Draw horizontal error bars on your data points representing the uncertainties in K_e . Try to estimate the uncertainty in Q by drawing lines of the greatest and smallest slope consistent with the error bars you have drawn.

QUESTIONS:

1. The radioactive source used in these experiments is ^{204}Tl . Write down the beta decay reaction, carefully noting A , N , Z for the parent and daughter. The decay has a Q -value of 0.765 MeV [<http://ie.lbl.gov/toi>]. Compare your value with this value and account for any differences.
2. Suppose you changed the value of your initial estimate of Q by 10-20%. How would this change affect the values you plot from Equation 5? How is the slope of your straight line affected?
3. Why do we not use the low-energy points in plotting the straight line from Equation 5? If you extrapolate the line to low energy, do the experimental points fall above the line or below the line? Try to suggest some reasons for this behavior. Consider what happens to the electrons as they leave the atom, as they leave the source, and as they travel from the source to the detector.
4. Why was the existence of the neutrino postulated by Pauli?